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On Power Factor Improvement by Lossless Linear Filters under Nonlinear Nonsinusoidal Conditions

Abstract. *Recently, it has been established that the problem of power factor compensation for nonlinear loads with nonsinusoidal source voltage can be recast in terms of the property of cyclodissipativity. The purpose of this brief note is to review and to illustrate the application of this framework to the practical of passive reactive compensation of linear loads with a nonsinusoidal source voltage. We give criteria for determining the optimal values of the compensator parameters that depend of the spectral line of load susceptance and voltage source.*

Streszczenie. *Stwierdzono ostatnio, że problem poprawy współczynnika mocy odbiorników nieliniowych zasilanych napięciem niesinusoidalnym może być rozpatrywany z punktu widzenia cyklo-rozpraszania (cyclodissipativity). Celem tego artykułu jest przegląd i pokazanie zastosowania takiego podejścia przy bezstratnej kompensacji odbiorników liniowych zsilanych napięciem niesinusoidalnym. Podane zostały kryteria do określenia optymalnych wartości parametrów kompensatora, które zależą od charakterystyki widmowej susceptancji obciążenie i źródła napięcia.* ***(Poprawa współczynnika mocy filtrami bezstratnymi w warunkach nieliniowych i niesinusoidalnych)***

Keywords: Power factor compensation; nonlinear network; nonsinusoidal conditions.
Słowa kluczowe: Kompensacji współczynnika mocy; sieć nieliniowych; warunkach niesinusoidalnych.

Introduction

Optimizing energy transfer from an alternating current (ac) source to a load is a classical problem in electrical engineering. In practice, the efficiency of this transfer is typically reduced due to the phase shift between voltage and current at the fundamental frequency. The power factor captures the energy-transmission efficiency for a given load. The standard approach to improving the power factor is to place a compensator between the source and the load.

The effectiveness of capacitive compensation in systems with nonsinusoidal voltages and currents has been widely studied by [1, 2] and [3]. Unfortunately, in [4] it has been illustrated that the capacitive compensation may not be effective for non-sinusoidal voltages. Therefore, a more complex compensator than only a capacitor is required for the reactive power minimization in such situations.

Recently, in [5] it has been established that the classical problem is equivalent to imposing the property of cyclodissipativity to the source terminals. Since this framework is based on the cyclodissipativity property, see [6, 7], the improvement of the power factor (PF) is done independent of the reactive power definition, which is a matter of discussions in the power community, see for instance [8] and its references. Most of the approaches used to improve the PF are based on different power definitions and a lack of a unified definition of reactive power produces misunderstanding of power phenomena in circuits with nonsinusoidal voltages and currents, [9]. Similarly, the task of designing compensators that aim at improving the PF for nonlinear time-varying loads operating in non-sinusoidal regimes is far from clear.

Using the cyclodissipativity framework the classical capacitor and inductor compensators were interpreted in terms of energy equalization, see [5] for more details. And we have presented an extension of this result in [10] where we considered arbitrary lossless linear time invariant (LTI) filters, and proved that for general lossless LTI filters the PF is reduced if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured.

Here we illustrate the application of this framework to the passive compensation of linear loads with a non-sinusoidal source voltage. We give criteria for improvement of PF with linear capacitors, LC filters, which determine the optimal values for the compensator parameters that depend of the spectral line of load susceptance and voltage source.

Cyclodissipativity of RLC nonlinear networks

In order to make this paper self-contained, the purpose of this section is to briefly review the meaning of cyclodissipativity, [6, 7], and some of its connections with the nonlinear circuit theory. Although the dissipativity theory applies to a wider classes of systems, let us consider dynamical systems modeled by ordinary differential equations:

Definition 1 (Input-State-Output Representation)**.** *The input-state-output representation of the dynamical systems $\mathcal{H}:\mathcal{U}\to\mathcal{Y}$, is of the form*

$$\begin{aligned} \dot{x} &= f\left(x,u\right), \quad x\in\mathcal{X}\subset\mathbb{R}^m \\ y &= g\left(x,u\right), \quad u\in\mathcal{U}\subset\mathbb{R}^n, y\in\mathcal{Y}\subset\mathbb{R}^n, \end{aligned} \tag{1}$$

where $f\colon\mathcal{X}\times\mathcal{U}\to\mathbb{R}^n$ and $g\colon\mathcal{X}\times\mathcal{U}\to\mathcal{Y}$ are vector functions of class C^k, with $0<k<\infty$. Let $\mathcal{X}$ be the set of reachable and controllable points. Furthermore, assume that system (1), for $u=0$, has an equilibrium point in $x=0$. That is, $f\left(0,0\right)=0$ and $g\left(0,0\right)=0$.

The definition of cyclodissipativity involves a function called *supply rate* $w:\mathcal{U}\times\mathcal{Y}\to\mathbb{R}$, which is locally integrable for every $u\in\mathcal{U}$, [6].

Definition 2 (Cyclodissipativity)**.** *We say that the system $\mathcal{H}$ is cyclodissipative on $\mathcal{X}$ with the supply rate $w\left(u,y\right)$ if there exists a function S, called storage function, such that*

$$S\left(x\left(t_0\right)\right)+\int_{t_0}^{t_1} w\left(u\left(t\right),y\left(t\right)\right)dt\geq S\left(x\left(t_1\right)\right), \tag{2}$$

is satisfied for all $u\in\mathcal{U}$ and all $t_0<t_1$, such that $x\left(t\right)\in\mathcal{X}$ for all $t\in\left[t_0,t_1\right]$.

The inequality (2) is similar to the usual dissipation inequality where additionally it is required that $S\geq 0$, and which expresses that the increase in energy stored cannot be larger than the energy supplied from the outside. Furthermore, if the system $\mathcal{H}$ does not produce energy at any time, namely that the energy stored from the system is finite, $S(x)\geq 0$ for all $x\in\mathcal{X}$, then the system is called dissipative. Typical examples of dissipative systems are: passive electrical networks, mechanical systems, viscoelastic materials, etc.

However, if we consider an electrical network with active elements, namely negative resistors, tunnel diodes, etc., then the interpretation of (2) leads to some difficulties because the storage energy function needs in general not be bounded from below or above. This merely involved a generalization of the concept of a dissipative system to that of a cyclodissipative system[1]. The concept of cyclodissipativity is inspired by the fact that cyclodissipative systems exhibit a dissipative behavior in cyclic motions. However, normally, autonomous systems cannot exhibit periodic (or recurrent) motions. Hence, we restrict the set of inputs of interest to those inputs that generate periodic trajectories, and we consequently define the following signal space.

Definition 3. *Let $\mathcal{L}_2^n$ be the signal space defined by $\mathcal{L}_2^n := \left\{x : [0,T) \to \mathbb{R}^n : \|x\|^2 := \frac{1}{T}\int_0^T |x(\tau)|^2 d\tau < \infty\right\}$, where $\|\cdot\|$ is the rms value, $|\cdot|$ is the Euclidean norm and the inner product in $\mathcal{L}_2^n$ is defined as $\langle x, y\rangle := \frac{1}{T}\int_0^T x^\top(t)y(t)dt$.*

We thus arrive at the following lemma:

Lemma 1 (Cyclodissipativity)**.** *Given a mapping $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \to \mathbb{R}$. The system $\mathcal{H}$ is cyclodissipative with respect to the supply rate $w(u, y)$ if and only if*

$$\int_0^T w(t)dt \geq 0. \tag{3}$$

for all $T \geq 0$, and $u \in \mathcal{L}_2^n$ wherever $x(0) = x(T)$.

Remark 1. *In words, a system is cyclodissipative when it can not create (abstract) energy over closed paths in the state-space. It might, however, produce energy along some initial portion of such a trajectory; if so, it is only cyclodissipative and not dissipative.*

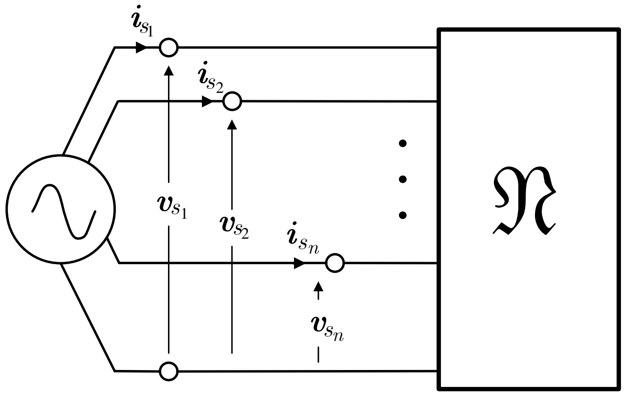

Fig. 1. Illustrating power delivered to a (possibly nonlinear and time varying) load from an n-phase ac ideal generator.

For instance, consider the n-port network $\mathfrak{N}$ of Figure 1 consists of arbitrary interconnection of n_L inductors, n_C capacitors with n_R passive resistors. From the principle of conservation of energy, regardless of the nature of the elements or the excitation, the instantaneous rate of energy transfer or instantaneous power at the input terminal is equal to

$$\sum_{p=1}^{n} i_{s_p} v_{s_p} = \sum_{\alpha=1}^{n_L} i_{L_\alpha} v_{L_\alpha} + \sum_{\beta=1}^{n_C} i_{C_\beta} v_{C_\beta} + \sum_{\gamma=1}^{n_R} i_{R_\gamma} v_{R_\gamma},$$

where we have adopted the standard sign convention for the (instantaneous) supplied power $i_s^\top v_s$, which can be rewritten as,

$$\sum_{p=1}^{n} i_{s_p} v_{s_p} = \frac{d\mathcal{E}}{dt} + i_R^\top v_R, \tag{4}$$

where

$$\mathcal{E}(t) := \sum_{\alpha=1}^{n_L} \mathcal{E}_{L_\alpha}(t) + \sum_{\beta=1}^{n_C} \mathcal{E}_{C_\beta}(t) \tag{5}$$

represents the total stored energy in the network. Time-integration of (4) over the interval $[t_0, t_1]$ yields

(6)
$$\int_{t_0}^{t_1} i_s^\top(\tau) v_s(\tau) d\tau = \mathcal{E}(t_1) - \mathcal{E}(t_0) + \int_{t_0}^{t_1} i_R^\top(\tau) v_R(\tau) d\tau.$$

Let us assume that all resistors are passive, namely the instantaneous dissipated power is always positive $i_R^\top(t) v_R(t) \geq 0$ for all $t \in [t_0, t_1]$, then we obtain the dissipation inequality,

$$\mathcal{E}(t_0) + \int_{t_0}^{t_1} i_s^\top(\tau) v_s(\tau) d\tau \geq \mathcal{E}(t_1), \tag{7}$$

which verifies for all $t_1 \geq t_0$. Indeed, we can conclude the following result.

Proposition 2. *A n-port network $\mathfrak{N}$ consists of arbitrary interconnection of n_L inductors, n_C capacitors with n_R passive resistors verify the dissipation inequality (7) for all $t_1 \geq t_0$ and, therefore, $\mathfrak{N}$ is cyclodissipative with respect to the supply rate $w(u_s, i_s) = i_s^\top(\cdot) v_s(\cdot)$ and the storage function is the total stored energy (5).*

A cyclodissipativity characterization of power factor compensation

This section introduces the identification of the key role played by cyclodissipativity in power factor compensation.

We consider the energy transfer from an n-phase ac generator to a load, see Figure 1. The voltage and current of the source are denoted by the column vectors $v_s(t), i_s(t) \in \mathbb{R}^n$ and the load is described by a (possibly nonlinear and time varying) n-port network $\mathfrak{N}$. We make the following assumption.

Assumption 1. *All signals are assumed to be periodic, $x(t) = x(t+T)$, and have finite power, that is, they belong to $\mathcal{L}_2^n$.*

Assumption 2. *The source is ideal[2], in the sense that v_s remains unchanged for all loads Y_ℓ.*

The universally accepted definition of PF is given as [11]:

Definition 4 (Power factor)**.** *The PF of the source is defined by*

$$PF := \frac{P}{S}, \tag{8}$$

where

$$P := \langle v_s, i_s\rangle, \tag{9}$$

is the active real power, also equal to average power [12], and $S := \|v_s\|\|i_s\|$ is the apparent power.

From (8) and the Cauchy–Schwartz inequality, it follows that $P \leq S$. Hence $PF \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

[1] As explained in [6], cyclodissipativity is understood here in terms of the available generalized energy. The idea is borrowed from thermodynamics, where the notion is formulated in a conceptually clearer manner than in circuits and systems theory.

[2] Under Assumption 2, the apparent power S is the highest average power delivered to the load among all loads that have the same rms current $\|i_s\|$.

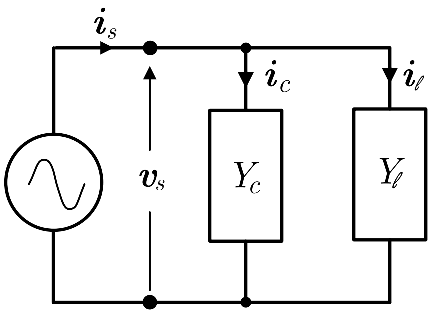

Fig. 2. Schematic diagram of shunt PF compensation configuration.

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c, Y_\ell : \mathcal{L}_2^n \to \mathcal{L}_2^n$ are the admittance operators of the compensator and the load, respectively. That is, $i_c = Y_c(v_s)$, $i_\ell = Y_\ell(v_s)$ where i_c,$i_\ell \in \mathcal{L}_2^n$, are the compensator and load currents, respectively. In the simplest LTI case the operators Y_c, Y_ℓ can be described by their admittance transfer matrices, which we denote by $\hat{Y}_c(s)$, $\hat{Y}_\ell(s) \in \mathbb{R}^{n\times n}(s)$, respectively, where s represents the complex frequency variable $s = j\omega$.

The uncompensated PF, that is, the value of PF when $Y_c = 0$, is clearly given by

$$PF_u := \frac{\langle v_s, i_\ell\rangle}{\|v_s\|\|i_\ell\|}, \tag{10}$$

where $i_s = i_\ell$.

Definition 5 (Power factor improvement)**.** *For a given compensator Y_c. Power Factor Improvement is achieved if and only if*

$$PF > PF_u. \tag{11}$$

Following standard practice, we consider only lossless compensators, that is,

Definition 6 (Lossless compensator)**.** *The power-factor compensator is lossless if*

$$\langle Y_c(v_s), v_s\rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{12}$$

Furthermore, if Y_c is LTI, this is equivalent to

$$\mathrm{Re}\{\hat{Y}_c(j\omega)\} = 0, \tag{13}$$

for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$, where $\mathrm{Re}\{\hat{Y}_c(j\omega)\}$ is the real part of the admittance transfer matrix $\hat{Y}_c(j\omega)$.

The following results from [5] are repeated here for convenience to place our results in context.

Proposition 3. *Consider the system of Figure 2 with fixed Y_ℓ. The compensator Y_c improves the PF if and only if the system is cyclodissipative with respect to the supply rate*

$$w(v_s, i_s) := (Y_\ell(v_s) + i_s)^\top (Y_\ell(v_s) - i_s). \tag{14}$$

The proof follows from (12) and the fact the compensator is lossless.

Remark 2. *Notice that we use scattering variables where the elements of $\mathcal{U}$ denote the magnitude of the incident wave and those of $\mathcal{Y}$ the magnitude of the reflected wave. The function w the equals $\|u\|^2 - \|y\|^2$, i.e., the difference of the power in the incident wave and the reflected wave with $u = i_\ell$ and $y = i_s$, where S still denotes the stored energy.*

The next result follows from Proposition 3 and it characterizes the set of all compensators Y_c that improve the power-factor for a given Y_ℓ.

Corollary 4. *Consider the system of Figure 2 Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies*

$$2\langle Y_\ell(v_s), Y_c(v_s)\rangle + \|Y_c(v_s)\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{15}$$

Dually, given Y_c, the PF is improved for all Y_ℓ that satisfy (15).

Weighted power equalization renders into power factor improvement

In this section we extend Proposition 5 in [5], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters.

We assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements (n_L inductors, n_C capacitors) and static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [12]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \tag{16}$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \tag{17}$$

respectively, where i_C, v_C, $q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages and charges, and i_L, v_L, $\phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages and flux–linkages, $H_L : \mathbb{R}^{n_L} \to \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C : \mathbb{R}^{n_C} \to \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable and for linear capacitors and inductors, $H_C(q_C) = \frac{1}{2}q_C^\top C^{-1} q_C$, and $H_L(\phi_L) = \frac{1}{2}\phi_L^\top L^{-1}\phi_L$, respectively, with $L \in \mathbb{R}^{n_L\times n_L}$, $C \in \mathbb{R}^{n_C\times n_C}$. To avoid cluttering the notation we assume L, C are diagonal matrices.

Finally, we distinguish between two sets of nonlinear static resistors: n_{R_i} current–controlled resistors and n_{R_v} voltage–controlled resistors, for which the characteristics are given by the following one-to-one real-valued functions:

$$v_{R_i} = \hat{v}_{R_i}(i_{R_i}), \tag{18}$$

$$i_{R_v} = \hat{i}_{R_v}(v_{R_v}), \tag{19}$$

respectively, where i_{R_i}, $v_{R_i} \in \mathbb{R}^{n_{R_i}}$ are the currents, voltages of the current-controlled resistors, and i_{R_v}, $v_{R_v} \in \mathbb{R}^{n_{R_v}}$ are the currents, voltages of the voltage-controlled resistors, with $n_R = n_{R_i} + n_{R_v}$.

Recalling the definition of real power (9) we introduce the following.

Definition 7. *(Weighted averaged power) Given a compensator admittance Y_c the weighted averaged power (WAP) of a single–phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by*

$$P^w := \langle Y_c(v), i\rangle. \tag{20}$$

If Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}[k]\hat{I}^*[k] \tag{21}$$

where $\hat{V}[k], \hat{I}[k]$ are the k-th spectral lines of v and i, respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := \frac{2\pi}{T}$.

Remark 3. *WAP is the sum of the power components of the circuit modulated by the frequency response of Y_c—hence the use of the "weighted" qualifier. Moreover, since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.*

The aforementioned definition motivates the next lemma.
Lemma 5. *Consider a nonlinear time invariant (TI) current-controlled $\{$voltage-controlled$\}$ one-port resistor characterized by (18) $\{$(19)$\}$ and a fixed LTI lossless compensator Y_c with $n = 1$. Let $\hat{Y}_c(j\omega)$ denote the associated admittance transfer function. If $\hat{Y}_c(j\omega)$ has a zero at the origin, then the weighted averaged power, along periodic trajectories,*

$$P^w_{R_i} := \langle Y_c v_{R_i}, i_{R_i}\rangle = 0, \tag{22}$$

$\{P^w_{R_v} := \langle Y_c v_{R_v}, i_{R_v}\rangle = 0\}$ *for all admissible pair $(v_{R_i}, i_{R_i}) \in \mathcal{L}_2 \times \mathcal{L}_2$ $\{(v_{R_v}, i_{R_v}) \in \mathcal{L}_2 \times \mathcal{L}_2\}$, and for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$.*

The proof is given in [14]. Lemma 5 suggests that the composition of an one-port resistor and a fixed one-port LTI defines a 1-port for which, in the case of periodic signals, the voltage and current are orthogonal to one another.
The following proposition gives the relation between WAP equalization and power-factor improvement by LTI lossless filters.
Proposition 6. *Consider the system of Figure 2 with $n = 1$,[3] a nonlinear RLC load, with linear resistors, and a fixed LTI lossless compensator Y_c with admittance transfer function $\hat{Y}_c(s)$.*
i) *PF is improved if and only if*

$$\frac{1}{2}V^w_s + \sum_{q=1}^{n_L} P^w_{L_q} + \sum_{q=1}^{n_C} P^w_{C_q} < 0 \tag{23}$$

where V^w_s is the rms value of the filtered voltage source, that is,

$$V^w_s := ||Y_c v_s||^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k)\hat{V}_s(k)|^2,$$

and

$$\begin{aligned} P^w_{C_q} &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{C_q}[k]\hat{I}^*_{C_q}[k] \\ P^w_{L_q} &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{L_q}[k]\hat{I}^*_{L_q}[k], \end{aligned}$$

are the weighted powers of the q–th capacitor and inductor, respectively.
ii) *Condition (23) may be equivalently expressed as*

$$\left\langle \left(\tfrac{1}{p}Y_c\right) v_L, \nabla^2 H_L v_L \right\rangle - \left\langle i_C, (\tfrac{1}{p}Y_c)\nabla^2 H_C i_C \right\rangle > \tfrac{1}{2}V^w_s \tag{24}$$

where $p := \frac{d}{dt}$.
iii) *If the capacitors and inductors are linear their weighted powers become*

$$\begin{aligned} P^w_{C_q} &:= 2\omega_0 \sum_{k=1}^{\infty} \left\{ k \ \mathrm{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_C} C_q |\hat{V}_{C_q}[k]|^2 \right\} \\ P^w_{L_q} &:= -2\omega_0 \sum_{k=1}^{\infty} \left\{ k \ \mathrm{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_L} L_q |\hat{I}_{L_q}[k]|^2 \right\}. \end{aligned} \tag{25}$$

where $\mathrm{Im}\{\hat{Y}_c[k]\}$ is the imaginary part of the admittance transfer function $\hat{Y}_c[k]$.
iV) *Furthermore, the results i-iii can be extended for load with nonlinear TI resistors, if the admittance transfer function $\hat{Y}(j\omega)$ of the LTI lossless compensator has a zero at the origin.*

The proof, which follows from Lemma 5 and the generalized form of Tellegen's theorem [13], is given in complete detail in [14].
Remark 4. *Condition (23) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.*
Remark 5. *From (24) (or replacing (25) in (23)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.*

Application of the framework

In this section, we focus our attention to the PFC of LTI loads by the use of a passive filter.

Consider an one-port $\mathfrak{N}$ with linear passive resistors, inductors, and capacitors supplied by a periodic nonsinusoidal voltage source $v_s(t)$. Considering periodicity, we can express the voltage source in terms of its (exponential) Fourier series as $v_s(t) = \sum_{k=-\infty}^{\infty} \hat{V}_s(k)\exp(jk\omega_0 t)$, where $\omega_0 := 2\pi/T$ is the fundamental frequency and, for integers k, $\hat{V}_s(k) := \frac{1}{T}\int_0^T v_s(t)\exp(-jk\omega_0 t)dt$ are the Fourier coefficients of the voltage source, also called spectral line or harmonics.

The LTI load Y_ℓ can be characterized in terms of the associated admittance, specified by its conductance $G[k]$ and susceptance $B[k]$ for the kth order harmonic, namely $\hat{Y}_\ell[k] = G[k] + jB[k]$. Additionally, it can be expressed in terms of the magnetic and electric energies of the kth harmonic as

$$\begin{aligned} \hat{Y}_\ell[k] = \ & \tfrac{2P(k\omega_0)}{|\hat{V}_s[k]|^2} + j\tfrac{k\omega_0}{|\hat{V}_s[k]|^2}\Big\{\textstyle\sum_{q=1}^{n_C} C_q|\hat{V}_{C_q}[k]|^2 \\ & - \textstyle\sum_{q=1}^{n_L} L_q|\hat{I}_{L_q}[k]|^2\Big\} \end{aligned} \tag{26}$$

where $P(k\omega_0) = \frac{1}{2}\sum_{p=1}^{n_R} G_{R_p}[k]|\hat{V}_{R_p}|^2$, $\hat{V}[k]$ is the kth spectral line of $v_s(t)$, and $\hat{V}_{C_q}[k]$, $\hat{I}_{L_q}[k]$ are the spectral lines of the qth capacitor voltage and inductor current.

From the Proposition 6, for LTI loads and a given compensator Y_c, we have that $PF > PF_u$ if and only if the following inequality verify

$$\sum_{q=1}^{n_L} P^w_{L_q} + \sum_{q=1}^{n_C} P^w_{C_q} + \frac{1}{2}V^w_s < 0$$

or, by Claim 3 of the proposition,

$$\begin{aligned} 2\omega_0 \sum_{k=1}^{\infty} k\mathrm{Im}\{\hat{Y}_c[k]\} \left\{ \sum_{q=1}^{n_C} C_q|\hat{V}_{C_q}[k]|^2 - \sum_{q=1}^{n_L} L_q|\hat{I}_{L_q}[k]|^2 \right\} \\ + \sum_{k=1}^{\infty} |\hat{Y}_c[k]\hat{V}_s[k]|^2 > 0 \end{aligned}$$

and, in comparison with (26), we obtain the following

$$\sum_{k=1}^{\infty} |\hat{Y}_c[k]\hat{V}_s[k]|^2 + 2\omega_0\sum_{k=1}^{\infty} k\mathrm{Im}\{\hat{Y}_c[k]\}B[k]|\hat{V}_s[k]|^2 < 0. \tag{27}$$

A. Maximum power factor for LIT load by capacitive compensator

A typical industrial approach for maximizing PF is sought for the connection of a variable, lossless capacitor C in par-

[3]This condition is imposed, without loss of generality, to simplify the presentation of the result.

allel with the load. The capacitive compensation in systems with nonsinusoidal voltages and currents has been widely studied, for instance [1, 2, 3] and [4]. In a similar way, consider that the compensator as $\hat{Y}_c(s) = sC$ in (27). Furthermore, if power-factor improvement is possible, the optimal value of the capacitance is obtained as follow. So, from (27) we define the function:

(28)

$$f(C) = C^2 \sum_{k=-\infty}^{\infty} (k\omega_0)^2 |\hat{V}_s[k]|^2 + 2\omega_0 C \sum_{k=-\infty}^{\infty} kB[k]|\hat{V}_s[k]|^2.$$

The problem becomes on optimization problem, i.e., the optimal values of the unknown C that minimize the right side of the inequality (27) which has the minimizer

$$C_{opt} = \frac{-\sum_{-\infty}^{+\infty} k\omega_0 B[k]|\hat{V}_s[k]|^2}{\sum_{-\infty}^{+\infty} k^2\omega_0^2 |\hat{V}_s[k]|^2}. \tag{29}$$

This result is also obtained in [5] and for $v_s(t) = V_s \sin\omega_0 t$ in [3].

However, the capacitive compensation may not be effective for distorted voltage, [4]. Therefore, alternative circuit topologies are studied in the circuits literature [2] but there seem to be many open problems for this solution. For instance, in [4] and [15], it is shown that for RL loads the optimal solution for parallel shunt LC compensator corresponds to a negative inductance, and thus a switched series LC circuits is suggested as an alternative solution. Furthermore, as it is shown in [3], if the supply bus has not an infinite power then a shunt reactive compensator changes the load voltage. This change could be particularly high when the resonance between the compensator and the supply source occurs in the systems. Indeed, usually it is the resonance between capacitance of the compensator and the inductance of the supply source. Moreover, in the practical applications most of the loads present impedance RL type, which increases with the frequency. Consequently, a circuit with a such load supplied from an inductive source and compensated by a capacitor behaves in the range of frequency $\omega \gg \omega_0$, see Fig. 3.

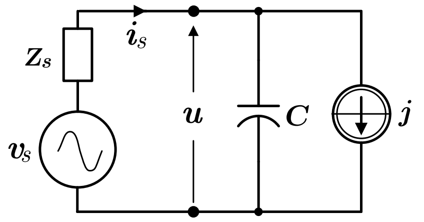

Fig. 3. Equivalent circuit of compensated load for $\omega \gg \omega_1$ where a current source j represents the harmonics generated by the load and $Z_s = j\omega L_s$ is the inductive impedance of the source.

The Load to Distribution Voltage (L/VD) transmittance, which expresses dependence of the load voltage on frequency in the circuit with a voltage resonance, is given by

$$\frac{U(j\omega)}{V_s(j\omega)} = \frac{1}{1-\omega^2 L_s C} = \frac{1}{1-\left(\frac{\omega}{\omega_r}\right)^2}$$

where $\omega_r = \frac{1}{\sqrt{L_s C}}$ is the resonant frequency. The bode plot of the magnitude of the L/VD transmittance $A(j\omega)$ is shown in Fig. 4. We observe that the magnitude of L/VD transmittance is higher than zero for frequency ω such $(\omega/\omega_r)^2 < 2$. It means that all distribution voltage harmonics of the frequency below $\sqrt{2}\omega_r$ are amplified by the compensator. This amplification increases to infinity at resonant frequency.

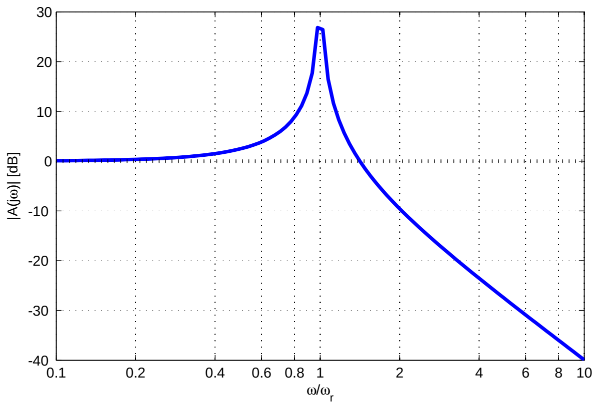

Fig. 4. Bode plot of L/DV transmittance versus relative frequency ω/ω_r

From the above shown, previously pointed out in [3], the capacitive compensation in systems with nonsinusoidal voltages and currents could contribute to an increase of voltage and current distortion and have low effectiveness. In addition, a possible total compensation of reactive power usually requires a compensator built of a high number of reactive components. Hence, such requirement could be fulfilled by the LC compensator shown in Fig. 5. Furthermore, a series LC compensator does not increase waveform distortion and it can be assumed that the voltage harmonics are not affected by the compensator if L and C are selected such that frequency range of amplification is below the 2^{nd}, i.e.,

$$\frac{1}{\sqrt{(L_s+L)C}} < \sqrt{2}\omega_0,$$

where ω_0 is fundamental frequency.

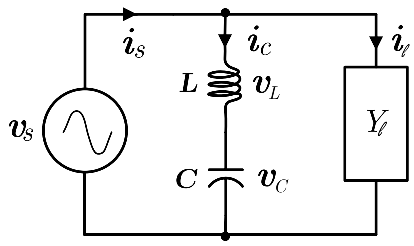

Fig. 5. Series LC-Compensation of Linear Network.

B. Maximum power factor for LIT load by series LC compensator

In [5] and [16] the PF improvement by cyclodissipativity condition was presented, but the improvement had been carried out by either capacitive or inductive compensation. Now, we consider for the power factor improvement of a linear load by a series LC compensator based on cyclodissipativity, see Fig. 5. We outline the calculations. By the Corollary **7** we have that series LC compensation improves the power factor, $PF > PF_u$, if and only if

$$0 > \sum_{k=-\infty}^{\infty} \left(\frac{k\omega_0 C}{1-k^2\omega_0^2 LC}\right)^2 |\hat{V}_s[k]|^2 + 2\sum_{k=-\infty}^{\infty} \frac{k\omega_0 C}{1-k^2\omega_0^2 LC} B(k\omega_0)|\hat{V}_s[k]|^2$$

where, the admittance transfer function of the compensator is given by

$$\hat{Y}_c(j\omega) = \frac{j\omega C}{1-\omega^2 LC},$$

and C and L are the capacitance and inductance, resp.

In order to find the optimal values of the unknown C and L, we define the function

$$\begin{aligned} f(L,C) = & \sum_{k=-\infty}^{\infty} \left(\frac{k\omega_0 C}{1-k^2\omega_0^2 LC}\right)^2 |\hat{V}_s[k]|^2 \\ & +2\sum_{k=-\infty}^{\infty} \frac{n\omega_0 C}{1-k^2\omega_0^2 LC} B[k]|\hat{V}_s[k]|^2. \end{aligned} \tag{30}$$

By assuming that $L_s << L$. The series inductance L must be selected such that the frequency range of amplification is below the 2^{nd} order harmonic, i.e. $\frac{1}{\sqrt{LC}} < \sqrt{2}\omega_0$, with ω_0 is the fundamental frequency. Hence we define the following constraint

$$g(L,C) = \frac{1}{\sqrt{LC}} - \sqrt{\alpha}\omega_0 = 0. \tag{31}$$

with $0 < \alpha < 2$ and $k \neq 1$.

Therefore, the problem becomes to find the local critical points of f restricted to S, and let S be the level set for g with value 0. By the method of Lagrange Multiplier and considering $c = 0$ we have f restricted to S which has a local minimum on S at

$$C_{opt} = \frac{\sum_{k=-\infty}^{\infty} \frac{n\omega_0}{1-\frac{k^2}{\alpha}} \frac{X(k\omega_0)}{R^2(k\omega_0)+X^2(k\omega_0)} |\hat{V}_s[k]|^2}{\sum_{k=-\infty}^{\infty} \frac{k^2\omega_0^2}{(1-\frac{k^2}{\alpha})^2} |\hat{V}_s[k]|^2} \tag{32}$$

and, using the constrain $g(L,C) = 1/\sqrt{LC} - \sqrt{\alpha}\omega_0 = 0$,

$$L_{opt} = \frac{1}{\alpha\omega_0^2 C_{opt}}, \tag{33}$$

where the susceptance $B(k\omega_0)$ is given by

$$B[k] = \frac{-X(k\omega_0)}{R^2(k\omega_0) + X^2(k\omega_0)}.$$

An analogous result can be found for parallel LC compensators. We now apply our result to an example system, [1].

Example 1. *A periodic nonsinusoidal voltage given by* $v_s = \sqrt{2}[200\sin\omega_0 t + 200\sin(5\omega_0 t + 30°)]$ *is applied to a series, linear, resistance-inductance load with resistance* $4\ \Omega$ *and fundamental frequency reactance* $10\ \Omega$*. Consider the fundamental frequency as* $f_1 = 50$ *Hz.*

The uncompensated circuit has the power factor $PF_u = 0.27$. From (29), the optimum capacitance that will give maximum power factor has the value $C_{opt} = 22.8\mu F$. The Power Factor after compensation is $PF = 0.292$. This result is the same as in [1]. The degree of the power factor improvement from 0.27 to 0.292 is seen to be very small. This is typical for circuits in which the voltage is grossly nonsinusoidal, see [1] for more cases. The optimal capacitor and inductor given by LC compensation, e.i. (32) and (33), resp., are $C_{opt} = 124.4\mu F$ and $L_{opt} = 40.7mH$, yielding an improved power factor $PF = 0.4987$, with $k = 1.999$. The power factor can be increased to a value which is approximately twice as high as the uncompensated power factor.

Conclusions

In this paper, extensions to the analysis of power factor compensation of nonsinusoidal networks based on cyclodissipativity were presented. We have studied the concept of cyclodissipativity property of electrical circuits and showed that the power factor by general LTI compensators is improved if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. Based on this condition, we have given criteria for improvement of PF with linear capacitors and LC filters. These criteria determine the optimal values for the compensator parameters. The analysis is carried out for the general periodic and nonsinusoidal supply voltage. Although we only study the linear case, this can be done for a nonlinear one as well.

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