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On the derivation of Piecewise-Linear Chaotic Oscillators using Simulated Annealing Method and Hspice

***Abstract**. The several novel structures of the chaotic oscillators with piecewise-linear vector field are derived and verified. For the synthesis of the linear part of the circuit the so-called simulated annealing method is utilized. For the rapid calculation of the fitness function the circuit simulator Hspice is used. Starting with the given mathematical model, namely the eigenvalues for each state space region, the final circuits consist of parallel connection of idealized nonlinear resistor and higher-order linear admittance.*

***Streszczenie.** Przedstawiono kilka nowych chaotycznych generatorów w częściowo liniowym wektorem pola. Do syntezy części liniowej obwodu wykorzystano metodę symulowanego wyżarzania. Do szybkich obliczeń funkcji sprawności użyto symulatora Hspice. Startując od modelu matematycznego wartości własne dla każdego regionu składają się z równoległych połączeń idealizowanych rezystorów nieliniowych i admitancji wyższego rzędu. (**Częściowo liniowy chaotyczny generator wykorzystujący symulowane wyżarzanie i Hspice**).*

Keywords: analog oscillator, chaotic motion, nonlinear dynamics, simulated annealing, global optimization.
Słowa kluczowe: generator, symulowane wyżarzanie..

Introduction

To date, the plenty methods for synthesis of the linear circuits already exist. The most of them are based on the matrix-pencil approach. Unfortunately it is hard to use them in the case of the large complex networks with potentially negative circuit elements. In such situations computer-aided design is preferred. The fundamental question which is faced here which method should be used to solve the particular problem. The main idea is to transfer the problem from circuit synthesis to the optimization task. Doing this, there is no need to know the analytical expression of the fitness function since some sort of stochastic optimization can be used. There are two basic algorithms and many different modifications, namely genetic algorithm (GA) and particle swarm optimization (PS). In each iteration step both methods solve the fitness function for the whole population. This property uncover the disadvantage of such approach, i.e. requirement of the rapid calculation of the fitness function. Another serious drawback is that the convergence itself is not guaranteed. For the purpose of precise frequency approximation the so-called simulated annealing method has been utilized. The corresponding routine is discussed in the chapter 2.

One of the most difficult analogue circuit design problem is a precise modeling of the higher-order nonlinear dynamics. It is known that some subset of the third-order dynamical systems can produce irregular noise-like (but deterministic) behavior. Such chaotic oscillators are extremely sensitive to the tiny changes in the circuit parameters. Assume now a parallel connection of linear third-order admittance network and piecewise-linear (PWL) resistor as shown in Fig.1. Having this the derivation process is provided in chapter 3. To preserve a certain structural stability of some desired state space attractor the admittance network must be approximated as accurately as possible in the large range of the frequencies. It has been verified by a number of the experiments that for denormalized impedance and frequency the sufficient range is from 100Hz up to 1MHz.

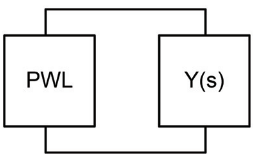

Fig.1. Principal configuration of PWL chaotic circuit

Ideas behind optimization

Synthesizing of topology and values of electronic circuit is hard optimization problem where global optimization method has to be used. There are many possible methods of global optimization, for example genetic algorithms, particle swarm optimization, ant colony optimization, messy genetic algorithm, simulated annealing, etc. The last mentioned method was used in this article.

Simulated annealing (SA) is a probabilistic method for finding of the global minimum in a large search space proposed by Kirk, Gelett and Vecchi in (1983) [1]. The method is inspired by annealing in metallurgy. For the purpose of searching of desired electronic circuit improved simulated annealing method was used [3]. This method was further modified to allow using of encoding of solution to sequences of integer numbers. In Fig. 2 there is sequence of integer numbers which encodes single electronic component.

type	node 1	node 2	exponent	values

Fig. 2. Encoding of single electronic component

Parameter “type” defines type of electronic component (1 – open circuit, 2 – short circuit, 3 – inductance, 4 – conductance, 5 - capacitor). The parameters “node 1” and “node 2” define connecting nodes of the components and last two parameters “exponent” and “value” define value of the component. This way the desired electronic circuit is encoded to sequence of integer numbers. For example electronic circuit consisting of 10 passive components is encoded to sequence of integer numbers which consists 10 x 5 = 50 variables (variable *number_of_variables* in Fig. 4). In the Fig. 3 there is shown an example of such sequence.

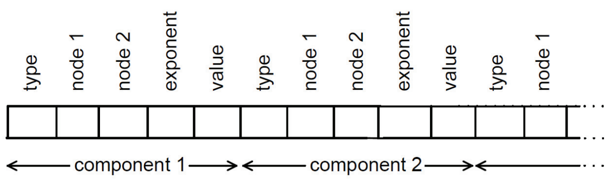

Fig. 3. Structure of encoding of a solution

In the Fig. 4 there is structure of used simulated annealing method.

```
procedure Simulated annealing
  for tries =1 to max_tries
      current_solution = random assignment
      iterations = 1
      repeat
         T = temp
         for k = 1 to number_of_variables do
           neighbourhood = current_solution
            if rand_1>0.5
                neighbourhood(k)=inc(current_solution(k))
           else
                neighbourhood(k)=dec(current_solution(k))
            end
            δ= fit(neighbourhood) – fit(current_solution)
            if rand_2 < Pa then
               current_solution = neighbourhood
            end
        end
         inc(iterations)
      until T<T_min
  end
```

Fig. 4. Structure of used simulated annealing method

After executing of the algorithm, *current_solution* is assigned randomly. After that the process of cooling starts (**repeat** – **until** loop). In the every single iteration (variable *iterations*) all the length of the encoding sequence is consequently modified in the second **for** - **end** loop. Based on the value of random generator rand_1 every single parameter of the encoding sequence *current_solution* is increased or decreased and resulting fitness of such *neighbourhood* is evaluated. Difference of the fitness of *neighbourhood* (fit(*neighbourhood*)) and *current_solution* (fit(*current_solution*)) is computed (*δ*). In the next step result of random generator rand_2 and result of the form Pa (1) are compared.

(1) $$P_a = \frac{1}{\left(1+\exp\left(\frac{\delta}{T}\right)\right)}$$

If the result of rand_2 is lower then result of Pa then *current_solution* is replaced by *neighbourhood*. In other case, the *current_solution* stays unmodified. In the second **for** – **end** loop the process described above is repeated *number_of_variables* times until all variables of the solution are modified. After modifying of all parameters of encoding sequence (second **for** – **end** loop finished) the number of iterations is incremented, according to (2) new value of temperature is calculated and the process described above is repeated for new (lower) value of temperature.

(2) $$temp = T_{\max} \cdot \exp(-\text{iterations} \cdot r)$$

The whole process is repeated until the value of minimal temperature T_{min} is reached. After reaching minimal value of T_{min} actual try is done and next try starts (first **for** – **end** loop).

Mathematical background

Suppose the general class of three-segment piecewise-linear vector fields which can be described by the following set of the differential equations [4]:

(3) $$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \dot{z}\\ \dot{z} &= q_1 z - q_2 y + q_3 x + h(.)\end{aligned}$$

where argument of the nonlinear feedback function is:

(4) $$h(.) = [(p_1-q_1)z - (p_2-q_2)y + (p_3-q_3)x]$$

and function itself:

(5) $$h(x) = 0.5(|x+1| - |x-1|)$$

Generally argument of the h(.) is a linear combination of all state variables. Having this mathematical model of some real physical system we have also some clues about geometry of the vector field. First of all the state space is separated by two parallel boundary planes into three linear segments. The equation for each plane can be considered as an argument of h(.). The dynamical motion in each segment is uniquely determined by eigenvalues, i.e. roots of the characteristic polynomials. For inner segment we get:

(6) $$\lambda^3 - p_1\lambda^2 + p_2\lambda - p_3 = 0$$

and analogically for both outer segments:

(7) $$\lambda^3 - q_1\lambda^2 + q_2\lambda - q_3 = 0$$

It is evident that the system behavior can be also uniquely specified by the so-called equivalent eigenvalues, p_i and q_i which are always real numbers. Typically chaotic solution is characterized by both stable and unstable manifolds for the individual fixed points. Recently it turns out that this is not a strict requirement for the existence of chaotic solution. For example the configuration:

(8) $$\begin{matrix} p_1 = 0.363 & p_2 = 1.063 & p_3 = 0.277\\ q_1 = -10.286 & q_2 = -1.2 & q_3 = -2.719\end{matrix}$$

as well as values:

(9) $$\begin{matrix} p_1 = 0.8 & p_2 = 100.21 & p_3 = 20.018\\ q_1 = -2.4 & q_2 = -0.71 & q_3 = -3.27\end{matrix}$$

lead to the following geometry of the vector field:

(10) $$D_0: \ \Re^3 \in E_u^2 \oplus E_u^1 \quad D_{\pm1}: \ \Re^3 \in E_u^2 \oplus E_s^1$$

where D_0 denotes inner segment and symbol $D_{\pm1}$ marks outer segments. Speaking in numerical terms the set (8) leads to the eigenvalues:

(11) $$\begin{aligned} D_0: &\ \lambda_{1,2} = 0.048 \pm 1.017j \quad \lambda_3 = 0.267\\ D_{\pm1}: &\ \lambda_{1,2} = 0.07 \pm 0.506j \quad \lambda_3 = -10.426\end{aligned}$$

Analogically for the set (7) results into the eigenvalues:

(12) $$\begin{aligned} D_0: &\ \lambda_{1,2} = 0.3 \pm 10j \quad \lambda_3 = 0.2\\ D_{\pm1}: &\ \lambda_{1,2} = 0.3 \pm 1j \quad \lambda_3 = -3\end{aligned}$$

It seems that dynamical system described by (3) and (4) with an fully unstable equilibria at origin is something special because the state space trajectory entering inner segment is repeled towards outer segments along every direction. Since by definition the trajectory can not cross an eigenspace system has two symmetrical chaotic attractors separated by an unstable eigenplane. From the viewpoint of circuit theory this property also suggest not only one but rather several grounded and/or floating one-ports. Negative resistors, capacitors and inductors are unwanted since it makes the final circuits much more complicated. The number of the negative elements can be minimalized as will be clarified later.

As the reference state space trajectory the numerical integration of the mathematical model together with a given set of the parameters can be considered. For this purpose the Mathcad and build-in fourth-order Runge-Kutta method has been used. The typical chaotic attractor for (8) is shown in Fig.5 and similarly for (9) it is demonstrated in Fig.6.

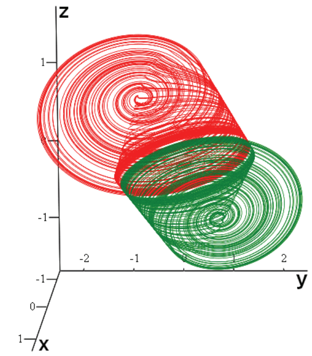

Fig.5. Numerical integration of the first set of parameters

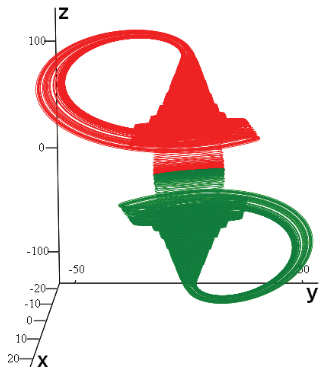

Fig.6. Numerical integration of the second set of parameters

Experimental results

According to block diagram in the Fig. 1 the chaotic oscillator of presented type consists of two parts, the PWL resistor and admittance network.

The three-segment PWL resistor is usually realized by diode limiter and imitance converters. Further details can be found in [5]. Since in our case the experimental verification is restricted to the Pspice circuit simulations the idealized approach can be adopted. One possible implementation of the function (5) is provided in Fig.7. The breakpoints are defined by the voltage controlled switches, where on and off states are represented by 1μΩ and 1TΩ resistor on the switch output port.

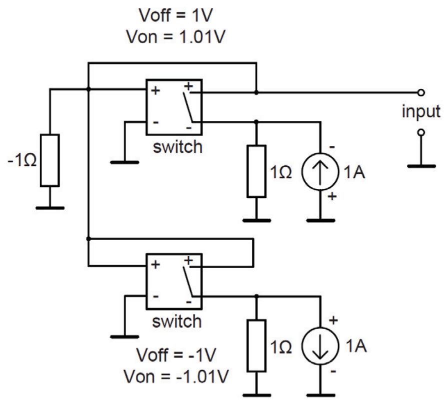

Fig.7. Idealized PWL resistor suitable for Pspice analysis

The second part, admittance network, was synthesized using presented simulated annealing method. Temperature coefficient were set to T_{max}= 100 and T_{min}= 1. Speed coefficient r (see (2)) was set to 0.01. The fitness function was specified according to form (13) as least mean square deviation of desired function (D) and actual solution (A).

(13) $$\sum_{i=1}^{n}\omega_i\left(D(f_i)-A(f_i)\right)^2 + nc\cdot size\_pressure$$

Fitness was calculated over the frequency range 100Hz – 1MHz at 20 frequency points per decade. In the frequency ranges 100Hz - 200Hz, 4kHz - 50kHz and 0.8MHz – 1MHz the weight factor ω was set to 15. Except weight factor, there was used another coefficient called size_pressure which was set to 10. This coefficient ensures preferring of solutions with less number of components (13). Variable nc is number of components of the solution. For evaluation of frequency responses of actual solution Hspice simulator was used.

The first and second examples of the synthesized admittance network for the first set of parameters are presented in Fig. 8 and Fig. 9. The admittance networks for the second set of parameters are presented in Fig. 10 and Fig. 11. Note that some resistances are negative.

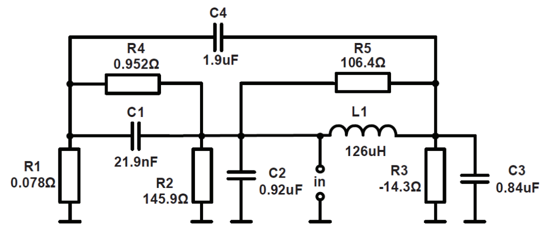

Fig. 8. First configuration of admittance network for the first set of parameters

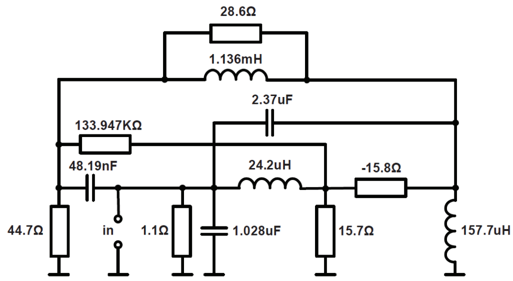

Fig. 9. Second configuration of admittance network for the first set of parameters

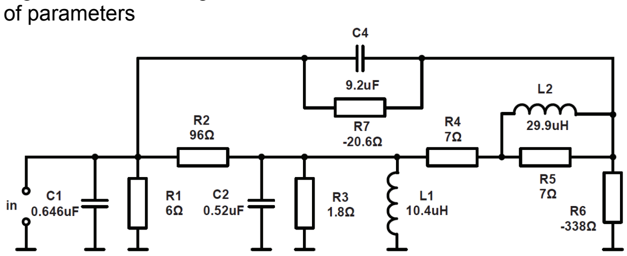

Fig. 10. First configuration of admittance network for the second set of parameters

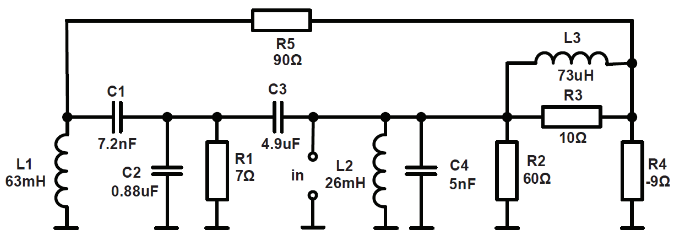

Fig. 11. Second configuration of admittance network for the second set of parameters

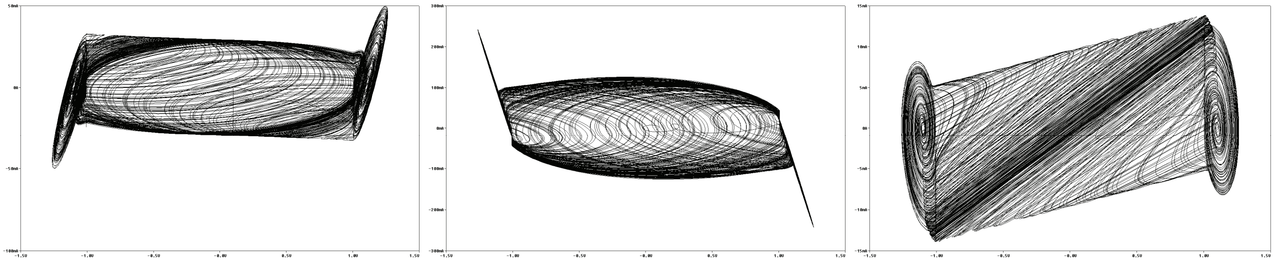

Fig. 12. Plane projections associated with the first admittance network for the first set of parameters: I(C3) vs V(in), I(L1) vs V(in), I(R5) vs V(in).

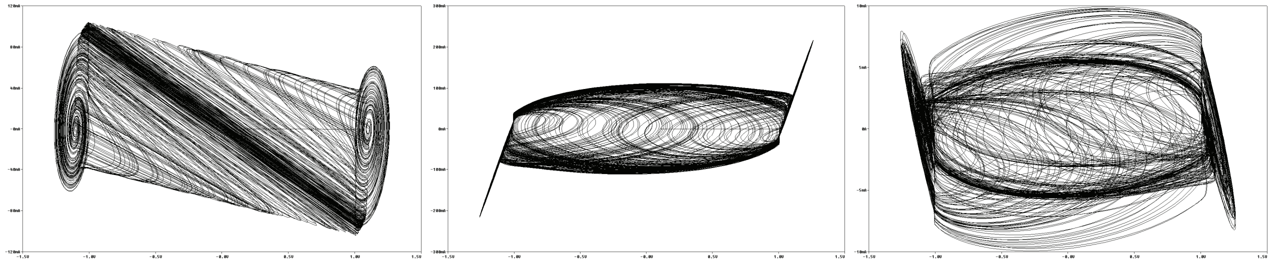

Fig. 13. Plane projections associated with the second admittance network for the first set of parameters: I(L1) vs V(in), I(L2) vs V(in), I(L3) vs V(in).

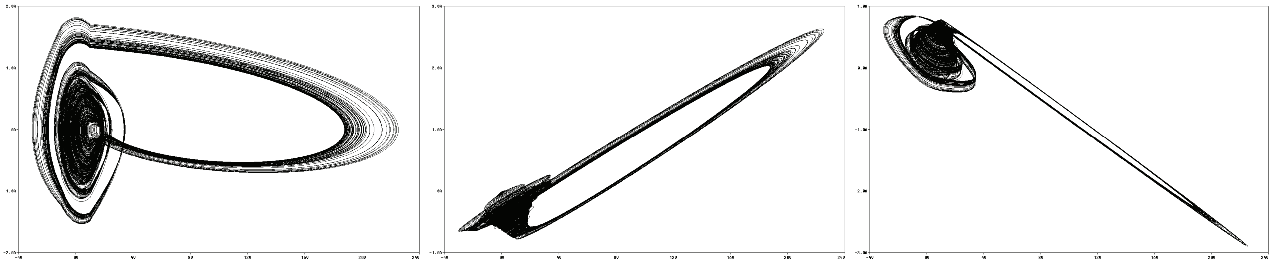

Fig. 14. Plane projections associated with the first admittance network for the second set of parameters: I(C1) vs V(in), I(L1) vs V(in), I(L2) vs V(in).

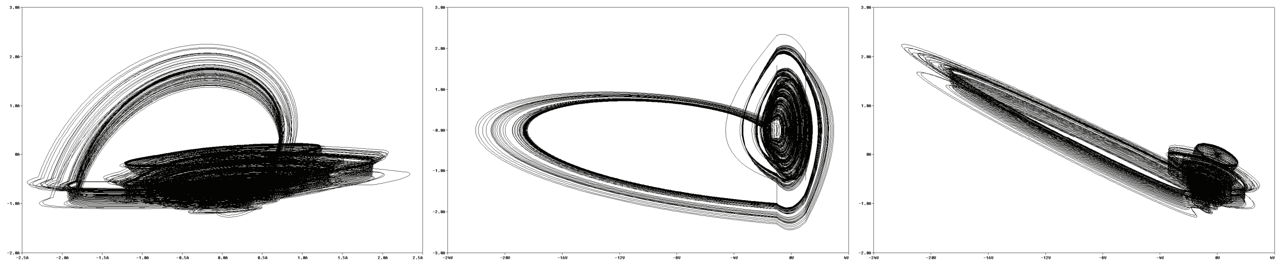

Fig. 15. Plane projections associated with the second admittance network for the second set of parameters: I(L3) vs I(C2), I(C2) vs V(in), I(L3) vs V(in).

Conclussion

It is demonstrated by means of several examples that it is effective to use SA optimization method for the synthesis of chaotic dynamics. The individual network configurations are tested using Pspice circuit simulator. The corresponding results are in good accordance with theoretical expectations, i.e. numerical integration of the given mathematical model.

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