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Analysis of permanent magnet motor with powder magnetic core using edge element method

***Abstract**. In the paper the potential application of soft magnetic composite material (Somaloy 500) in the permanent magnet motor (PMM) has been presented. The influence of rotor material parameters on electromagnetic torque and electromotive forces (emfs) of PMM has been discussed. Two types of permanent magnet motors have been considered: (a) rotor with powder magnetic core and (b) rotor with laminated magnetic core. Parameters of motors have been calculated using reluctance network method. Reluctance network has been formed using the interpolation functions of edge and facet element.*

***Streszczenie.** W artykule rozważono możliwość zastosowania kompozytu magnetycznie miękkiego typu Somaloy 500 w silnikach magnetoelektrycznych. Zbadano wpływ parametrów materiałowych wirnika na moment elektromagnetyczny i wartość sił elektromotorycznych rozpatrywanych silników. Parametry całkowe silnika wyznaczono na podstawie rozwiązań równań oczkowych siatki reluktancyjnej. Siatkę reluktancyjną utworzono na podstawie funkcji interpolacyjnych elementów krawędziowych i ściankowych. (**Analiza silnika magnetoelektrycznego o rdzeniu z kompozytu proszkowego metodą elementów krawędziowych**).*

Keywords: permanent magnet motor, powder magnetic materials, electromagnetic torque, edge element method.
Słowa kluczowe: silnik magnetoelektryczny, materiał proszkowy, moment elektromagnetyczny, metoda elementów krawędziowych.

Introduction

Permanent magnet machines (PMMs) are widely applied in industrial applications. Recently, in order to improve the performance of PMM the new powder magnetic materials have been proposed. Currently, two types of those materials are available, respectively with soft and hard magnetic properties [1]. In opposite to laminated cores (made of traditional magnetic sheets) the soft magnetic composite (SMC) powder materials have 3D-flux carrying capabilities. Application of those 3D magnetic properties is a key to success with an SMC core motors. This technology gives the possibilities for the designer to use new topologies with shape, winding and assembly solutions which are beyond today's standards and opens up for benefits such as better performance, reduced size and weight, fewer parts and lower cost [1].

In this paper the PMM with rotor core made of SMC material Somaloy 500 and laminated stator and PMM with laminated magnetic circuit have been investigated and compared. The 3D edge element method (EEM) has been applied for calculations of magnetic field in the motors. The edge element equations represent the loop equations of reluctance network (RN) [2]. RN is formed using the interpolation functions of edge and facet element [3, 4].

Edge element equations

In order to describe the distribution of the magnetic field the edge element method using the vector magnetic potential $\boldsymbol{A}$ has been applied. In the presented approach the considered motors have been subdivided into curved rectangular parallelepipeds (Fig.1).

The equations of used EEM represent the loop equations of reluctance network. The branches of RN connect the centres of neighbouring elements. The branch fluxes of RN represent the facet values of the magnetic flux density vector $\boldsymbol{B}$. In the used method the facet values of vector $\boldsymbol{B}$ are defined by the edge values of vector potential $\boldsymbol{A}$, i.e. loop fluxes φ around edges. For example the loop flux $\varphi_{3,7}$ that represents the edge value of $\boldsymbol{A}$ for edge P_3P_7 is shown in Fig.1. In this method the branch reluctances are calculated on the basis of the interpolation functions of facet element.

(1) $$R_{\mu i,j} = \int_{V_e} \mathbf{w}_{si}^T \boldsymbol{\nu} \mathbf{w}_{sj} dv$$

where, $\boldsymbol{w}_{si}$ and $\boldsymbol{w}_{sj}$ are interpolation functions of a facet element for the face $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$ [2], V_e is the volume of element. In contradistinction to the classical RN, the reluctance network formed by EEM has mutual reluctances.

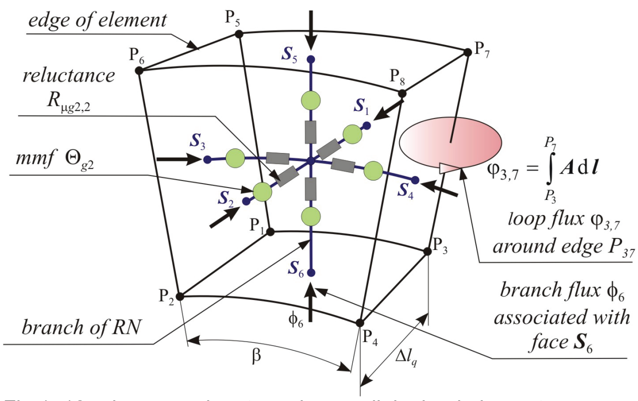

Fig.1. 12-edge curved rectangular parallelepiped element

The equations that describe loop fluxes φ of RN may be written in the following matrix form

(2) $$\boldsymbol{k}_e^T \boldsymbol{R}_{\mu g} \boldsymbol{k}_e \varphi = \boldsymbol{k}_e^T \theta_g = \theta$$

Here, $\boldsymbol{R}_{\mu g}$ is the matrix of branch reluctances, $\boldsymbol{k}_e$ is the transposed full loop matrix for RN, θ_g is the vector of branch magnetomotive forces (mmfs). The product $\boldsymbol{k}_e^T\theta_g$ represents the vector θ of loop mmfs. In the considered problem θ_g is a sum of two components: (a) component θ_m represents branch mmfs in area of permanent magnets and (b) component θ_0 that represents branch mmfs in area of windings. To define the field sources the electrical vector potentials $\boldsymbol{T}_m$ and $\boldsymbol{T}_0$ have been applied. In this approach the $\boldsymbol{T}_m$ potential describes magnetization of the permanent magnets [2], and the $\boldsymbol{T}_0$ is additionally introduced electric vector potential describing field of conduction current determined by direction of the current density vectors in the windings [5, 6]. Additionally, in the considered problem the homogenous magnetized magnets have been considered.

In the applied approach the vector of branch mmfs θ_m is solved using the edge values of magnetization vector $\boldsymbol{T}_m$. Those edge values have been represented by loop magnetizations currents $\boldsymbol{i}_m$ around of the element edges [7].

The value of vector $\boldsymbol{T}_m$ is determined by demagnetization curve of the applied material of permanent magnet [7]. In the proposed approach the loop current $\boldsymbol{i}_m$ for the $N_{i,j}$ -th edge is calculated from the following integral

$$\boldsymbol{i}_{mN_{i,j}} = \int_{P_i}^{P_j} \boldsymbol{T}_m \mathrm{d}\boldsymbol{l} \tag{3}$$

where, P_i and P_j are the nodes of $N_{i,j}$ -th edge.
Next, the values of the branch mmfs vector θ_m are calculated as follows

$$\theta_m = \kappa \boldsymbol{i}_m \tag{4}$$

where, κ is matrix that transposes the loop values around element edges to the loop values associated with element facets [2], and is also called weight parameters matrix. In the approach the homogenous magnetized magnets have been considered.

In the winding areas the values of branch mmfs θ_0 may be calculated using the $\boldsymbol{T}_0$ values that represent loop currents $\boldsymbol{i}_0$ around element edges intersecting surfaces created by surrounding turns [2]. Thus, the loop currents can be expressed by integral of the vector $\boldsymbol{T}_0$ over the edges of the FE mesh in the winding area

$$\boldsymbol{i}_{0N_{i,j}} = \int_{P_i}^{P_j} \boldsymbol{T}_0 \mathrm{d}\boldsymbol{l} \tag{5}$$

In opposite to the values of $\boldsymbol{T}_m$, the values of potential $\boldsymbol{T}_0$ are not known a priori. In order to describe the edge values of $\boldsymbol{T}_0$ potential, it is necessary find relation between those values and currents in the windings of the motors $\boldsymbol{i}_c$. It can be done by using following expression

$$\boldsymbol{i}_0 = \boldsymbol{z}_k \boldsymbol{i}_c \tag{6}$$

where z_k is the matrix that represents the coil turns agreement in the edge element space [2]. This matrix was also used by authors to solutions of fluxes linked with windings and *emfs* [8].
Elements of the vector θ_0 are calculated the same as elements of vector θ_m, because the relationship between elements of the vector θ_0 and loop currents $\boldsymbol{i}_0$ is the same as between θ_m and $\boldsymbol{i}_m$. In this case the vector of branch mmfs in winding regions can be calculated from

$$\theta_0 = \kappa \boldsymbol{i}_0 \tag{7}$$

Using relation (6) the vector that describes the branch mmfs θ_0 of RN can be written as

$$\theta_0 = \kappa \boldsymbol{z}_k \boldsymbol{i}_c \tag{8}$$

The details of describing the magnetic field sources in the magnets and windings region have been described in [2] and [8], respectively.

The electromagnetic torque for RN is obtained from finite difference approximation of the magnetic energy derivative versus the virtual moving [9]. The finite difference approximation gives

$$T(\alpha) = -\frac{1}{2\beta}\left(W(\alpha+\beta) - W(\alpha-\beta)\right) \tag{9}$$

Where, $W(\alpha\pm\beta)$ is the magnetic energy for discrete rotor positions $\alpha\pm\beta$, α is an considered rotor position, β is the angular width of element, see Fig.1.
To obtain $T(\alpha)$ characteristics the band with curved rectangular parallelepipeds has been placed in the air-gap of 3D PMM model. The element edges are parallel to the axis of a cylindrical co-ordinate system r, z, ψ. The trace of the elements in the plane perpendicular to the axis z is the grid with quadrangles of identical angular length of the base β. The band is subdivided into m layers of thickness Δl_q in direction z (Fig.1). For the band, formula (9) has been expressed as

$$T(\alpha) = \frac{1}{2\beta}\left\{\sum_{q=1}^{m} R_{p,q} \sum_{i=1}^{n} \phi_{\psi q,i}\left(\varphi_{q,i-1} - \varphi_{q,i+1}\right)\right\} + \frac{1}{2\beta}\left\{\sum_{q=1}^{m-1} R_{z,q} \sum_{i=1}^{n} \phi_{zq,i}\left(\varphi_{\psi q,i-1} - \varphi_{\psi q,i+1}\right)\right\} \tag{10}$$

Here, $\phi_{\psi q,i\pm1}$, is the value of flux in branch P_{bq} for position $\alpha\pm\beta$ and $\phi_{zq,i\pm1}$ is the value of flux in branch P_{zq} for position $\alpha\pm\beta$, see Fig.2. R_{bq} and R_{zq} are the reluctances associated with the band, see also Fig.2. Subscript q denotes the reluctances and the branch fluxes related to the q-th layer.
As a result the equation that represents the stress tensor formula for torque calculation using RN and EEM has been obtained [9]. To calculate RN equations the block over-relaxation method has been employed.

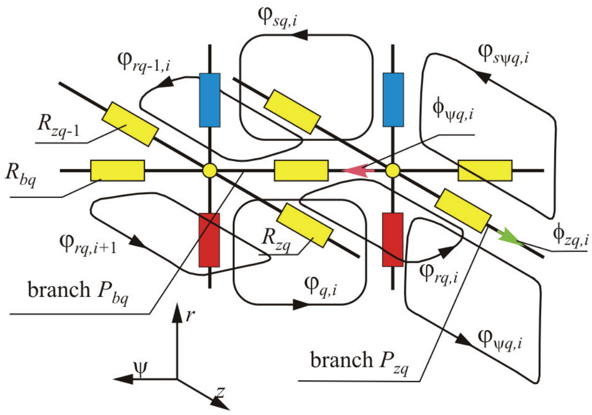

Fig.2. Part of the band with branches P_{bq} and P_{zq} [9]

Permanent magnet motors

Two types of 4 pole motors with 6 slots per pole have been designed. The difference between motors is only in rotor materials. The cases with SMC material and laminated steel sheets have been examined. In the both motors permanent magnets are not skewed and are mounted on rotor (Fig.3).

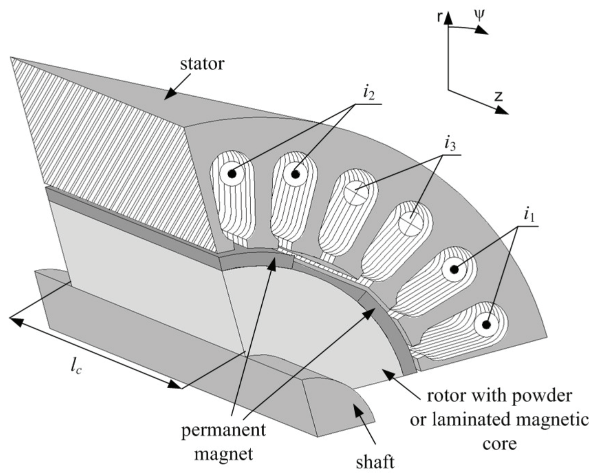

Fig.3. Construction of PMM

Figure 4 shows the magnetic property comparison between the ferromagnetic sheet ST3 and SMC material – Somaloy 500.

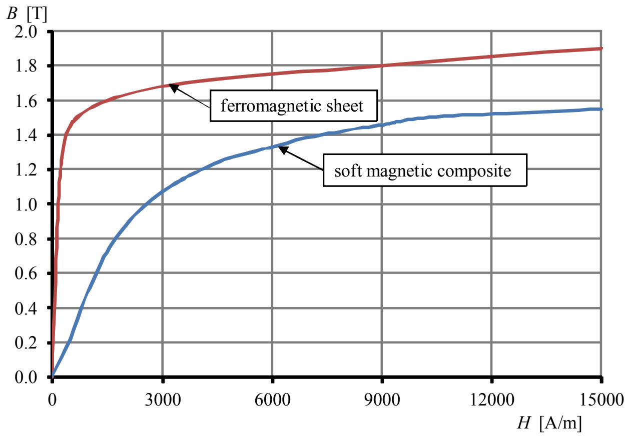

Fig.4. Magnetization curves $B=f(H)$ for soft magnetic composite and ferromagnetic sheet

A single-layer winding is composed of 4 multi-turn coils per phase. The coils are connected in series. The 3-phase stator windings are star connection. The motors are equipped with rare-earth $SmCo_{18}$ type permanent magnets.The stator cores of both machines are laminated, see Fig.5.

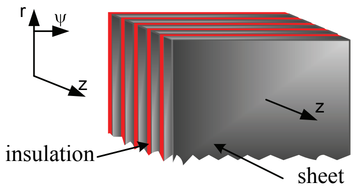

Fig.5. Anisotropic structure of laminated stator core

In order to model the laminated core in 3D it is assumed that reluctivity ν_z in the direction parallel to shaft axis z differs from the reluctivity $\nu_{r,\psi}$ in the direction orthogonal to axis z, i.e. the core reluctivity is considered to be orthogonally anisotropic. The values of ν_z and $\nu_{r,\psi}$ have been obtained from the formulas that describe the equivalent reluctivity of the system composed of two reluctances: the reluctance for the flux which passes through the ferromagnetic sheets and the reluctance for the flux that penetrates the isolation. This approach gives

(11) $$\nu_{r,\psi} = \frac{\nu_0 \nu_{Fe}}{k_z \nu_0 + (1-k_z)\nu_{Fe}} \approx \frac{\nu_{Fe}}{k_z}$$

(12) $$\nu_z = (1-k_z)\nu_0 + k_z \nu_{Fe} \approx (1-k_z)\nu_0$$

where, k_z is the stacking factor, $\nu_{Fe} = \nu(\boldsymbol{B})$ is the reluctivity of isotropic ferromagnetic sheet.

In order to model the powder magnetic core it is assumed that reluctivity ν_z and reluctivity $\nu_{r,\psi}$ are equal to reluctivity of SMC materials (ν_{SMC}).

Results

The presented method has been used in the analysis of two types of PMMs: (a) rotor with powder magnetic core, (b) rotor with laminated magnetic core. The considered region has been subdivided into 200 000 elements per pole.

In this paper the results of electromagnetic torque and *emfs* calculation are given. In the case of torque calculation it was assumed that the phase currents are sinusoidal, i.e. $i_q = 13.85 \sin(\omega t + (q-1)120)$ $(q = 1,2,3)$, and torque angle δ=90°. The calculations have been performed for different values of core length l_c (see Fig.3) i.e. for the motors of $0.166r_s \le l_c \le 2r_s$, where r_s is the external stator radius. Here, the results for the motor of core length l_c equal to $0{,}33r_s$ are presented. The 3D and 2D models have been considered (2D – see Fig.6a and 3D – see 6b). The relative waveforms of electromagnetic torque and the cogging torque are shown in Figs. 6 and 7, respectively. The torque waveforms are related to the rotor with powder magnetic core T_{SMC}, T_{c_SMC} and rotor with laminated magnetic core T_{FS}, T_{c_FS}.

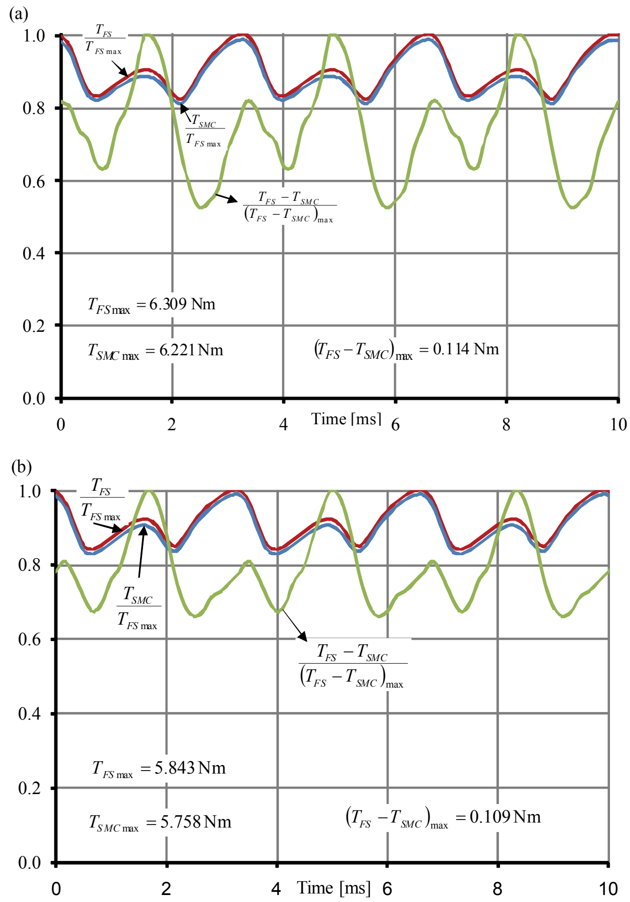

Fig.6. The waveforms of electromagnetic torque (a) 2D model (b) 3D model ($l_c = 0{,}33r_s$)

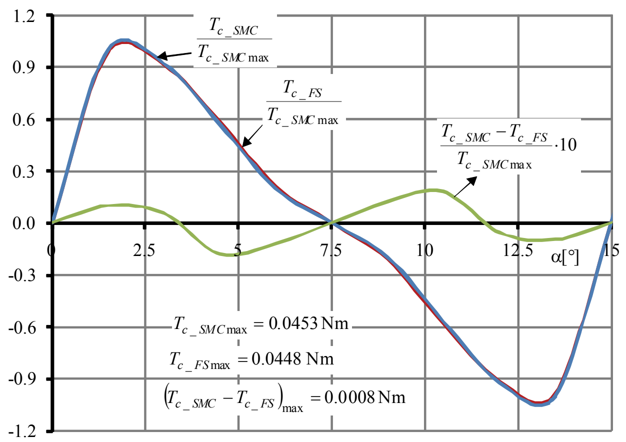

Fig.7. The waveforms of cogging torque (3D model, $l_c = 0{,}33r_s$)

The differences between the calculated values of cogging torque for the motor with SMC rotor and motor with laminated rotor are smaller than 1 %. Additionally, the more results, i.e. the obtained values of maximum cogging torque T_{cmax}; average T_{av} and maximum T_{max} electromagnetic torque values for the motors of different values of core length l_c, can be found in [10].

Figure 8 shows the relative difference between the maximum values of average torque for the 2D and 3D analysis as a function of core length. The 3D calculations have been performed for the rotor with powder magnetic core T_{av3D_SMC} and rotor with laminated magnetic core T_{av3D_FS}. The results of torque for 3D analysis, i.e. T_{av3D_SMC} and T_{av3D_FS} are related to the torque for 2D analysis and rotor with laminated magnetic core T_{av2D_FS}. It is interesting to notice that the difference between the results of 2D and 3D models are bigger then difference between the results of SMC and laminated core.

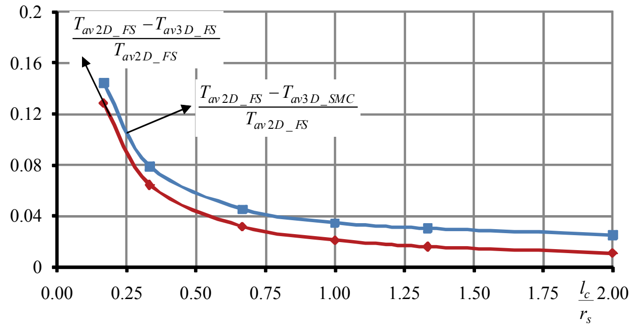

Fig.8. Difference between the results of electromagnetic torque calculation for the 2D and 3D analysis as a function of core length

Next, the results of emfs calculation are given. In Fig.9 the emf waveforms are presented. The emf characteristics have been interpolated by FFT method. The amplitudes of calculated harmonics are shown in Fig.10.

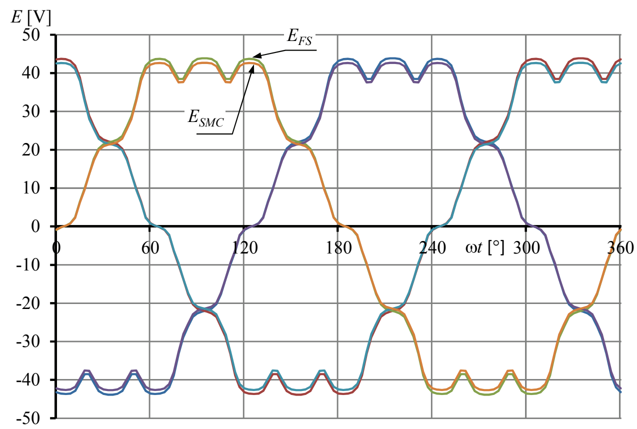

Fig.9. *Emf* waveforms, (n=1500 rpm)

The results of electromagnetic torque and emfs calculations in PMM with powder magnetic rotor compared to conventional PMM with laminated magnetic rotor are given. From the obtained results it can be notice that the differences between SMC and laminated core are not significant and the performance is similar. For example the differences between the calculated values of electromagnetic torque for SMC and laminated core are smaller than 2%.

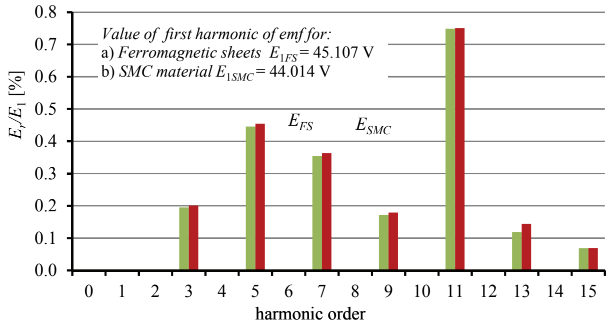

Fig.10. Percentage of harmonics E_r related to the first harmonic of emf

Conclusions

Soft magnetic composites materials have the potential to be applied to electrical machines. Analysis of permanent magnet motor with powder magnetic core shows that it is possible to design motor with such a soft magnetic circuit. The benefits of replacing the conventional laminated cores in the electrical motors with the powdered iron composites are as follows: significant reduced production costs, due to the simplified design and essentially unity iron stacking factor.

The used method in the paper is universal and can be successfully applied in the analyses isotropic, anisotropic materials and non-homogenous regions.

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