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A New Efficient Method for Load-Flow Solution for Radial Distribution Networks

***Abstract:** A new efficient method is proposed for load-flow solution of radial distribution networks. Simple transcendental equations are used to relate the sending-end voltage, receiving-end voltage and voltage drops in each branch of the distribution system. The effect of charging capacitance of the line has been incorporated in load-flow solution. A computer algorithm is developed in such a way that there is no need to adopt any sequential node numbering scheme for the solution of the networks. The angle of the receiving-end voltage is also computed along with the magnitude of the voltage. It is an iterative method. The flat voltage (1pu) start from substation to every end-node is considered. The voltage magnitude and angle are updated after each successive iteration and the voltage drops are then computed by using the new obtained values of voltage magnitude and angle. The comparison of speed and memory requirement by the proposed method with the other methods has been verified to show its efficiency.*

***Streszczenie.** Zapropowano nową metodę kontroli przepływu mocy w radialnej sieci rozproszonej. Użyto prostego równania transcedentalnego do opisu napięcia wysyłanego, otrzymanego i spadki napięcia w każdej z gałęzi sieci. Opracowano odpowiedni algorytm komputerowy a obliczenia są przeprowadzane iteracyjne, gdzie amplituda i kąt napięcia są sukcesywnie zmieniane. (**Nowa skuteczna metoda obliczania rozpływu mocy w radialnej sieci rozproszonej**)*

Keywords- Constant current, constant impedance, constant power, distribution networks, load, Radial, voltage drops.
Słowa kluczowe: dystrybucyjna sieć rozproszona, obciążenie, spadki napięcia.

Introduction

The load-flow analysis has an important role for the design of distribution network. An efficient and good converging load-flow of distribution system is not only useful to obtain voltage and power loss of the network but also necessary for accurate selection of branch conductor and other aspect of planning. Load-flow methods like Newton-Raphson, and fast decoupled load-flow method proposed by Tinny and Hart [1] and Scott and Alsac [2] can be efficiently used for transmission systems. But the systems having high R/X ratio, the above methods failed to converge. Some researchers Iwamoto and Tamura [3] and Rajcic and Tamura [4] tried to modify the conventional methods to solve the distribution networks. Ladder network theory was used by Kersting [5]. A compensation based method was proposed by Shirmohammadi *et al.* [6] to solve the weakly meshed networks. The branch numbering scheme was developed to enhance the numerical performance. Baran and Wu [7] proposed a Newton – Raphson based method, which required the computation of Jacobian matrix to solve the radial distribution networks. So this method needed more computational efforts. Renato [8] used the bi-quadratic equations to relate the sending-end and receiving-end voltages. But the angle of the voltage was not considered in his method. Goswami and Basu [9] proposed a direct method for solving the radial networks but they had put the limit that no node could be a junction of more than three branches. Jasmon and Lee [10] reduced the network in a single line equivalent model and used the power-flow equations in which the sending-end voltage was involved. Das *et al.* [11] proposed a unique numbering scheme for nodes and branches. They had used the simple algebraic equations to solve the radial distribution system. Haque [12] had proposed a solution method for radial and mesh power distribution networks. Backward and forward sweep technique was used for final solution. Initially the substation voltage was taken as the voltage of each node, which was updated after each successive iteration. Ghosh and Das [13] proposed a method for load-flow solution using forward backward technique but only the magnitude of voltage at each node was computed. Aravindhababu *et al.* [14] proposed a technique to compute the branch currents and loads from the node branch matrix, voltage drops and node voltages were then computed. Mekhamer *et al.* [15] used the equations of Baran and Wu [7] for load-flow solution of radial distribution system but the solution methodology was different. The load of the lateral branches was supposed as the single equivalent load. After computation of the voltages of the main feeder nodes then the laterals were expanded and the voltage of the first node of the lateral was taken equal to the node voltage to which the lateral was connected. Same solution technique was used for the laterals. Eminoglu and Hocaoglu [16] developed a method for load-flow solution based on Kirchhoff's law equations and applied the proposed method on different load models. Satyanarayana *et al.* [17] developed the load-flow equations from the A B C D parameter model of short transmission line and applied the method on the different load models. Nagaraju *et al.* [18] proposed a method for load flow of radial distribution networks using sparse technique. But this method is only suitable for the sequentially numbered networks. If the network is not sequential numbered, manual work has to be done to make it sequentially numbered. Hamouda and Zehar [20] proposed an algorithm for load-flow solution of radial distribution networks in which branch-to-node matrix is constructed and then inverse of this matrix is formed to get the node-to-branch network. From node-to-branch matrix a branch matrix is formed. All this procedure needs ample amount of computational effort and computer memory to store the matrices.

The aim of the authors in this paper is to propose a new method for load-flow solution of radial distribution networks in which following features are incorporated:

1. No sequential numbering of nodes is required. So one or more load points can be added or eliminated from the network without the requirement of renumbering of nodes. Considerable effort for data preparation is reduced.
2. No need to store the nodes beyond the branch, the branch to node incident matrix. So the computer memory requirement is considerably reduced.
3. This method gives node voltages as well as their angle and can handle the effect of charging capacitances of the network. So this method can be used for reactive power compensation studies.

The proposed method has been tested with different types of load modeling and convergence has been obtained satisfactorily.

List of symbols

N: Total number of nodes.
SN, RN: Sending-end node and receiving-end node.
k: Node Number :1, 2, 3,…………. N.
jj: Branch number :1, 2, 3,…………. N-1.
V_k: Sending-end voltage.
V_{k+1}: Receiving-end node's voltage.
δ_k: Sending-end node's voltage angle.
δ_{k+1}: Angle of Receiving-end node's voltage.
I_{jj} : Branch current.
Z_{jj} : R_{jj} + jX_{jj}, Branch impedance.
y_m^c : Admittance of the charging capacitance.
P_m and Q_m : Real load and reactive load respectively connected to the node (m).
L_1, L_2, L_3……. are the arrays to store node numbers of feeder and laterals the distribution network.
JN and LN are two dimensional arrays.

Assumptions

It is assumed that the three-phase radial distribution networks are balanced and can be represented by their equivalent single-line diagrams. Line shunt capacitances are considered even at the distribution voltage levels.

Formulation of load-flow method

Fig. 1 shows the electrical equivalent of the radial distribution system.

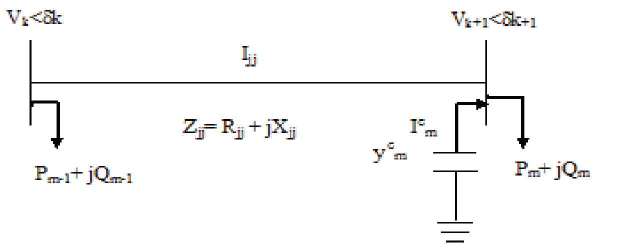

Fig. 1. Two-bus equivalent of distribution network

It is assumed that the charging capacitance of each branch is lumped at the receiving-end node of that branch.

Formulation of voltage and angle of the voltage at each node

The sending-end voltage can be related to the receiving-end voltage as:

(1) $$V_k\angle\delta_k = V_{k+1}\angle\delta_{k+1} + I_{jj}Z_{jj}$$

The current in a particular branch is the sum of all load currents due to the loads connected to the nodes minus the charging current due to line shunt capacitances beyond that branch and the receiving-end node of that very branch.

(2) $$I_{jj} = \sum_{m=k+1}^{N}\left[\left(\frac{P_m - jQ_m}{V_m\angle-\delta_m}\right) + \left\{j\left(V_m\angle\delta_m\right)y_m^c\right\}\right]$$

(3) $$I_{jj} = I_{jj}^a + jI_{jj}^r$$)

(4) $$I_{jj}^a = \sum_{m=k+1}^{N}\left\{\frac{P_m\cos\delta_m + \left(Q_m - V_m^2 y_m^c\right)\sin\delta_m}{V_m}\right\}$$

(5) $$I_{jj}^r = \sum_{m=k+1}^{N}\left\{\frac{P_m\sin\delta_m - \left(Q_m - V_m^2 y_m^c\right)\cos\delta_m}{V_m}\right\}$$

Real and imaginary parts of the node current can be expressed as

(6) $$I_m^a = \frac{P_m\cos\delta_m + \left(Q_m - V_m^2 y_m^c\right)\sin\delta_m}{V_m}$$

(7) $$I_m^r = \frac{P_m\sin\delta_m - \left(Q_m - V_m^2 y_m^c\right)\cos\delta_m}{V_m}$$

(8) Let $Q_m^{'} = Q_m - V_m^2 y_m^c$

Here P_m and Q_m, are the real and reactive loads connected to the mth node. V_m and δ_m are the voltage and its angle respectively at mth node. I_{jj}^a and I_{jj}^r are active and reactive components of the branch current I_{jj}. If the lengths of branches of the distribution system are considerably small, the value of y_m^c will be very low. In Eq. (2), the charging current will also be zero. Hence the proposed method can be applied to both the cases if the charging currents are considered or neglected.

The voltage drop in each branch can be expressed by

(9) $$I_{jj}Z_{jj} = \left(I_{jj}^a + jI_{jj}^r\right)\left(R_{jj} + jX_{jj}\right)$$

So Eq. (1) can be expressed as

(10) $$V_{k+1}\angle\delta_{k+1} = V_k\angle\delta_k - \left(I_{jj}^a + jI_{jj}^r\right)\left(R_{jj} + jX_{jj}\right)$$

For the simplicity

(11) $$C_{jj} = I_{jj}^a R_{jj} - I_{jj}^r X_{jj}$$

(12) $$D_{jj} = I_{jj}^a X_{jj} + I_{jj}^r R_{jj}$$

(13) $$V_{k+1}\left(\cos\delta_{k+1} + j\sin\delta_{k+1}\right) = V_k\left(\cos\delta_k + j\sin\delta_k\right) - \left(C_{jj} + jD_{jj}\right)$$

Separating real and imaginary parts of Eq. (13), we have

(14) $$V_{k+1}\cos\delta_{k+1} = V_k\cos\delta_k - C_{jj}$$

(15) $$V_{k+1}\sin\delta_{k+1} = V_k\sin\delta_k - D_{jj}$$

Squaring on both sides of Eq. (14) and Eq. (15), we have after addition

(16) $$V_{k+1}^2 + 2V_{k+1}(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1}) + (C_{jj}^2 + D_{jj}^2 - V_k^2) = 0$$

It is a quadratic equation in V_{k+1} so that V_{k+1} can be computed as

(17) $$V_{k+1} = \frac{-2(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1})}{2} \pm \frac{\sqrt{\left\{2(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1})\right\}^2 - 4(C_{jj}^2 + D_{jj}^2 - V_k^2)}}{2}$$

(18) i.e. $$V_{k+1} = -(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1}) \pm\sqrt{(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1})^2 - (C_{jj}^2 + D_{jj}^2 - V_k^2)}$$

Since the receiving-end voltage has positive value, the following root is considered.

(19) $$V_{k+1} = -(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1}) + \sqrt{(C_{jj}\cos\delta_{k+1} + D_{jj}\sin\delta_{k+1})^2 - (C_{jj}^2 + D_{jj}^2 - V_k^2)}$$

Divide Eq. (15) by Eq. (14) we have,

(20) $$\delta_{k+1} = \tan^{-1}\left(\frac{V_k\sin\delta_k - D_{jj}}{V_k\cos\delta_k - C_{jj}}\right)$$

Formulation of power losses

From Eq. (2) and Eq. (8) we have,

(21) $$I_{jj}^2 = I_{jj}I_{jj}^* = (I_{jj}^a + jI_{jj}^r)(I_{jj}^a - jI_{jj}^r) = \sum_{m=k+1}^{N}\left(\frac{P_m^2 + Q_m^{'2}}{V_m^2}\right)$$

From Eq. (6) and Eq. (7) we have,

$$I_m^2 = I_m I_m^* = (I_m^a + jI_m^r)(I_{jj}^a - jI_m^r) = \left(\frac{P_m^2 + Q'^2_m}{V_m^2}\right)$$

(22)

Branch Real power loss of branch jj

$$(23) \qquad = \left(\sum_{m=k+1}^{N} \frac{P_m^2 + Q'^2_m}{V_m^2}\right) R_{jj}$$

Branch Reactive power loss of branch jj

$$(24) \qquad = \left(\sum_{m=k+1}^{N} \frac{P_m^2 + Q'^2_m}{V_m^2}\right) X_{jj}$$

Total real power loss of the network

$$(25) \qquad = \sum_{jj-1}^{N-1}\left(\sum_{m=k+1}^{N} \frac{P_m^2 + Q'^2_m}{V_m^2}\right) R_{jj}$$

Total reactive power loss of the network

$$(26) \qquad = \sum_{jj-1}^{N-1}\left(\sum_{m=k+1}^{N} \frac{P_m^2 + Q'^2_m}{V_m^2}\right) X_{jj}$$

Load modelling

In constant power type of load model it is supposed that the power demand of the load remains same irrespective to the change in terminal voltage. But it is not true for most of the loads. Power demand changes with the change of terminal voltage of the loads. Due to the change in power demand of the loads with the change in terminal voltage, there is a worth noting effect on the convergence of the load-flow solution. The characteristics of exponential load models can be expressed as:

$$P = P_0\left(V/V_0\right)^{n_p} \quad Q = Q_0\left(V/V_0\right)^{n_q}$$

Here P_0 and Q_0 stand for active and reactive power respectively at nominal voltage, V and V_0 stand for node voltage and nominal load voltage respectively and n_p and n_q are the respective load exponents. The values of the load exponents for various types of load are shown by Eminoglu and Hocaoglu [16]. To show the goodness of the method the composite load and practical type load are also consider. Their composition is given in TABLE 1

Table 1. The types of loads and values of there constituents

Sr. No.	Type of load	Composition
1	Composite load	30% constant power + 30% constant current + 40% constant impedance load.
2	Practical load	Battery Charge 2% + Fluorescent lamps 11% + AC 8% + resistance space heating 5% + Pump, fans and other motors 18% + CFL 5% + Incandescent lamp 6% + Small motors 25% + Large motors 20%.

Convergence criteria

The difference of the voltage calculated for any node in two successive iterations should not be more than 0.00001 or 10^{-5}. Proof of convergence of load-flow solution is shown in Appendix-A for the proposed method.

Application of the proposed method

To show the effectiveness of the proposed method two examples, 33-node and 41-node distribution system are considered. In Example 1 constant power load is taken and effect of line shunt capacitance is neglected, the results are compared with the existing methods to show reliability of the method. Second example is taken to show the effect of load characteristics on convergence of load-flow solution for various types of loads and to show the effect of charging capacitances.

The line data and load data of 33- node radial distribution network (Example 1) is available in Baran and Wu [19]. The base voltage and base power are taken as 12.66kV and 10MVA respectively. The load-flow results are compared with the existing method proposed by Hamouda and Zehar [20] and shown in Table 2. It is shown in Table 2 that the results for the node voltages and their angles, real and reactive power losses are comparative to the existing method and it is one of the latest published work in this field. Example 2 is 41-node radial distribution system. In Appendix-B Line data, load data and single line diagram for 41-node distribution system are shown in Table B1 and Fig. B1respectively. The technical data regarding conductors of 41-node distribution system is shown in Table B2. The voltage and power base are 11kV and 100 MVA respectively. Load-flow solution is carried out for various types of loads without considering the capacitance of the branches. Only the voltage and angle of the voltage of the far most node which is node no. 13 are shown in Table 3. However number of iterations for the convergence of load-flow solution, real and reactive power losses for various types of loads is also shown in Table 3.

The proposed load-flow method is capable to handle the effect of charging capacitances of the branches. The line shunt capacitance is calculated from the line data is shown lumped at the receiving end node of the respective branch in Table B1 in Appendix-B. Table 4 compares the load-flow results with charging capacitance and without charging capacitance for constant power load only. Table 4 shows that not only the voltage magnitude of each node, has improved but also the angle of voltage of each node has improved due to the inclusion of shunt capacitance. The real and reactive power losses have been reduced when the shunt capacitances are in action.

Solution methodology

For the load-flow solution of radial distribution network either sequential numbering scheme is adopted or not, first a two-dimensional array is formed which contains in its each rows the node number of link node between two interlinked laterals and the node number of first node of the linked lateral. The arrays storing lateral node numbers are then formed. The number of lateral arrays is equal to the number of laterals present in the network. Main feeder laterals and sub-laterals all are consider as laterals. Once the lateral arrays are formed, the load-flow solution can be easily carried out.

Formation of lateral arrays

For any type of numbering scheme, sequential or non-sequential the brief procedure is defined here.

1. Read the sending end nodes and receiving end nodes.
2. Define an array JN[ ][ ] to store the junction nodes. It's a two dimensional array.
3. For every sending end node check if it appear more than once. For each repeated entry store the repeated node and its receiving end node in successive rows of the JN[ ][ ] array. The number of rows of the JN [ ][ ] array shows the number of lateral of the network.
4. Start from the substation node. Define a lateral array and put the substation node number as the first entry in the array. Check if the latest node stored in the array is the sending end node of the next branch if yes put receiving-end node of this branch as the next entry in the lateral array. Repeat it till the condition is false.
5. Define a lateral array to store the node numbers of the lateral. Put node numbers stored in the row under consideration of JN[ ][ ].

6. Check if last node in lateral array is the sending end node of the next branch. If yes, put receiving-end node of this branch it in the lateral array and repeat step 6. If no, go to step 5 and repeat the above procedure till all the rows of JN[ ][ ] are considered. The detailed procedure is defined in Fig 2. That shows the Flow Chart of above procedure.

Load-flow solution

Start from the first lateral and its first branch.

1. Consider a lateral. Compute the node current for each of its nodes after the branch under consideration.
2. While computing the node current it is checked whether the node under consideration is the source node of any other array. If yes, go to step 3. Otherwise repeat step 2 till the last node in array under consideration and go to step 4.
3. Store first two nodes of the linked array in LN[ ][ ] array and repeat step 2.
4. Consider the array that's first two nodes are stored in LN[ ][ ] and repeat steps 1-3 till all the entries of LN[ ][ ] are considered.
5. Compute voltage and its angle for the receiving end node of the branch under consideration.
5. Steps 1-4 are repeated for each branch of the network.
6. Repeat the above procedure till the convergence of the load-flow solution is occurred.
7. Print the results.

The detailed procedure of the above steps is shown in Fig 3.

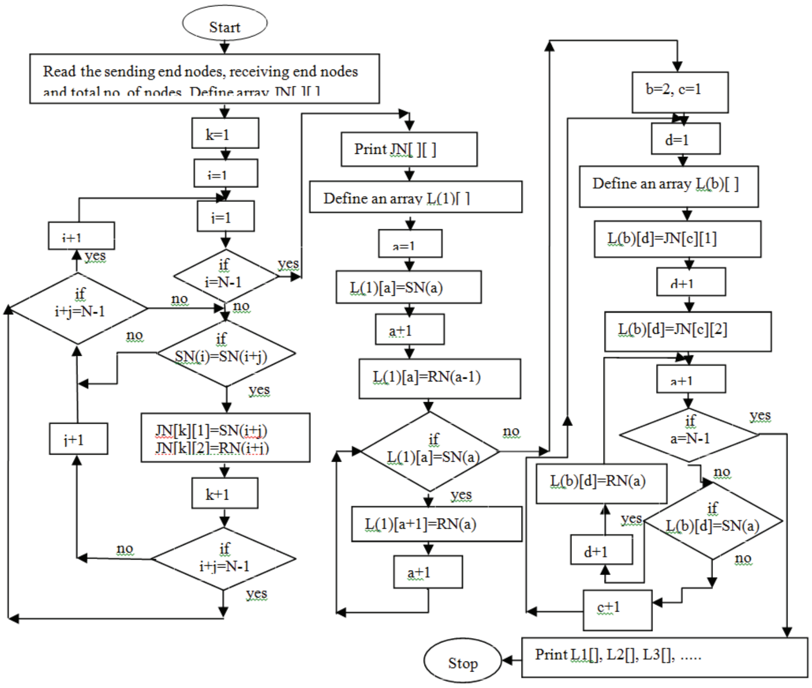

Fig 2. Flow Chart Shows algorithm to form the lateral arrays.

Table 2. The load-flow results of 33-node distribution system.

Node No.	Proposed method		Exist. Method [19].		Node No.	Proposed method		Exist. Method [19].	
	Voltage (pu)	Angle in radian	Voltage (pu)	Angle in radian		Voltage (pu)	Angle in radian	Voltage (pu)	Angle in radian
1	1.000000	0.000000	1.000000	0.000000	17	0.904391	-0.011944	0.904728	-0.011963
2	0.997015	0.000238	0.997170	0.000218	18	0.903778	-0.012115	0.904115	-0.012133
3	0.982882	0.001673	0.983040	0.001652	19	0.996486	0.000049	0.996584	0.000016
4	0.975373	0.002827	0.975566	0.002799	20	0.992909	-0.001120	0.993007	-0.001151
5	0.967946	0.003999	0.968170	0.003964	21	0.992204	-0.001458	0.992303	-0.001489
6	0.949468	0.002356	0.949754	0.002323	22	0.991567	-0.001813	0.991665	-0.001844
7	0.945943	-0.001688	0.946236	-0.001713	23	0.979297	0.001132	0.979486	0.001129
8	0.932287	-0.004367	0.932605	-0.004390	24	0.972625	-0.000417	0.972816	-0.000418
9	0.925955	-0.005667	0.926279	-0.005690	25	0.969300	-0.001179	0.969492	-0.001181
10	0.920097	-0.006789	0.920426	-0.006811	26	0.947539	0.003045	0.947870	0.003004
11	0.919229	-0.006660	0.919558	-0.006682	27	0.944974	0.004025	0.945310	0.003983
12	0.917714	-0.006455	0.918044	-0.006478	28	0.933532	0.005474	0.933883	0.005430
13	0.911538	-0.008081	0.911872	-0.008102	29	0.925312	0.006834	0.925669	0.006790
14	0.909248	-0.009482	0.909583	-0.009503	30	0.921754	0.008672	0.922113	0.008626
15	0.907821	-0.010153	0.908157	-0.010173	31	0.917592	0.007198	0.917952	0.007154
16	0.906439	-0.010568	0.906775	-0.010587	32	0.916676	0.006796	0.917037	0.006752
					33	0.916392	0.006661	0.916754	0.006617

Power losses using the proposed method		Power losses using the existing method [19]	
Active power losses	Reactive power losses	Active power losses	Reactive power losses
205.25kW	**139.31kVAr**	**205.19kW**	**139.23kVAr**

The ratio of CPU time of proposed the method with the existing method [19] is 1: 1.25.
The ratio of memory requirement of proposed the method with the existing method [19] is 1: 1.98.

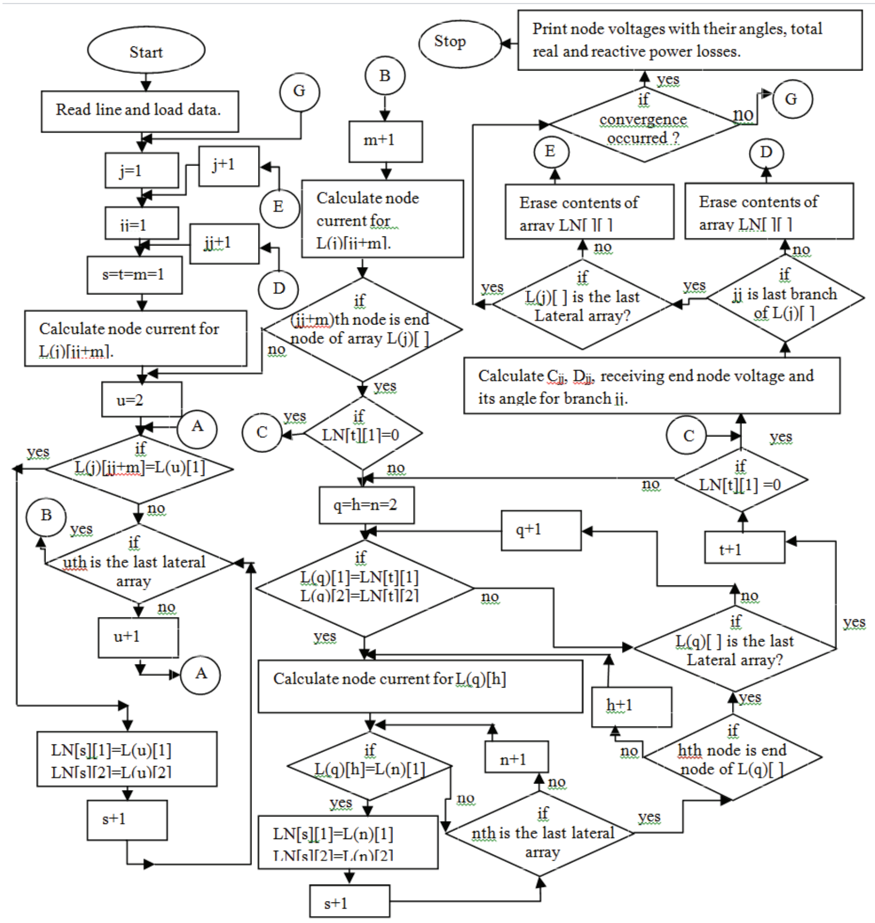

Fig. 3. Flow Chart for load-flow

Table 3. Load-Flow results for various types of loads

THE REA(	Load model	No. of iterations	Real power losses in kW	Reactive power losses in kVAr	Min. voltage of the network at node no.13pu	Angle of voltage at node no.13 in radians
1	Battery charge	4	172.8015	192.3338	0.92595	-0.01489
2	Fluorescent lamps	4	173.3655	192.9465	0.92338	-0.01485
3	Constant impedance	3	173.8920	193.5195	0.92107	-0.01275
4	Air conditioner	3	174.6066	194.2929	091765	-0.01909
5	Constant current	3	174.9854	194.7066	0.91610	-0.01343
6	Resistance space heater	3	174.8633	194.5766	0.91681	-0.00821
7	Pumps, fans other motors	3	175.3406	195.0899	0.91435	-0.01769
8	Incandescent lamps	3	175.1611	194.8993	0.91542	-0.00945
9	Compact fluorescent lamps	3	175.3353	195.0871	0.91457	-0.01189
10	Small industrial motors	3	175.8709	195.6667	0.91203	-0.01539
11	Large industrial motors	4	175.9700	195.7743	0.91159	-0.01532
12	Constant power	4	176.3188	196.1534	0.91005	-0.01414
13	Composite load*	3	174.8930	194.6062	0.91651	-0.01337
14	Practical load**	3	175.1937	194.9317	0.91509	-0.01503

Table 4. Load-flow results of 41-node system by considering the charging capacitance of the branches and without considering the charging capacitance.

Node No.	Results when capacitance is considered.		Results when capacitance is not considered.		Node No.	Results when capacitance is considered.		Results when capacitance is not considered.	
	Voltage pu	Angle (rad.)	Voltage pu	Angle (rad.)		Voltage pu	Angle (rad.)	Voltage pu	Angle (rad.)
1	1.00000	0.00000	1.00000	0.00000	12	0.93396	-0.00964	0.93390	-0.00969
27	0.98922	-0.00529	0.98920	-0.00530	15	0.98350	-0.00610	0.98347	-0.00611
35	0.97331	-0.00856	0.97327	-0.00859	30	0.98118	-0.00641	0.98115	-0.00643
8	0.96016	-0.01028	0.96009	-0.01034	29	0.97553	-0.00612	0.97549	-0.00615
33	0.95109	-0.01150	0.95100	-0.01157	41	0.97153	-0.00496	0.97149	-0.00500
20	0.94391	-0.01247	0.94381	-0.01255	28	0.96910	-0.00426	0.96906	-0.00430
39	0.93622	-0.01353	0.93611	-0.01362	2	0.96645	-0.00320	0.96640	-0.00324
4	0.93001	-0.01439	0.92989	-0.01449	26	0.96878	-0.00832	0.96872	-0.00836
34	0.92620	-0.01493	0.92607	-0.01504	24	0.96339	-0.00675	0.96334	-0.00680

32	0.92004	-0.01580	0.91990	-0.01592	23	0.95980	-0.00531	0.95975	-0.00537
16	0.91437	-0.01549	0.91422	-0.01562	37	0.95849	-0.00960	0.95841	-0.00966
21	0.91194	-0.01474	0.91179	-0.01487	10	0.94733	-0.00998	0.94724	-0.01006
17	0.91020	-0.01400	0.91005	-0.01414	5	0.94487	-0.00897	0.94478	-0.00906
18	0.97588	-0.00717	0.97585	-0.00719	9	0.94342	-0.00837	0.94333	-0.00847
22	0.96598	-0.00848	0.96594	-0.00851	3	0.93487	-0.01297	0.93476	-0.01307
11	0.95570	-0.00987	0.95565	-0.00991	36	0.91820	-0.01504	0.91806	-0.01516
13	0.95051	-0.01058	0.95046	-0.01062	31	0.91138	-0.01424	0.91123	-0.01437
14	0.94449	-0.01142	0.94444	-0.01146	38	0.91052	-0.01414	0.91037	-0.01427
40	0.94037	-0.01120	0.94031	-0.01125	7	0.94084	-0.00987	0.94078	-0.00993
6	0.93737	-0.01104	0.93730	-0.01109	19	0.97912	-0.00556	0.97909	-0.00559
					25	0.96754	-0.00783	0.96749	-0.00788

When capacitance is considered. Total real power loss = **176.28550 kW**
Total reactive power loss = **196.11720 kVAr**

When capacitance is not considered. Total real power loss =**176.3188kW**
Total reactive power loss =**196.1534kVAr**

Conclusion

In this paper an efficient load-flow method is proposed. The formulation of the method is capable of considering the effect of charging capacitance of the branches and the load-flow solution of the system can be obtained without considering the effect of charging capacitance. It is a forward sweep method in which voltage drop in each branch is computed at a voltage then the voltage of each node is computed with the recent values of voltage drop, so the convergence is conformed in this method. The solution algorithm do not needs the numbering of the nodes in sequential order. Convergence of the method is shown with different types of the load. To show the reliability of the method the load flow results are compared with the existing methods for 33-node distribution systems. The CPU time and the memory requirement is less for the proposed method.

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Appendix-A

Proof of convergence: It is mathematically established in this section of the paper that the formulation proposed in section 3 has a surety of convergence. This is because the voltage that is being calculated in the ongoing iteration depends upon the voltage and the voltage drop calculated in the previous iteration. The voltage drop in any branch is also a function of the previous iteration voltage. Fig. A1 graphically shows the convergence of the load-flow solution.

From Eq.(14) and Eq. (15)

$$\left(V_{k+1}\cos\delta_{k+1}+C_{jj}\right)^2+\left(V_{k+1}\sin\delta_{k+1}+D_{jj}\right)^2= \left(V_k\cos\delta_k\right)^2+\left(V_k\sin\delta_k\right)^2 \tag{A1}$$

By rearranging Eq. (A1)

$$V_{k+1}^2+2V_{k+1}(C_{jj}\cos\delta_{k+1}+D_{jj}\sin\delta_{k+1})+(C_{jj}^{\ 2}+D_{jj}^2)=V_k^2 \tag{A2}$$

Let $V_{k+1}^2+\Delta V_{jj}=V_k^2$ (A3)

$$V_{k+1}^2\left(1+\frac{\Delta V_{jj}}{V_{k+1}^2}\right)=V_k^2 \tag{A4}$$

$$V_{k+1}\left(1+\frac{\Delta V_{jj}}{V_{k+1}^{2}}\right)^{\frac{1}{2}}=V_k \qquad \text{(A5)}$$

$$V_{k+1}\left(1+\frac{\Delta V_{jj}}{2V_{k+1}^{2}}\right)=V_k \qquad \text{(A6)}$$

$$V_{k+1}=V_k-\frac{\Delta V_{jj}}{2V_{k+1}} \qquad \text{(A7)}$$

$$V_{k+1}=V_k-\frac{\Delta \mathrm{v}_{jj}}{V_{k+1}} \qquad \text{(A8)}$$

Here

$$\Delta \mathrm{v}_{jj}=\frac{V_{jj}}{2}$$

$$=V_{k+1}(C_{jj}\cos\delta_{k+1}+D_{jj}\sin\delta_{k+1})+\frac{1}{2}(C_{jj}{}^{2}+D_{jj}^{2}) \qquad \text{(A9)}$$

$$\frac{\Delta \mathrm{v}_{jj}}{V_{k+1}}=C_{jj}\cos\delta_{k+1}+D_{jj}\sin\delta_{k+1}+\frac{C_{jj}{}^{2}+D_{jj}^{2}}{2V_{k+1}} \qquad \text{(A11)}$$

From Eq. (A8) and Eq. (A11)

$$V_{k+1}=V_k-(C_{jj}\cos\delta_{k+1}+D_{jj}\sin\delta_{k+1})-\left(\frac{C_{jj}{}^{2}+D_{jj}^{2}}{2V_{k+1}}\right) \qquad \text{(A12)}$$

C_{jj} and D_{jj} has link with V_{k+1} and the nodes beyond the node under consideration. Hence convergence of V_{k+1} gives assurance the convergence of the V's of the other nodes beyond this node. In this case the solving of V_{k+1}, the present value of V_k and the past value of V's of m=k+1th node and nodes beyond this are used.

Let

$$g(V_{k+1})=V_k-(C_{jj}\cos\delta_{k+1}+D_{jj}\sin\delta_{k+1})-\left(\frac{C_{jj}{}^{2}+D_{jj}^{2}}{2V_{k+1}}\right) \qquad \text{(A13)}$$

Hence C_{jj} and D_{jj} are functions of $f\left(\frac{1}{V_m}\right)$ where m=k+1, k+2, ………

In the above graph shown in Fig. A1 $V_{k+1}(0)$ is the initial value of V_{k+1}. $V_{k+1}(n)$ and $V_{k+1}(n+1)$ are the value of V_{k+1} at the nth and n+1th iteration respectively ξ_{k+1} is the final value of V_{k+1}.

Here $\varepsilon_{k+1}(n)=V_{k+1}(n)-\xi_{k+1}$ (A14)

and $\varepsilon_{k+1}(n+1)=V_{k+1}(n+1)-\xi_{k+1}$ (A15)

$V_{k+1}(n)=\varepsilon_{k+1}(n)+\xi_{k+1}$ (A16)

And $V_{k+1}(n+1)=\varepsilon_{k+1}(n+1)+\xi_{k+1}$ (A17)

$$\mathrm{V}_{k+1}(1)=\mathrm{g}\{\mathrm{V}_{k+1}(0)\} \qquad \text{(A18)}$$

$$\mathrm{V}_{k+1}(2)=\mathrm{g}\{\mathrm{V}_{k+1}(1)\} \qquad \text{(A19)}$$

$$\mathrm{V}_{k+1}(3)=\mathrm{g}\{\mathrm{V}_{k+1}(2)\} \qquad \text{(A20)}$$

:

:

$$\mathrm{V}_{k+1}(n+1)=\mathrm{g}\{\mathrm{V}_{k+1}(n)\} \qquad \text{(A21)}$$

Table b1. Line data and load data for 41-node distribution system

$$\xi_{k+1}+\varepsilon_{k+1}(n+1)=\mathrm{g}\{\xi_{k+1}+\varepsilon_{k+1}(n)\}=$$

$$\xi_{k+1}+\varepsilon_{k+1}(n)\mathrm{g}'\{\xi_{k+1}\}+\frac{1}{2!}\varepsilon^{2}{}_{k+1}(n)\mathrm{g}''\{\xi_{k+1}\}+\text{--}$$

(A22)

$$\mathrm{g}'\{\xi_{k+1}\}=\frac{\Delta \mathrm{v}_{jj}}{V_{k+1}^{2}} \qquad \text{(A23)}$$

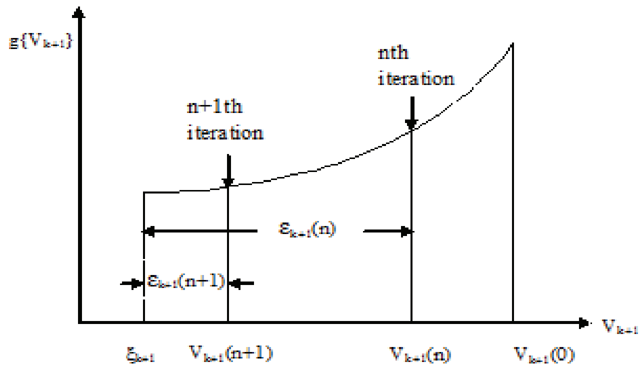

Fig. A1 Plot of $g\{V_{k+1}\}$ v/s V_{k+1}

$$\mathrm{g}''\{\xi_{k+1}\}=\frac{-2\Delta \mathrm{v}_{jj}}{V_{k+1}^{3}} \qquad \text{(A24)}$$

For $\mathrm{V}_{k+1}=\xi_{k+1}$, (A25)

$$\mathrm{g}'\{\xi_{k+1}\}=\frac{\Delta \mathrm{v}_{jj}}{\xi_{k+1}^{2}} \qquad \text{(A26)}$$

$$\mathrm{g}''\{\xi_{k+1}\}=\frac{-2\Delta \mathrm{v}_{jj}}{\xi_{k+1}^{3}} \qquad \text{(A27)}$$

So $\varepsilon_{k+1}(n+1)=\varepsilon_{k+1}(n)\left(1-\frac{\varepsilon_{k+1}(n)}{\xi_{k+1}}\right)\frac{\Delta \mathrm{v}_{jj}}{\xi_{k+1}^{2}}$

$$=\varepsilon_{k+1}(n)\left(1-\frac{\varepsilon_{k+1}(n)}{\xi_{k+1}}\right)g'\{\xi_{k+1}\} \qquad \text{(A28)}$$

Since $\frac{\varepsilon_{k+1}(n)}{\xi_{k+1}}$ can be neglected,

$$\varepsilon_{k+1}(n+1)=\varepsilon_{k+1}(n)g'\{\xi_{k+1}\} \qquad \text{(A29)}$$

Since the $\sin\delta_{k+1}$ and $\cos\delta_{k+1}$ are approximately equal to 0 and 1. $\Delta \mathrm{v}_{jj}$ becomes $\Delta \mathrm{v}_{jj}=V_{k+1}C_{jj}+\frac{1}{2}\left(C_{jj}^{2}+D_{jj}^{2}\right)$. Which is function of R, X, P and Q. Hence the convergence of the proposed method becomes linear in this case and $\Delta \mathrm{v}_{jj}$ becomes very small and hence $\mathrm{g}'\{\xi_{k+1}\}$ less than 1. This gives the guarantee of the convergence of the proposed method.

Appendix-B

Fig. B1. Shows Single line diagram of 41-node distribution system. Line and node data for 41-node distribution is given in Table B1. The technical data regarding the conductor used and spacing between the line conductors is shown in Table B2.

(A21)

S N	R N	Cond Type	Len. in km	kVA load on RN	Cap. at RN in µF	SN	RN	Cond Type	Len. in km	kVA load on RN	Cap. at RN in µF
1	27	6	1	175	0.005153	27	15	4	1.5	150	0.006857
27	35	5	1.87	185	0.008786	15	30	4	0.73	165	0.003337
35	8	4	1.76	60	0.008046	30	29	3	2.10	150	0.009250
8	33	4	1.34	165	0.006126	29	41	2	1.39	80	0.005848
33	20	4	1.41	85	0.006446	41	28	2	1.11	113	0.004670
20	39	4	1.63	95	0.007452	28	2	1	1.66	140	0.006743
39	4	4	1.58	90	0.007223	35	26	3	1.5	155	0.006607
4	34	4	1.08	85	0.004937	26	24	2	1.88	163	0.007910
34	32	4	1.96	85	0.008960	24	23	1	1.88	166	0.007637
32	16	3	1.87	90	0.008237	08	37	1	1.46	100	0.005931
16	21	2	1.08	73	0.004544	33	10	1	1.5	75	0.006093
21	17	1	1.75	82	0.007109	10	5	1	1.5	75	0.006093
27	18	4	1.9	155	0.008686	5	9	1	1.91	65	0.007758
18	22	4	1.56	175	0.007132	39	3	1	1.43	80	0.005809
22	11	4	1.84	158	0.008412	32	36	1	1.7	90	0.006905
11	13	4	1.06	155	0.004846	16	31	1	1.4	176	0.005687
13	14	4	1.43	196	0.006537	21	38	1	1.3	90	0.005281
14	40	3	1.22	170	0.005133	14	7	1	1.9	163	0.007718
40	6	3	1.25	195	0.005259	30	19	1	2.02	90	0.008205
6	12	1	1.31	220	0.005321	26	25	1	1.96	55	0.007962

Table b2. Technical data of the conductors used in 41-node distribution system

Cond. type.	Code name.	Dia. Of cond. (mm)	Area of X-section.(mm^2) Nominal Copper area Sq. mm.	Resistance (ohm/km)	Reactance (ohm/km)	Max. Current carrying capacity (amp.) at 45° C ambient temp.
1	Squirrel	6.33	12.90	1.3760	0.3896	107
2	Weasel	7.77	19.35	0.9810	0.3797	139
3	Rabbit	10.05	32.26	0.5441	0.3673	193
4	Raccoon	12.27	48.46	0.3657	0.3579	250
5	Dog	14.15	65	0.2745	0.3112	300
6	Lion	22.26	140	0.1223	0.2446	515

The spacing between the two conductors is 1.2 meter. System voltage is 11kV. Power factor is 0.8. Base voltage = 11kV, base power = 100MVA.

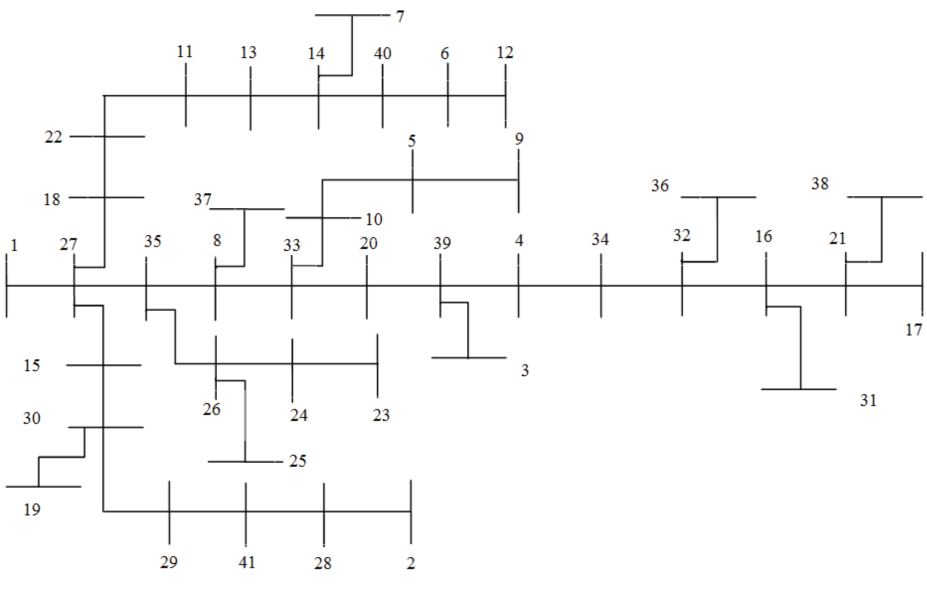

Fig. B 1. Single line diagram of 41-node distribution system

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