Why $\alpha$-activity diagrams, $\alpha$-nets and s-nets are good models of production processes?

Abstract. The paper proposes combined use of UML and Petri nets to model production processes which can be useful for designers of production processes in any area of industry. Modeling, analysis and verification of a wide range of parallel processes is important for the correct production process which should be cyclic. The formal verification allows to check whether the process is well-formed. The paper describes why transformation of $\alpha$-activity diagram into $\alpha$-net is useful to modeling of production processes. Applying s-nets to modeling of such processes is also discussed.

Introduction

Already in the 1980ths Petri nets [1] were used for modeling of production processes [2]. A wide range of Petri net classes defined since 1970ths [3][4] allow to model, analyze and verify various event systems, parallel and sequential. UML modeling language, since it has become developed enough, is also used for modeling of production processes [5]. Experience of the engineers has shown that analysis of such processes requires object approach to the problem. On the other hand, formal methods of analysis are not developed well for the object specifications of real processes. That created the motivation for works trying to integrate object vision and mathematical verification [6][7].

It is essential for using UML language [8] that it often allows to obtain better results by using a reasonable subset of its elements than by exaggerate using of all available ones. That is the reason of focusing the research on the activity diagrams. It is worth noting that activity diagrams of UML version 2.0 are based on Petri nets. The changes in comparison with the previous version are remarkable. In the previous UML versions (UML 1.x) the activity diagrams were a kind of state machines. In version UML 1.4 possibility of synchronization of parallel processes was added. In version 2.0 the state diagrams (essentially the statecharts) and the activity diagrams are two different kinds of behavior diagrams with different functions.

An essential reason to use UML for modeling of production processes is the fact that this language is accepted by the industry because of its clarity. UML allows easy communication and understanding between all members of a project team independently of their specialization. Other formal models, such as Petri nets in their traditional form, are usually rejected by industrial engineers.

Activity diagrams from UML 2.0 provide a convenient and unambiguous way to present sequential and parallel flows of control and work. Since UML activity diagrams are based on Petri nets, they have become an adequate language for modeling the production processes in Petri net style. We use a restricted subset of the elements of activity diagrams for modeling of production processes; the diagrams constructed from such elements we call the $\alpha$-activity diagrams. For verification of the modeled processes we use two classes of Petri nets: $\alpha$-nets and s-nets. Structure of proposed representations reflects structure of the real-world production processes.

Keywords: production processes, $\alpha$-activity diagrams, s – nets, $\alpha$ - nets

Preliminaries: production processes, $\alpha$-activity diagrams, s – nets, $\alpha$ - nets

The considered production processes belong to the class of working and assembling processes to which finite sets of the machine tools, devices and resources are involved. It is supposed that every machine tool cannot execute several operations at the same time (it has to finish previous operation before starting the next one). When a process is initiated, all necessary materials and elements are taken from a magazine.

$\alpha$-activity diagram is a UML activity diagram consisting only of the elements which kinds are presented in table 1. Petri net is a tuple $PN = (P, T, F, M_0)$, where $P$ is a set of places, $T$ is a set of transitions, $F$ is a flow relation, $M_0$ is an initial marking, and the following conditions are kept:

a) $P \cap T = \emptyset$,
b) $F \subseteq (P \times T) \cup (T \times P)$,
c) $\text{In}(F) \cup \text{Out}(F) = P \cup T$, where $\text{In}(F) = \{ x \mid \exists y : (x,y) \in F \}$, $\text{Out}(F) = \{ x \mid \exists y : (y,x) \in F \}$,
d) $M_0: P \rightarrow N \{0\}$.

Petri net can be represented as a bipartite oriented graph having two kinds of nodes: places $(P)$ and transitions $(T)$. Detailed definitions of the Petri nets and related notions see in [4].

Petri net with single token in its initial marking is called an s-net [9].

Petri net is called an $\alpha$-net, if it has single token in the initial marking and any two its places having the same output transition have equal sets of output transitions [10]: $\forall p_1, p_2 \in P : (t \in \text{Out}(p_1) \Rightarrow t \in \text{Out}(p_2)) \Rightarrow (\text{Out}(p_1) = \text{Out}(p_2))$. $\alpha$-nets are a subclass of s-nets.

The following behavioral properties of Petri nets are important for verification of the modeled production processes:

- A net is live, if for every transition from every reachable marking a marking is reachable in which this transition can fire.
- A net is safe if in every reachable marking no place contains more than one token.
- A net is repetitive if the initial marking is reachable from every reachable marking.
- A net is well-formed if it is live, safe and repetitive.
Mapping between \( \alpha \)-activity diagrams and \( \alpha \) or \( s \) – nets

\( \alpha \)-activity diagrams are mapped (figure 1) into the Petri nets belonging to the class of \( \alpha \)-nets because of their structural similarity. Backward transformation is from \( \alpha \)- or \( s \)-net to an \( \alpha \)-activity diagram. Limitation of the used elements of UML activity diagrams to the elements listed in table 1 is motivated by our intention to provide maximal similarity to the \( \alpha \)-nets (table 2). When such restrictions are kept, dual transformation (table 3) can be automated. On the other hand, selected elements allow specifying the production processes in detailed and adequate way.

As far as production processes can be modeled by \( \alpha \)-activity diagrams, and such diagrams can be transformed into \( \alpha \)-nets, methods of Petri net analysis can be used for verification of production processes. Structural similarity of \( \alpha \)- and \( s \)-nets (table 2) to the production processes is caused first of all by the fact that the considered processes start from single initial step, identified as taking the materials from a magazine, which corresponds to single initially marked place of an \( \alpha \)- or \( s \)-net. It is not reasonable to use for modeling the considered processes by the nets with several tokens in initial marking, because it would mean that a process starts from several concurrent threads.

![Figure 1. The field of the mapping](image)

Table 1 Grammar of \( \alpha \)-activity diagram

<table>
<thead>
<tr>
<th>Components of ( \alpha )-activity diagrams</th>
<th>Graphical presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Action</td>
</tr>
<tr>
<td>Activity</td>
<td>Activity</td>
</tr>
<tr>
<td>Flow</td>
<td></td>
</tr>
<tr>
<td>Initial node</td>
<td></td>
</tr>
<tr>
<td>Activity final</td>
<td></td>
</tr>
<tr>
<td>Flow final</td>
<td></td>
</tr>
<tr>
<td>Fork node / Join node</td>
<td></td>
</tr>
<tr>
<td>Decision node / Merge node</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Grammar of Petri nets

<table>
<thead>
<tr>
<th>Components of ( \alpha )-nets / ( s )-nets</th>
<th>Graphical presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place</td>
<td></td>
</tr>
<tr>
<td>Transition</td>
<td></td>
</tr>
<tr>
<td>Arc</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Correspondence between elements of \( \alpha \)-activity diagrams and elements of \( \alpha \)- or \( s \)-nets

![Diagram showing correspondence between elements](image)

If modeling Petri net is not a live and safe (well-formed) \( \alpha \)- or \( s \)-net, then the process is not designed (or modeled)
properly. It means that parallel threads are not implemented correctly or do not interact correctly. Hence structure of the production processes can be verified by means of checking well-formeness of the modeling Petri nets. If behavioral analysis shows that a net is not well-formed (for example, not reversible or can be deadlocked), then the net can be modified to provide its correctness, and backward transformation to an activity diagram allows to correct mistakes in the structure of the production process.

α-activity diagrams are mapped (figure 1) into the Petri nets belonging to the class of α-nets because of their structural similarity. Backward transformation is from α- or s-net to an α-activity diagram. Limitation of the used elements of UML activity diagrams to the elements listed in table 1 is motivated by our intension to provide maximal similarity to the α-nets. When such restrictions are kept, dual transformation can be automated. On the other hand, selected elements allow specifying the production processes in detailed and adequate way.

As far as production processes can be modeled by α-activity diagrams, and such diagrams can be transformed into α-nets, methods of Petri net analysis can be used for verification of production processes. Structural similarity of α- and s-nets to the production processes is caused first of all by the fact that the considered processes start from single initial step, identified as taking the materials from a magazine, which corresponds to single initially marked place of an α- or s-net. It is not reasonable to use for modeling the considered processes by the nets with several tokens in initial marking, because it would mean that a process starts from several concurrent threads. If modeling Petri net is not a live and safe (well-formed) α- or s-net, then the process is not designed (or modeled) properly. It means that parallel threads are not implemented correctly or do not interact correctly. Hence structure of the production processes can be verified by means of checking well-formeness of the modeling Petri nets. If behavioral analysis shows that a net is not well-formed (for example, not reversible or can be deadlocked), then the net can be modified to provide its correctness, and backward transformation to an activity diagram allows to correct mistakes in the structure of the production process.

**Design of production processes**

Main model-related steps of design of production processes are the following ones: modeling [11][12], verification [13][14][15], modification [13][15], variation [14]. The step of modeling consists of the following sub-steps:

a) model the production process applying the proposed methodology [12][13][15], using the technical documentation;

b) model the alternative variants of implementation of the designed production process [11][15];

c) verify correspondence of the created models to the technical documentation;

d) select the optimal variant among the designed models.

At different steps of process design alternative variants of its structure can be considered. Of course all such variants should correspond to the technical documentation and must be formally correct. Selection of the final variant of the process among several considered variants is performed by the board of managers of the company responsible for the process in cooperation with the project team. Main parameters taken into account during such selection are costs of the production, time consuming and availability of the machine tools and human resources.

Creating a model, we should suppose that some processes and sub-processes will be implemented in parallel way.

![Fig.2. Steps of modeling of a production process](image)

Fig.2. Steps of modeling of a production process

Which part of a process can be implemented in parallel, depends on many conditions, such as technological recommendations, availability of machines and human resources. Taking into account conditions for every operation, defining proper ordering of the technological actions, including assembling of the elements and subsystems, belong to the tasks of the engineer who models the production process using UML. Lack of reasonable ordering and chronology of the operations belonging to a designed process can lead to mistakes in realization of the process and makes impossible obtaining a good with needed properties. Technical specification contains information on which modeling of the production process should be performed. Such specification contains complete description of the technological process presented with the help of technical drawings and additional information.

Technical description of the process is transformed into an object form presented by means of an α-activity diagram, which is easily understandable for all members of the design team. Then the activity diagram is transformed into an α-net, to make possible its formal verification.
If verification of the production process demonstrates some mistakes in its structure, then it is modified. To a well-formed $\alpha$-net, representing the production process according to its technical specification, the intermediate quality control steps are introduced. A process, which is structurally correct and is modeled by a net belonging to the class of $\xi$-nets, is transformed backward to an $\alpha$-activity diagram. Block diagram of modeling of a production process is presented in figure 2. Realization of a production process can be modified if the design team decide to perform its variation. Variation of a process can be performed when it is verified that the modeling $\alpha$-net or $\xi$-net is well-formed, i.e. before or after implementing the intermediate quality control steps. Moment in which a process is modified depends on the kind of the modifications. As far as proposed design methodology joins independent areas of knowledge (mechanics and computer science), it is necessary to eliminate possible structural mistakes which can appear when the operations are projected at different abstraction levels.

**Conclusion**

Formal and object methods of modeling are constantly developing; however methods integrating both approaches still virtually do not exist. The proposed method of modeling the production processes allows filling a lack in the considered area by integration of an object language of description of reality, which is UML, with simple formal methods allowing reducing and verifying modeled systems by using the selected classes of Petri nets.

The author is a scholar within Sub-measure 8.2.2 Regional Innovation Strategies, Measure 8.2 Transfer of knowledge, Priority VIII Regional human resources for the economy Human Capital Operational Programme co-financed by European Social Fund and state budget.

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