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Determination of parameters to define a resultant inertial moment of the drive system for polymerization reactor based on asynchronous induction motor with a pipe body

***Abstract**. In the paper the problems concerning the operation of the drive system for a polymerization reactor based on asynchronous induction motor with a pipe body are presented. The range of the presented problems is a resultant inertial moment of the system determined by specificity of the operation of a polymerization reactor. A simplified kinematic structure of the drive system and a calculation kinematic diagram are presented. Formulas for calculation of the resultant inertial moment of the drive system are given.*

***Streszczenie**. W artykule przedstawiono zagadnienia dotyczące pracy układu napędowego reaktora polimeryzacji z asynchronicznym silnikiem indukcyjnym w wykonaniu rurowym w zakresie wypadkowego momentu bezwładności układu wynikającego ze specyfiki pracy reaktora polimeryzacji. Przedstawiono uproszczony schemat kinematyczny układu napędowego oraz obliczeniowy schemat kinematyczny w odniesieniu do którego przedstawiono wzory dotyczące wyznaczania wypadkowego momentu bezwładności układu napędowego. (**Wyznaczanie parametrów do określenia wypadkowego momentu bezwładności układu napędowego reaktora polimeryzacji z asynchronicznym silnikiem indukcyjnym w wykonaniu rurowym**).*

Keywords: polymerization, induction motor, kinematics, inertial moment.
Słowa kluczowe: polimeryzacja, silnik indukcyjny, kinematyka, moment bezwładności.

Introduction

There are phenomena during operation of the drive system for a polymerization reactor exceeding an operation of the standard drive systems. There are examples of the abovementioned phenomena having a direct influence on the operation of the drive system in the rage of its load considering various phases of the polymerization process:
- agglutination of rotor and stator of the asynchronous motor from the driving side as a result of polymerization in this area of a polymerization reactor chamber
- sliding friction in the large-size slide bearing made of sintered carbides an cooled with ethylene stream via stall fences of a cooling set in the lower cover of the motor
- filling with polyethylene the construction of the mixer in the lower part of a polymerization reactor chamber
- sliding friction of the mixer filled with polyethylene in the area of the filled mixing chamber of a polymerization reactor
- mixing the ethylene stream by mixer in the lower chamber of a polymerization reactor in the part uncovered by polyethylene or for the work of the drive system with the unfilled mixer.

The drive system for a polymerization reactor is the particular example of drive systems because a change of the resultant inertial moment during operation of the drive system is a very rare example among various drive systems. The change of an inertial moment has direct influence on transient responses of the system. The transient responses of the specially designed drive systems (systems having high operational reliability) are analyzed in detail in order to determine foredesigning assumptions for these systems [1, 2].

Calculation oriented kinematic structure of the drive system

The simplified kinematic structure of the drive system for the polymerization reactor based on asynchronous induction motor with a pipe body is presented in Fig. 1.

The calculation oriented kinematic structure of the drive system is determined (Fig. 2) as a result of consideration of the simplified kinematic structure of the exemplary drive system for a polymerization reactor based on the asynchronous induction motor with a pipe body (Fig. 1).

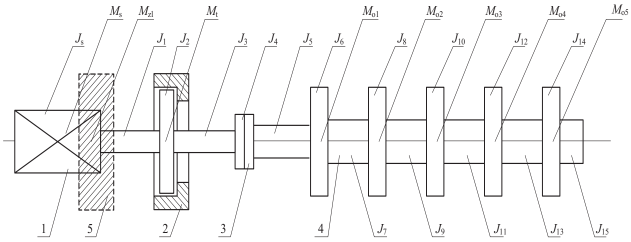

Fig. 1 The simplified kinematic structure of the drive system, where: 1 is motor, 2 is set of large-size slide bearing, 3 is clutch of mixer, 4 is mixer, 5 is area of occurrence of agglutination torque concerning stator and rotor of the motor

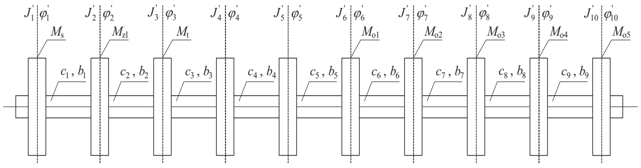

Fig. 2 The calculation oriented kinematic structure of the drive system for polymerization reactor based on induction motor with a pipe body

The mixer working vertically in the lower part of a polymerization reactor chamber is a main operating element of the drive system for a polymerization reactor. The cross-sections of mixers have various shapes depending on technological requirements. The area components of a mixer cross-section of the drive system for polymerization reactor based on the prototypical specially designed asynchronous motor with a pipe body SAR-55/1500/09 are shown in Fig. 3 [1, 2].

The area components of a mixer cross-section (Fig. 3) may be determined assuming various approximations. The area component ϑ_m'' of a mixer cross-section (Fig. 3) may be determined as follows:
- by determining values of arcs: Ł_{max}, Ł_{min} and $\text{Ł}_{śr}$ in accordance with the assumed description (Fig. 4 - Example P1)
- by determining values of radiuses: r_{min} and r_{max} in accordance with the assumed description (Fig. 4 - Example P2)

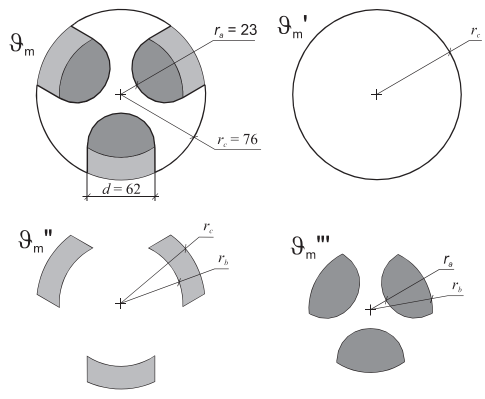

Fig. 3 The area components of a mixer cross-section of the drive system for a polymerization reactor based on the prototypical specially designed asynchronous motor with a pipe body SAR-55/1500/09 [1, 2]

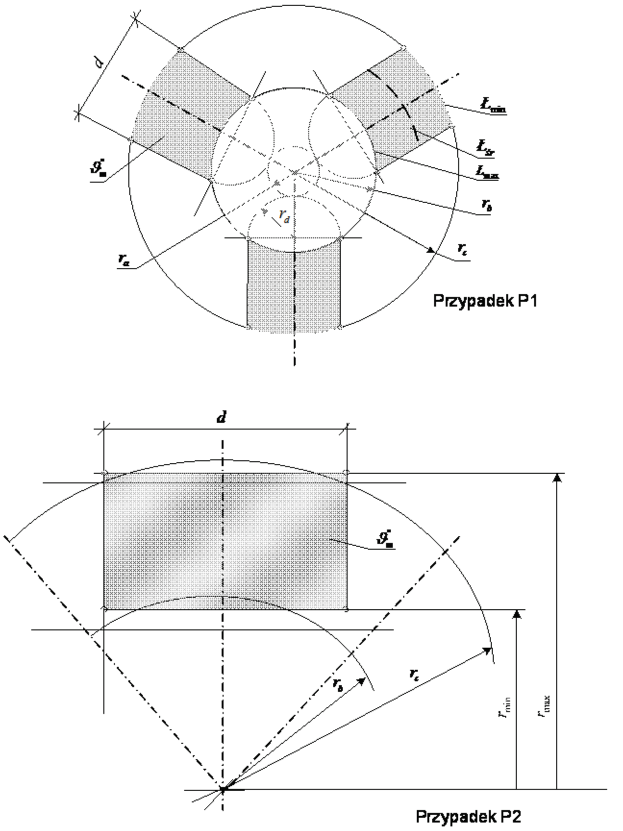

Fig. 4 The auxiliary figure for determination of the area of the segment ϑ_m'' for the examples P1 and P2

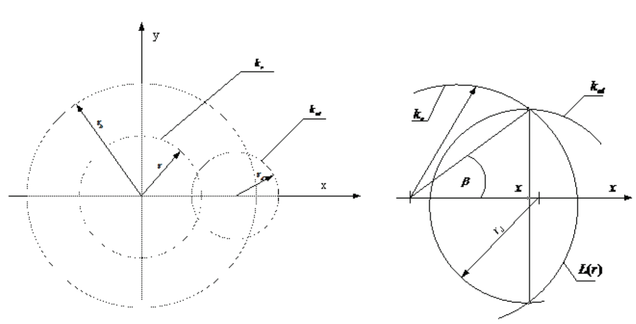

Fig. 5 The auxiliary figure for determination of the length of the arc $Ł(r)$ in order to determine the area of the segment ϑ_m'' (Fig. 3)

Coordinates of the intersection points of the circles K_r and K_{rd} with the radiuses r and r_d (Fig. 5) are solution of the system of equations (1):

$$(1)\qquad x^2 + y^2 = r^2\,, \qquad (x - r_a - r_d)^2 + y^2 = r_d^2$$

Determination of the coordinate of the intersection points of the circles K_r and K_{rd} on the axis x is sufficient in order to determine the length of arc on the basis of the system of equations (1):

$$(2)\qquad x = \frac{r^2 + r_a(r_a + 2r_d)}{2(r_a + r_d)} = \frac{r^2 + r_a(r_a + d)}{2r_a + d}$$

$$\L(r) = 2\pi r\frac{2\beta}{2\pi} = 2\beta r\,, \qquad \beta = \arccos\frac{x}{r}$$

The area components required to determine the inertial moment may be calculated with the accuracy depending on the assumed simplifications concerning their shapes. The components of mixer inertial moment are given as follows:

$$(3)\qquad \vartheta_{m_1}' = 0{,}5\pi l_m\rho_1\left(\frac{d_m}{2}\right)^4, \qquad \vartheta_{m_1}'' = 6l_m\rho_1\int_{r_b}^{r_c} r^3\alpha(r)dr$$

$$\vartheta_{m_1}''' = 3l_m\rho_1\int_{r_a}^{r_b} r^2\L(r)dr$$

where: d_m is external diameter of the mixer, l_m is length of the mixer.

Determination of parameters

Inertial moments of the drive system for a polymerization reactor are functions of several variables as a result of the polymerization process. The polymerization process causes variation of the abovementioned moments during the process as a result of filling the respective considered parts of the mixer by polyethylene. Inertial moments of the mixer differ from inertial moments of mixer fillings due to the different areas considered during determination of the inertial moments and different values of mass density concerning materials of the mixer and mixer fillings [3, 4].

The dependencies (3) may be simplified considering assumptions for the examples P1 and P2, respectively (Fig. 4). The simplified form of dependencies (3) are given as follows:

$$(4)\qquad \vartheta_{m_{1P1}}''' = 3l_m\rho_1\int_0^{r_b}\L(r)dr\,, \qquad \vartheta_{m_{1P1}}'' = 3l_m\rho_1\L_{śr}\int_{r_b}^{r_c} r^2dr$$

$$\vartheta_{m_{1P2}}''' = 6l_m\rho_1\gamma\int_0^{r_b} r^3dr$$

$$\vartheta_{m_{1P2}}'' = 2l_m\rho_1 r_d^3(r_{\max} - r_{\min}) + l_m\rho_1 d\left(r_{\max}^3 - r_{\min}^3\right)$$

The auxiliary quantities for calculation of the variables given by the dependencies (3) and (4) are as follows:

$$(5)\qquad r_{\max} = 0{,}5\left(\sqrt{r_c^2 - r_d^2} + r_c\right)$$

$$r_{\min} = 0{,}5\left(r_a + r_d + \sqrt{(r_a + r_d)^2 + r_d^2}\right)$$

$$\text{Ł}_{\text{śr}} = 0{,}5\left(\text{Ł}_{\min} + \text{Ł}_{\max}\right),\ \text{Ł}_{\min} = 2\alpha r_c\ ,\ \text{Ł}_{\max} = 2\alpha_1 r_b$$

$$\text{Ł}(r) = 2r\gamma\ ,\ \gamma = \arccos\frac{r_d}{r_b}$$

$$\alpha(r) = \arcsin\frac{d}{2r}\ ,\ \alpha = \arcsin\frac{r_d}{r_c}\ ,\ \alpha_1 = \arcsin\frac{r_d}{r_b}$$

Inertial moments concerning fillings of the notches along the axis of the mixer are given as follows:

(6) $$\vartheta''_{m_2} = 6l_m{}'\rho_2\int_{r_b}^{r_c} r^3\alpha(r)dr\ ,\ \vartheta'''_{m_2} = 3l_m{}'\rho_2\int_{r_a}^{r_b} r^2\text{Ł}(r)dr$$

The dependencies (6) may be simplified considering assumptions for the examples P1 and P2, respectively (Fig. 4). The simplified form of the dependencies (6) are given as follows:

(7) $$\vartheta'''_{m_{2P1}} = 3l_m{}'\rho_2\int_0^{r_b}\text{Ł}(r)dr\ ,\ \vartheta''_{m_{2P1}} = 3l_m{}'\rho_2\text{Ł}_{\text{śr}}\int_{r_b}^{r_c} r^2dr$$

$$\vartheta'''_{m_{2P2}} = 6l_m{}'\rho_2\gamma\int_0^{r_b} r^3dr$$

$$\vartheta''_{m_{2P2}} = 2l_m{}'\rho_2 r_d^3\left(r_{\max} - r_{\min}\right) + l_m{}'\rho_2 d\left(r^3_{\max} - r^3_{\min}\right)$$

Considering different values of mass density for mixer and fillings the resultant inertial moment of the working mixer in the case of the whole lower reactor chamber covered by polyethylene is given as follows:

(8) $$J_k^* = \vartheta'_{m_1} - \vartheta''_{m_1} - \vartheta'''_{m_1} + \vartheta''_{m_2} + \vartheta'''_{m_2}$$

Considering the dependencies for calculation of the respective areas of the mixer cross-section and particularizations for the respective areas (Fig. 3) as well as considering the simplifications assumed for the first example, the resultant inertial moment is given as follows:

(9) $$\begin{aligned} J_{kP1}^* &= 0{,}5\pi l_m\rho_1\left(\frac{d_m}{2}\right)^4 - \\ &- 3l_m\rho_1\text{Ł}_{\text{śr}}\int_{r_b}^{r_c} r^2dr - 6l_m\rho_1\gamma\int_0^{r_b} r^3dr + \\ &+ 3l_m{}'\rho_2\text{Ł}_{\text{śr}}\int_{r_b}^{r_c} r^2dr + 6l_m{}'\rho_2\gamma\int_0^{r_b} r^3dr \end{aligned}$$

$$\begin{aligned} J_{kP1}^* = l_m\rho_1&\left[0{,}5\pi\left(\frac{d_m}{2}\right)^4 - 3\text{Ł}_{\text{śr}}\int_{r_b}^{r_c} r^2dr - 6\gamma\int_0^{r_b} r^3dr\right] + \\ &+ l_m{}'\rho_2\left[3\text{Ł}_{\text{śr}}\int_{r_b}^{r_c} r^2dr + 6\gamma\int_0^{r_b} r^3dr\right] \end{aligned}$$

Considering the simplifications assumed for the second example, the dependencies (9) are transformed as follows:

(10) $$\begin{aligned} J_{kP2}^* &= 0{,}5\pi l_m\rho_1\left(\frac{d_m}{2}\right)^4 - 6l_m\rho_1\gamma\int_0^{r_b} r^3dr - 6l_m{}'\rho_2\gamma\int_0^{r_b} r^3dr - \\ &- 2l_m\rho_1 r_d^2\left[0{,}5\left(\sqrt{r_c^2 - r_d^2} + r_c\right) - 0{,}5\left(r_a + r_d + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right)\right] - \\ &- l_m\rho_1 d\left[0{,}5^3\left(\sqrt{r_c^2 - r_d^2} + r_c\right)^3 - 0{,}5\left(r_a + r_d + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right)^3\right] + \\ &+ 2l_m{}'\rho_2 r_d^2\left[0{,}5\left(\sqrt{r_c^2 - r_d^2} + r_c\right) - 0{,}5\left(r_a + r_d + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right)\right] + \\ &+ l_m{}'\rho_2 d\left[0{,}5^3\left(\sqrt{r_c^2 - r_d^2} + r_c\right)^3 - 0{,}5\left(r_a + r_d + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right)^3\right] \end{aligned}$$

$$\begin{aligned} J_{kP2}^* &= 0{,}5\pi l_m\rho_1\left(\frac{d_m}{2}\right)^4 + \\ &+ \left(l_m\rho_1 l_m{}'\rho_2\right)\cdot\left\{r_d^2\left(r_a + r_c + r_d + \sqrt{r_c^2 - r_a^2} + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right) - \right. \\ &\left. - \frac{d}{8}\left[\left(\sqrt{r_c^2 - r_d^2} + r_c\right)^3 - \left(r_a + r_d + \sqrt{\left(r_a + r_d\right)^2 + r_d^2}\right)^3\right] - 6\gamma\int_0^{r_b} r^3dr\right\} \end{aligned}$$

The dependencies (9) and (10) may be given in short as a sum of the inertial moments coming from the mixer and mixer filling for the examples P1 and P2. The shortened presentation of the final dependencies (9) and (10) is given as follows:

(11) $$J_{kP1}^* = c_{k_1}\rho_1 l_m + c_{k_2} l_m{}'\rho_2\left(\frac{l_m{}'}{l_m}\right)$$

$$J_{kP2}^* = c_{k_1}{}'\rho_1 l_m + c_{k_2}{}'\left(l_m\rho_1 + l_m{}'\rho_2\right)\left(\frac{l_m{}'}{l_m}\right)$$

where: c_{k_1} , c_{k_2} are constructional constants resulting from mechanical parameters of the drive system for the example P1, c'_{k_1} , c'_{k_2} are constructional constants resulting from mechanical parameters of the drive system for the example P2.

Conclusions

The knowledge of variation in the polyethylene level in a polymerization chamber is required in order to determine the variation in the resultant inertial moment of the mixer. This variation may be determined practically on the basis of the measurements made for each type of a polymerization reactor and the working parameters determined for the reactor. The relative level of polyethylene in a polymerization reactor versus time for the rated working parameters and parameters exceeding the rated ones is the most commonly found dependency for calculation of the speed of the polyethylene level increase. The dependencies are given in a form of areas what allows averaging in time the increasing level of polyethylene as a result of determining the equation of a line for a given area. The exemplary dependency between the increasing level of polyethylene and time is shown in Fig. 6.

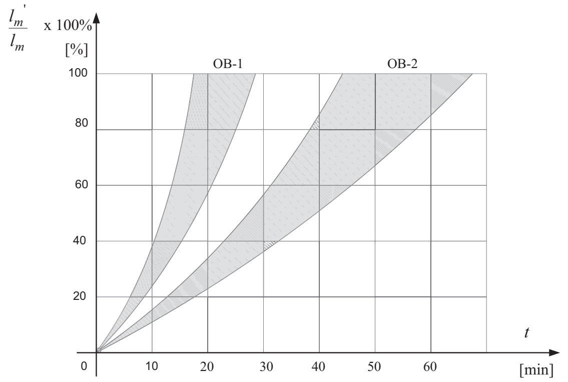

Fig. 6 Increase of the level of polyethylene in a polymerization reactor chamber versus time, where: OB-1 is area of polymerization corresponding to parameters exciding the increased ones, OB-2 is area of polymerization corresponding to parameters exciding the rated ones

Variations in the increase of the polyethylene level in a polymerization reactor chamber in terms of dynamical changes in time determined for the drive system are slow and they do not have any influence on the dynamical processes occurring in the drive system. Thus, the polyethylene level in a polymerization reactor chamber may be considered as a constant value for any calculation case of simulation of the drive system. Transient responses of the drive system for the parts determining steady states or beginnings of steady states may be related to the static characteristics in order to evaluate them. This evaluation is often made for the specially designed drive systems because of specific requirements in the range of the designing assumptions concerning specially designed motors and systems of motion transmission [5].

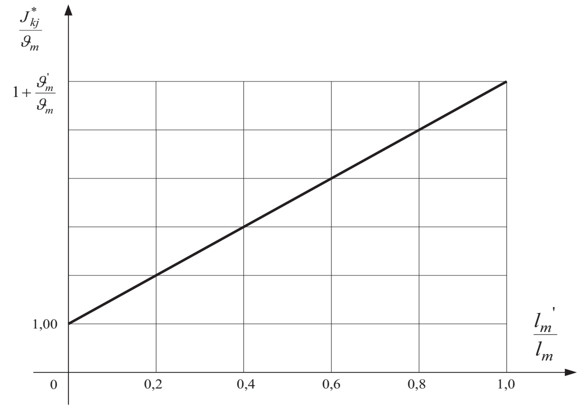

Fig. 7. Relative resultant inertial moment versus relative polyethylene level in a reactor chamber

The inertial moment concerning the cross-section of the mixer is constant but the resultant inertial moment considering the increasing polyethylene level in a reactor chamber is the sum of the inertial moment of the mixer and inertial moments of the fillings along the axis of the mixer. The relative resultant inertial moment versus the relative polyethylene level in the reactor chamber is shown in Fig. 7.

Comparing results of computational analysis based on the dependencies for the examples P1 and P2 and results of numerical calculations made for the true shapes of cross-sections of mixers, the following error of approximation for both examples was determined: for the Example P1 - $\delta_{\vartheta P1} < 0{,}30\%$ and for the Example P2 - $\delta_{\vartheta P2} < 0{,}86\%$.

On the basis of the presented numerical analysis the following final conclusions may be formulated:

- the derived analytical dependencies allow calculating the resultant inertial moment of mixers in drive systems of polymerization reactors and the other drive systems where the similar kinetic structures occur; the accuracy of the abovementioned dependencies is sufficient from the point of view of designing requirements
- the relative resultant inertial moment of the mixer changes itself significantly as a result of influence of operating conditions (Fig. 7); the relative resultant inertial moment of the mixer depends on the charge of reactor chamber
- the increase of a polyethylene level in polymerization chambers is slow and does not exert an influence on transient responses of the drive systems; the polyethylene levels may be considered as parameters determined for each computational case in terms of the given reactor and polymerization process (Fig. 6).

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