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An algorithm for electric circuits calculation based on instantaneous power component balance

Abstract. *A method for nonlinear circuit calculation based on the algorithm of formulation and solution of instantaneous power components balance equations has been developed and verified. The offered algorithm of instantaneous power components formation is grounded on the discrete convolution of two series and implemented using MathCad mathematical package. The presented algorithm has been applied for calculation of a nonlinear inductance circuit at sinusoidal supply voltage.*

Streszczenie. *W pracy rozwinięto oraz zweryfikowano metodę obliczania obwodów elektrycznych nieliniowych, bazującą na równowadze mocy chwilowej. Pokazany algorytm formowania składników mocy chwilowej ma podstawę w dyskretnym splocie dwóch szeregów i jest zrealizowany w pakiecie MathCad. Przedstawiony algorytm służy do rozwiązywania obwodu z nieliniowymi indukcyjnościami.* ***,(Algorytm obliczania obwodów elektrycznych bazujący na równowadze mocy chwilowej)***

Key words: nonlinear electric circuit, nonlinear inductance, instantaneous power, harmonic balance, discrete convolution.
Słowa kluczowe:nieliniowy obwód elektryczny, indukcyjność nieliniowa, moc chwilowa, równowaga harmonicznych, splot dyskretny

Introduction

Calculation of nonlinear electric circuits underlies the analysis of various electric devices. An efficient and easily applied calculation method is necessary for attainment of sufficient accuracy of determining the parameters of electric circuits of electrical devices containing various nonlinear elements [1].

Calculation of nonlinear electric circuits is known to be reduced to calculation of nonlinear equations describing physical phenomena in electric circuits. Analysis of the literature revealed the small parameter method [2] to be a common representative of classic methods of such equations solution [2]. The use of this method is limited by the following faults: complication of calculation dependences with every consequent approximation and with increase of the number of nonlinear elements, as well as limited accuracy [3].

There appeared new methods taking into consideration the listed faults of the small parameter method. These approaches make it possible to analyze both steady-state and transient processes in nonlinear electric circuits. Methods of calculation of nonlinear circuit steady-state processes belong to a broad class of methods of time- or frequency-domain calculations [4].

There are three major limitations for nonlinear circuits calculation in time-domain. Firstly, these methods are inoperative for calculations with wide frequency range. One of such methods is discrete singular convolution method [5], based on shooting and wavelet-balance methods [6]. Secondly, there is complexity of using mentioned methods for circuits with several frequency exciters calculation [7]. Thirdly, these methods lead to decreasing of numerical efficiency for calculation of circuits containing a great number of induction coils and capacitors.

Harmonic balance (HB) method is most commonly used for calculation of steady-state circuit conditions in frequency-domain. A significant advantage of this method consists in the fact that the solution directly provides harmonic composition of the researched circuit parameters. A Fourier transformation is used for getting harmonic series of unknown parameters in the circuit under consideration. This method has a number of modifications:

a) HB method based on Galerkin-Urabe's approach [7] extensively used for solution of various applied problems;

b) a hybrid method consisting of Newton's HB method for solution of a combined circuit using Newton-Rafson's method [8];

c) HB relaxation hybrid method based on relaxation method [8].

However, to achieve an accurate solution using the listed HB method modifications it is necessary to apply a great number of harmonic components and correct initial approximations [9]. Discrete Fourier transform and inverse discrete Fourier transform are also to be performed repeatedly in the process of its implementation. This involves considerable amount of time and errors [8]. Numerical efficiency also decreases sharply when the number of nonlinear elements increases, as the equation system for determination of Fourier coefficients gets rather cumbersome [8].

Hence, none of the existing analytical methods gives exhaustive information about processes in nonlinear electric circuits. So in practice a combination of several methods is used. This, in its turn, leads to complication of calculation dependences and causes increase of calculation errors [3].

Currently it has been offered to use instantaneous power (IP) method based on calculation of balance equations of IP harmonic components for power supply and equivalent circuit elements [10]. It makes it possible to calculate the parameters of nonlinear electric circuits when it is incorrect to use superposition principle for current harmonics [11]. However, formation of balance equations of IP harmonic components involves such negative aspects as calculation complexity and awkwardness.

Problem statement

Development and verification of the algorithm for formation and solution balance equations of instantaneous power components. This algorithm allows one to reduce the amount of time and work content consumed during its implementation, and, consequently, improve accuracy and efficiency of nonlinear circuit calculation.

Fundamentals of method based on the instantaneous power components balance equations

Dependences for instantaneous values of voltage u and current i on time t can be presented as follows:

$$u(t)=\sum_{n=0}^{N}U_n\,cos(n\Omega t-\varphi_n);\qquad i(t)=\sum_{m=0}^{M}I_m\,cos(m\Omega t-\varphi_m) \quad (1)$$

where: I_m, U_n – amplitudes of current and voltage harmonics, correspondingly; m, n – current and voltage harmonics numbers, correspondingly; φ_m, φ_n – shear angles of current and voltage harmonics, correspondingly; Ω – angular frequency; M, N – the number of analyzed current and voltage harmonics.

Instantaneous power (IP):

(2)
$$p(t)=\sum_{m=0}^{M}I_m\cos(m\Omega t-\varphi_m)\sum_{n=0}^{N}U_n\cos(n\Omega t-\varphi_n)= =\sum_{k=0}^{M+N}P_k\cos(k\Omega t-\varphi_k)$$

where: P_k – power harmonics amplitude, k – power harmonic number, φ_k – power harmonics shear angles.

IP presents a complicated dependence including a complex of vector product of voltage and current components. Circular frequency of every vector is determined by the sum and difference of vector-factor frequencies.

IP consists of the sum of active power P_0, and sign-changing real $p_a(t)$ and imaginary $p_b(t)$ components:

(3)
$$p(t)=P_0+\sum_{k=1}^{K}p_{ak}(t)+\sum_{k=1}^{K}p_{bk}(t).$$

To solve nonlinear electric circuits it is necessary to make a system of balance equations of IP harmonics components on the power supply and equivalent circuit elements:

(4)
$$\begin{cases} P_0=\sum_{j}^{J}P_{0Rj}; \\ P_{ak}=\sum_{j}^{J}P_{akRj}+\sum_{i}^{I}P_{akLi}; \\ P_{bk}=\sum_{j}^{J}P_{bkRj}+\sum_{i}^{I}P_{bkLi} \end{cases}$$

where: P_0, P_{ak}, P_{bk} – active power, IP real and imaginary components of power supply, correspondingly; $\sum_{j}^{J}P_{0Rj}$, $\sum_{j}^{J}P_{akRj}$, $\sum_{j}^{J}P_{bkRj}$ – sums of IP active, real and imaginary components according to *j*-th active elements of equivalent circuit, correspondingly; $\sum_{i}^{I}P_{akLi}$, $\sum_{i}^{I}P_{bkLi}$ – sums of IP real and imaginary components according to *i*-th inductive elements of equivalent circuit, correspondingly.

Right parts of the IP balance equations (4) correspond to the expressions for the IP components on the equivalent circuit's elements [12], the formation of which proceeds on the basis of expressions for IP:

on the power supply

(5)
$$p(t)=u(t)i(t),$$

on the linear inductance L

(6)
$$p_L(t)=e(t)i(t)=L\frac{d(i(t))}{dt}i(t)$$

where $e(t)$ – instantaneous value of inductance electromotive force (EMF);

on linear active resistance R

(7)
$$p_R(t)=e(t)i(t)=(i(t))^2R,$$

on nonlinear inductance $L(i)$ dependent on current

(8)
$$p_L(t)=e(t)i(t)=\frac{d(i(t)L(i))}{dt}i(t),$$

on nonlinear active resistance in the function of current $R(i)$ and in the function of time $R(t)$

(9)
$$p_{R(i)}(t)=R(i)i^2(t);$$

(10)
$$p_{R(t)}(t)=R(t)i^2(t);$$

on linear capacity C

(11)
$$p_{C(t)}(t)=\frac{1}{C}i(t)\int i(t)dt,$$

on nonlinear capacity in the function of current $C(i)$ and in the function of time $C(t)$

(12)
$$p_{C(t)}(t)=i(t)\int\frac{1}{C(t)}i(t)dt;$$

(13)
$$p_{C(i)}(t)=i(t)\int\frac{1}{C(i)}i(t)dt.$$

Development of calculation algorithm

The developed algorithm is based on the discrete convolution theorem known from the signal theory. It consists in the fact that Fourier transform (FT) of two signals product is their FT convolution [12]. IP real and imaginary components can be written down in the general form of:

$$Re(P_k)=\left(\sum_{k=0}^{K-1} if\ \left(k-m\ge 0,\ Re(I_m)Re(U_{k-m}),\ Re(I_m)Re(U_{m-k})\right)\right)+ +\left(\sum_{k=0}^{K-1} if\ \left(k-m\ge 0,\ Im(I_m)Im(U_{k-m}),\ -Im(I_m)Im(U_{m-k})\right)\right),$$

$$Im(P_k)=\left(\sum_{k=0}^{K-1} if\ \left(k-m\ge 0,\ Im(I_m)Re(U_{k-m}),\ Im(I_m)Re(U_{m-k})\right)\right)+ +\left(\sum_{k=0}^{K-1} if\ \left(k-m\ge 0,\ Re(I_m)Im(U_{k-m}),\ -Re(I_m)Im(U_{m-k})\right)\right)$$

where: $Re(P_k)$ – IP real component; $Im(P_k)$ – IP imaginary component; $Re(U_m)$, $Re(I_m)$ – real voltage and current components, correspondingly; $Im(U_m)$, $Im(I_m)$ – imaginary voltage and current components, correspondingly.

Calculation according to the general form was made with the help of MathCad mathematical package symbolic computing subsystem. This calculation provides the possibility to assign data both in a symbolic and a numerical form:

```
Re(P_m) := | for k ∈ 0, 1..K
           |   | P_k ← 0
           |   | for m ∈ 0..K
           |   |   | P_k ← P_k + Re(I_m)Re(U_{k−m}) −
           |   |   |       − Im(I_m)Im(U_{k−m}) if k − m ≥ 0
           |   |   | P_k ← P_k + Re(I_{|k−m|})Re(U_m) +
           |   |   |       + Re(I_m)Re(U_{m−k}) + Im(I_m)Im(U_{m−k}) +
           |   |   |       + Im(I_{|k−m|})Im(U_m) otherwise
           |   | P_k ← P_k/4 if k = 0
           |   | P_k ← P_k/2 if k > 0
           | P
```

$$
Im(P_m) := \left|
\begin{array}{l}
for \quad k \in 0, 1..K \\
\quad \left| \begin{array}{l}
P_k \leftarrow 0 \\
for \quad m \in 0..K \\
\quad \left\| \begin{array}{l}
P_k \leftarrow P_k + Im(I_m)\,Re(U_{k-m}) + \\
\quad + Re(I_m)\,Im(U_{k-m}) \;\; if \;\; k-m \geq 0 \\
P_k \leftarrow P_k + Im(I_m)\,Re(U_{m-k}) - \\
\quad - Im\left(I_{|k-m|}\right) Re(U_m) + Re\left(I_{|k-m|}\right) Im(U_m) - \\
\quad - Re(I_m)\,Im(U_{m-k}) \;\; otherwise
\end{array} \right. \\
P_k \leftarrow \frac{P_k}{2}
\end{array} \right. \\
P
\end{array}
\right.
$$

An example of calculation using the developed algorithm

Calculation of IP components was done at sinusoidal supply for nonlinear electric circuit (Fig.1) containing a coil with steel having a nonlinear dependence of inductance on current.

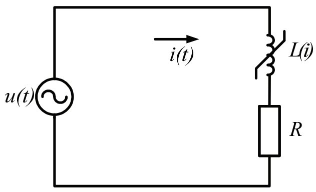

Fig.1. Equivalent circuit of an electric circuit containing a coil with steel

To simplify the process of harmonic components formation on non-linear inductance, according to the presented algorithm, the following sequence was used:

1. Formation of square current $(I_m)^2$ components:

$$(I_m)^2 = [Re(I_m) \quad Im(I_m)] * [Re(I_m) \quad Im(I_m)].$$

2. Formation of dependence non-linear inductance L_p on current components:

$$L_p = \left[a_0 + a_2 (Re(I_m))^2 \quad a_2 (Im(I_m))^2\right].$$

3. Formation of product $I_m dL_p$ of harmonic components of current input arrays and nonlinear inductivity input harmonic components derivative:

$$I_m dL_p = [Re(I_m) \quad Im(I_m)] *$$
$$* \left[a_2 (Im(I_m))^2 \omega j \quad - a_0 + a_2 (Re(I_m))^2 \omega j\right].$$

4. Formation of product $L_p dI_m$ of harmonic components of nonlinear inductance and current input harmonic components derivative:

$$L_p dI_m = \left[a_0 + a_2 (Re(I_m))^2 \quad a_2 (Im(I_m))^2\right] *$$
$$* [Im(I_m)\omega j \quad - Re(I_m)\omega j].$$

5. Formation of IP components P_{kL} for non-linear inductance:

$$P_{kL} = [I_m dL_p + L_p dI_m] * [Re(I_m) \quad Im(I_m)].$$

In the above formulas, the sign * denotes a convolution of two series operation.

Analytic expressions for calculation of IP components of non-linear inductance are not represented in this paper because of their awkwardness. But it makes it possible to estimate directly contribution of the voltage and current harmonic components to the IP components formation.

Symbolic calculations based on the IP harmonics components balance equations automation allows to obtain the required current harmonics of the electric circuit without intermediate transformations using the proposed algorithm.

This algorithm requires only initial voltage harmonics and expressions for approximating the nonlinear characteristics. At the same time, usage of the analytical IP balance equations allows: to evaluate the contribution of certain current and voltage harmonics into the IP components during the evaluation of the operating conditions of electric devices operation; to realize the current and voltage signals processing presented in the form of spectrum; to identity the parameters of equivalent circuits of electric machines an apparatus.

To estimate the accuracy of the offered method, current and IP harmonic components values, obtained by means of a numerical computation of a mathematical model (Fig. 2) of the considered circuit in Matlab environment, were taken as a master data. It was compared with the results of computation according to the offered method.

A second order polynomial was used for approximation of nonlinear characteristic $L(i)$ for the given inductance:

(14) $$L(i) = a_0 + a_2 i^2(t)$$

where a_0, a_2 – polynomial coefficients.

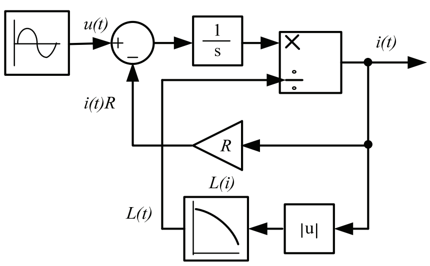

Fig.2. Block diagram of mathematical model of a nonlinear inductance system with sinusoidal supply

The parameters of the considered circuit are $u(t)=780cos(\omega t)$ V, $R=0.5$ Ohm, approximation coefficients: $a_0 = 0.014289$, $a_2 = 7.835 \cdot 10^{-8}$. Application of expression (14) made it possible to achieve the sufficient accuracy of nonlinear approximation while its implementation was rather simple.

As a result of numerical modeling and calculation of a nonlinear circuit using the offered algorithm current curves (Fig.3) were obtained and compared.

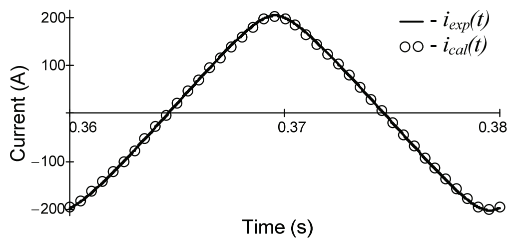

Fig.3. Current signal curves with sinusoidal supply

Fig.3 represents current curves $i_{exp}(t)$, obtained by the calculation of a mathematical model of the considered circuit in Matlab, and current $i_{cal}(t)$, obtained using the developed algorithm of formation and solution balance equations of IP harmonic components.

Analysis of the obtained results

Current harmonics relative error $\delta(I_m)$ and power harmonics relative error $\delta(P_k)$, current signal effective value relative error $\delta(I_{RMS})$ and the same for power $\delta(P_{RMS})$ were chosen as criteria for estimation of the accuracy of nonlinear inductance electric circuit calculation. Received results are presented in Table 1, 2.

As can be seen from tables 1 and 2 calculation error for the main harmonics of the current (1, 3) and power (2, 4) is within permissible limits. These errors occur the significant values only for higher harmonics (5) and power (6), constituting about 1 % of the corresponding based harmonics.

The analysis of the calculation results according to (18) and (19) showed that power and current signal effective $\delta(P_{RMS}) = 0.07\%$, and $\delta(I_{RMS}) = 0.85\%$, respectively.

Table 1. Results of the current harmonics components and relative error calculation

Component		Harmonic number		
		1	3	5
Real	$Re(I_{mcal})$, A	–91.47	–5.7	–1.41
	$Re(I_{mexp})$, A	–92.49	–5.84	–1.08
	$\delta\ (Re(I_m))$,%	+0.91	+1.3	–31
Imaginary	$Im(I_{mcal})$, A	–12.02	–2.73	+0.7
	$Im(I_{mexp})$, A	–12.23	–2.83	+2.5
	$\delta\ (Im(I_m))$,%	+0.7	–1.01	– 64.7

Table 2. Results of the power harmonics components and relative error calculation

Component		Harmonic number		
		2	4	6
Real	$Re(P_{mcal})$, W	$-8.149 \cdot 10^3$	$-1.266 \cdot 10^3$	$-4.63 \cdot 10^2$
	$Re(P_{mexp})$, W	$-8.15 \cdot 10^3$	$-1.269 \cdot 10^3$	$-3.46 \cdot 10^2$
	$\delta\ (Re(P_m))$,%	+0.01	+0.3	– 31.9
Imaginary	$Im(P_{mcal})$, W	$2.968 \cdot 10^4$	$1.288 \cdot 10^3$	$2.63 \cdot 10^2$
	$Im(P_{mexp})$, W	$2.9681 \cdot 10^4$	$1.287 \cdot 10^3$	$1.46 \cdot 10^2$
	$\delta\ (Im(P_m))$,%	+0.003	– 0.078	– 80.5

Conclusions

A method of nonlinear electric circuit calculation using the new algorithm of instantaneous power components formation is offered. Verification of the offered algorithm of instantaneous power harmonic components formation and solution of instantaneous power balance equations revealed high accuracy and efficiency for the nonlinear electric circuit analysis. The described algorithm can be developed for solution of more complicated bifurcated circuits.

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