Mathematical model of drive system for metallurgical roller table unit with rotation of rollers transmitted by chain transmission

Abstract. The paper discusses the drive system with the chain transmission driven by induction motor through the multi-stage gear-reducer. The mathematical model is formulated taking into account the load at both endings of each roller of drive set. The equations describing the drive system are derived on the basis of variational Hamilton’s principle and Lagrange’s equation of the second type. The final system of equations was transformed to the linear differential equations. The aforementioned system of equations allows formulating the computer model for digital simulations.

Streszczenie: W artykule zaprezentowano układ napędowy z łańcuchowym napędem rolek z zastosowaniem wielostopniowego reduktora zębatego napędzanego silnikiem indukcyjnym. Model matematyczny sformułowany z uwzględnieniem obciążenia na obu końcach każdej rolki zestawu napędowego. W oparciu o wariacyjną zasadę Hamiltona oraz równanie Lagranga’a drugiego rodzaju wyprowadzono równania opisujące układ napędowy. Ostateczną postać układu równań sporządzono do układu liniowych równań różniczkowych umożliwiającego sformułowanie modelu komputerowego do obliczeń symulacyjnych. Układ napędowy z łańcuchowym napędem rolek z zastosowaniem wielostopniowego reduktora zębatego napędzanego silnikiem indukcyjnym

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Słowa kluczowe: napęd łańcuchowy, samotokowa linia transportowa, modelowanie matematyczne, obciążenia technologiczne.

Introduction
The drive systems for roller-table sets with rotation of the rollers transmitted by chain transmission are example of a constructional solution of the drive systems for roller-table transporting lines. The aforementioned drive systems are applied in transporting lines in which due to technological problems the instalment of gear-motors with self-transporting construction on each roller is not possible. It concerns also these cases where transmission of motion among rollers through the shaft with gears is not possible. These cases also appear in technological lines in which elements have sections transported perpendicular to the roller-table by the axis of the chain gears fixed on the endings of the rollers of the drive is depicted in Figure 1.

In the presented kinematic diagram of the abovementioned drive system the gears of the chain tension are shown as elements with inertia [15]. Three gears in the other constructional solutions may be replaced by eight gears of the chain tension in such a way that the gears of the chain tension shown as elements with inertia J17, J19 are not in the axis of the chain gears fixed on the endings of the transport rollers shown as elements with inertia J16, J18, J20.

The advantage of the presented drive system is the possibility of whichever forming of the rollers’ span depending on the constructional requirements.

![Fig. 1. The kinematic diagram of the drive for three rollers of the transporting roller-table with chain transmission, where: 1 – the induction motor, 2 – two-stage reduction gear, 3 – the clutch, 4 – the chain drive set for rollers, 5 – the transporting roller](image-url)
Determination of the output quantities

Taking into consideration a kinematic diagram of the drive system illustrated in Figure 1 this system was transformed to the reduced kinematic diagram given in Figure 2.

The distributed load torques shown in Figure 1 for respective rollers appear in the simplified kinematic diagram (Fig. 2) as follows:

- for the first roller the load torques \( M_{11} \), \( M_{12} \) at reduced shield with moments of inertia \( J_{5}, J_{10} \),
- for the second roller the load torques \( M_{12}^{(1)}, M_{22}^{(1)} \) at reduced shield with moments of inertia \( J_{11}, J_{12} \),
- for the third roller the load torques \( M_{13}^{(1)}, M_{23}^{(2)} \) at reduced shield with moments of inertia \( J_{13}, J_{14} \).

The gear ratios of the transmissions shown in Figure 2 are determined by dependencies (1):

\[
\begin{align*}
    p_{12} &= \frac{d_2}{d_5}, & p_{23} &= \frac{d_7}{d_10}, & p_{34} &= \frac{d_{15}}{d_{16}} = \frac{d_{15}}{d_{18}} = \frac{d_{16}}{d_{17}} = \frac{d_{20}}{d_{19}}.
\end{align*}
\]

The moments of inertia reduced to the shields shown in Fig. 2 for respective elements of the system are expressed by dependencies (2): for the driving motor \( J_{1}' \) for the reduction gear \( J_{2}' = J_{3}' \), for the chain set of the driven rollers \( J_{5}' = J_{6}' \), for the transporting rollers \( J_{13}' = J_{14}' \):

\[
\begin{align*}
    J_{1}' &= J_{s} + 0.5J_{1}, & J_{2}' &= 0.5J_{1} + J_{2} + J_{3}, & J_{3}' &= J_{4} + J_{5} + J_{6} + J_{7} + J_{8}, & J_{4}' &= J_{9} + J_{10} + J_{11} + J_{12}, \\
    J_{5}' &= J_{13} + J_{14} + J_{15}, & J_{6}' &= J_{16} + 0.5p_{25}J_{17} + 0.5J_{21}, & J_{7}' &= J_{18} + 0.5p_{26}J_{17} + 0.5p_{25}J_{19} + 0.5J_{24}, & J_{8}' &= J_{20} + 0.5p_{25}J_{19} + 0.5J_{27}, \\
    J_{9}' &= 0.5J_{21} + 0.5J_{22}, & J_{10}' &= 0.5J_{22} + J_{23}, & J_{11}' &= 0.5J_{24} + 0.5J_{25}, & J_{12}' &= 0.5J_{25} + J_{26}, & J_{13}' &= 0.5J_{27} + 0.5J_{28}, & J_{14}' &= 0.5J_{28} + J_{29}.
\end{align*}
\]

In the kinematic diagram (Fig. 2) the rotating masses are transformed to the shields with moments of inertia \( J_{i}' \) \( (i = 1,2,\ldots,14) \). The position angles \( \varphi_{i} \) \( (i = 1,2,\ldots,14) \) and coefficients of elasticity \( c_{i} = M_{i}/\Delta\varphi_{i} \) \( (k = 1,2,\ldots,13) \) were assigned to the shields, where \( M_{i} \) is an internal torque resulting in torsion of any elastic element by angle \( \Delta\varphi_{i} \).

Assuming addition of the moments of inertia of the halves the clutch to the moments of inertia of the shafts with the gears as well as considering serial connection of the shafts with coefficients of elasticity \( c_{2,3} \) and \( c_{3} \) the equivalent coefficient of elasticity \( c_{2,3} \) is defined as follows:

\[
    c_{2,3} = \frac{c_{2}c_{3}}{c_{2} + c_{3}}.
\]

For the chains of the drive set the coefficient of elasticity \( c_{i} = F_{i}/M_{i} \) \( (k = 4,\ldots,7) \) is defined as the ratio of the force \( F_{i} \) and the chain elongation \( \Delta l_{i} \). The following external torques are in the drive system:

- the electromagnetic torque of driving motor \( M_{1} \),
- the load torques at the \( i \)-th transporting roller occurring in the places of its settlement \( M_{i}^{(1)}, M_{i}^{(2)} \).

In the drive system presented as the kinematic diagram (Fig. 2) the fourteen elements with the determined moments of inertia are indicated. There are 12 degrees of freedom determined as independent coordinates of angles of shafts' rotation in the system. These coordinates are expressed as the column matrix (4):

\[
    q^{i} = [q_{1}, q_{2}, q_{3}, q_{4}, q_{5}, q_{6}, q_{7}, q_{8}, q_{9}, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}]
\]

In the drive system presented as the kinematic diagram (Fig. 2) the transformed moments of inertia given by the diagonal matrix (5) correspond with the considered independent coordinates given by dependency (4):

\[
    D = \text{diag}[J_{1}', J_{2}', J_{3}', J_{4}', J_{5}', J_{6}', J_{7}', J_{8}', J_{9}', J_{10}', J_{11}', J_{12}', J_{13}', J_{14}'],
\]

where: \( J_{1}' = J_{1}, J_{2}' = J_{2} + p_{25}^{2}J_{3} + p_{26}^{2}J_{4}, J_{k}' = J_{k}, k = 5,\ldots,14. \)
Mathematical model

The mathematical model of the drive system (Fig. 2) is based on Lagrange's equation of a second type taking into consideration the Rayleigh's dissipation function [1], [2]. In general form the Lagrange's equation of a second type is expressed by the following dependency:

\[
\frac{d}{dt} \left( \frac{\partial \Phi}{\partial \dot{q}_k} \right) - \frac{\partial \Phi}{\partial q_k} + \frac{\partial}{\partial \dot{q}_k} \left( \frac{\partial \Phi}{\partial \dot{q}_k} \right) = Q_k, \quad k = 1, \ldots, n
\]

where: \( \Phi \) is total potential energy, \( Q_k \) is the total power of energy dissipation in the system (Rayleigh's dissipation function), \( \dot{q}_k \) is the k-th independent generalized coordinate, \( Q_k \) is the non-potential generalized force affecting on a part of the system with k-th coordinate, \( n \) is a number of independent generalized coordinates of motion.

The total kinetic energy of the analyzed drive (Eq. 2) is given as follows:

\[
T = 0.5 \sum_{i=1}^{14} J_i \frac{d^2 \dot{\phi}_i}{dt^2} + \sum_{i=1}^{14} \int \left( \sum_{j=5}^{8} J_j \frac{d^2 \dot{\phi}_j}{dt^2} \right) \, dt
\]

In the chain drive set for the rollers the deformation potential energy of the stretched chain between driving chain gear and driven chain gear is collected. The deformation between these gears may be expressed as follows:

\[
\Delta l_{i-1(i-1)} = 0.5(d_{\phi_i} - d_{\phi_{i-1}}) > 0
\]

The value of the potential energy collected in the stretched chain between the driving chain gear and driven chain gear is given by the dependency (9):

\[
V_{i-1(i-1)} = 0.5 \sum_{i=1}^{14} \Delta l_{i-1(i-1)}^2, \quad \text{dla} \quad \Delta l_{i-1(i-1)} > 0
\]

\[
\text{dla} \quad \Delta l_{i-1(i-1)} \leq 0
\]

Differentiation of the potential energy given by (9) over generalized coordinate results in the dependency (10):

\[
\frac{\partial V_{i-1(i-1)}}{\partial \phi_i} = \frac{\partial}{\partial \phi_i} \left( \sum_{i=1}^{14} \Delta l_{i-1(i-1)}^2 \right) = 0.5 \sum_{i=1}^{14} \Delta l_{i-1(i-1)} \frac{d^2 \phi_i}{dt^2} + \sum_{i=1}^{14} \int \left( \sum_{j=5}^{8} \Delta l_{i-1(i-1)} \frac{d^2 \phi_j}{dt^2} \right) \, dt
\]

where the unit step function: \( \mathbf{1}(\Delta l_{i-1(i-1)}) = \begin{cases} 1, & \text{for} \ \Delta l_{i-1(i-1)} > 0 \\ 0, & \text{for} \ \Delta l_{i-1(i-1)} \leq 0 \end{cases} \)

The torque which deforms (stretches) the chain in the chain drive set for the rollers transformed to the shaft of the driving chain gear may be expressed by dependency (11):

\[
M_i = \begin{cases} 0.5 \sum_{i=1}^{14} \Delta l_{i-1(i-1)} \frac{d^2 \phi_i}{dt^2} + b_i (d_{\phi_i} - d_{\phi_{i-1}}), & \text{for} \ \Delta l_{i-1(i-1)} > 0 \\ 0, & \text{for} \ \Delta l_{i-1(i-1)} \leq 0 \end{cases}
\]

where: \( b_i \) is the general damping coefficient of deformed chain.

Considering the value of potential energy collected in the deformed chain, the total potential energy of the analyzed drive system (Fig. 2) may be expressed by dependency (12):

\[
V = 0.5 \sum_{i=1}^{14} \left( c_i (\phi_i - \phi_{i-1})^2 + c_{i+3} (\phi_{i+3} - \phi_{i+2})^2 \right) + \sum_{i=5}^{8} \left( c_i (\phi_i - \phi_{i-1})^2 + c_{i+3} (\phi_{i+3} - \phi_{i+2})^2 \right)
\]

The Rayleigh's dissipation function of the analyzed drive system (Fig. 2) is given by the dependency (13):

\[
\Phi = \sum_{i=1}^{14} \left( b_i (\phi_i - \phi_{i-1})^2 + b_{i+3} (\phi_{i+3} - \phi_{i+2})^2 \right) + \sum_{i=5}^{8} \left( b_i (\phi_i - \phi_{i-1})^2 + b_{i+3} (\phi_{i+3} - \phi_{i+2})^2 \right)
\]

where: \( b_{i+3} = \frac{b_i b_{i+3}}{b_i + b_{i+3}} \) – the generalized damping coefficient.
\[ J_0^s \dot{\phi}_0 - 0.25 \cdot \mathbf{1}(\Delta_{L_{-5}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + b_3 \left( \phi_3' - \phi_3 \right) - 0.25 \cdot \mathbf{1}(\Delta_{L_{-6}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + c_6 \left( \phi_6 - \phi_6' \right) + J_0^s \dot{\phi}_0 - 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + b_10 \left( \phi_{10}' - \phi_{10} \right) - 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + c_{10} \left( \phi_{10}' - \phi_{10} \right) = 0, \]

\[ J_0^s \dot{\phi}_0 - 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + b_2 \left( \phi_2' - \phi_2 \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + c_2 \left( \phi_2' - \phi_2 \right) + J_0^s \dot{\phi}_0 - 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_2 b_1' \left( d_2 \phi_2 - d_6 \phi_6' \right) + b_7 \left( \phi_7' - \phi_7 \right) - 0.25 \cdot \mathbf{1}(\Delta_{L_{-7}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + 0.25 \cdot \mathbf{1}(\Delta_{L_{-8}}) d_4 b_2' \left( d_4 \phi_4 - d_6 \phi_6' \right) + c_7 \left( \phi_7' - \phi_7 \right) = 0, \]

\[ J_0^s \dot{\phi}_0 - b_1 \left( \phi_1' - \phi_1 \right) - b_3 \left( \phi_3' - \phi_3 \right) - b_4 \left( \phi_4' - \phi_4 \right) - c_0 \left( \phi_0' - \phi_0 \right) + c_0 \left( \phi_0' - \phi_0 \right) = M_{s1}', \]

\[ J_0^s \dot{\phi}_0 - b_1 \left( \phi_1' - \phi_1 \right) - b_3 \left( \phi_3' - \phi_3 \right) - b_4 \left( \phi_4' - \phi_4 \right) - c_0 \left( \phi_0' - \phi_0 \right) = M_{s2}', \]

The final linear equations given by the dependencies (15) were obtained as a result of the successive transformations of the equations given by (14).
moments of inertia of the roller axis, the transport chain in the drive system among the chain gears with the transported element may be considered in the system of equations (15) expressed by dependency (16). Additionally, the system of equations expressed by dependency (15) allows for the computable analysis of the drive system for various constructional requirements and dimensions of driving chain route may be assumed for this drive set dependently to: the distance between rollers as well as the shape of the transported element measured along the roller axis, $a$, is the distance between the edge of transported element and the roller ending, $L$ is the roller length.

The system of equations (15) takes into account the dependencies dealing with free damping of elastic elements of the analyzed drive system and allows for analysis of this system for different materials applied in true drive systems. It expands significantly the possibility of practical application of the mathematical model in analysis of the drive systems at the stage of their designing or setting up the prototypical systems. It was not considered in previous mathematical models [5, 6]. The significant practical meaning has the proper selection of the driving chain in the systems with precise positioning of the transported element which is the most damage element of the system and it is exposed to the significant deformation during the operation of the drive system in frequent dynamical states.

Conclusions

The analysis of the drive system presented as computational kinematic diagram (Fig. 2) results in the system of equations (15). This system of equations allows for the wide and optional analysis of the dynamical states of the drive system. The obtained mathematical model allows analyzing, inter alia, the following cases:

- the computable analysis of the system in dynamical states considering or not the dissipation energy,
- the computable analysis of the drive system for various configurations of the chain gears contained in the chain drive set for the rollers,
- the computable analysis of dynamical states of the drive system for different sorts and lengths of the chain working in the chain drive set for the rollers.

The analysis presented in the paper and developed mathematical model does not consider an influence of the placement and elasticity of the transported element on the operation of the transporting rollers in analyzed sections of operation of the transporting lines.

REFERENCES


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