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Comparison of different experimental methods for determining the magnetically nonlinear iron core characteristics of transformers

***Abstract**. This paper is divided into two sections. In the first section, different experimental methods for determining magnetically nonlinear iron core characteristics of transformer are described and evaluated. In the second section the obtained characteristic were used in nonlinear dynamic model of a single-phase transformer. The comparison of the measured and by the dynamic model calculated results is given for the case of transformer steady state operation at rated load and for the case of switch–on of unloaded transformer.*

***Streszczenie.** Artykł jest podzielony na dwie części: w pierwszej doświadczalne metody są opisane i ewaluowane, w dtugiej otrzymane doświadczalnie charakterystyki są uzyte w modelu jednofazowego transformatora. Porównanie pomierzonych i obliczonych z modelu wartości zostało przeprowadzone dla warunków pracy ustalonej i dla przypadku właczenia transformatora do sieci. (**Porównanie różnych metod doświadczalnych dla okresślenia charakterystyki magnetycznie nieliniowego rdzenia transformatora**)*

Keywords: dynamic model, magnetic nonlinearities, parameters, power transformer
Słowa kluczowe: model dynamiczny, magnetyczne nieliniowości, parametry, transformator mocy

Introduction

This paper deals with determining parameters of a magnetically nonlinear dynamic model of power transformer. The paper focuses on determination of magnetically nonlinear iron core characteristic $\psi(i)$. It is determined by the classical methods based on no load test and by the alternative methods based on numerical integration of measured currents and voltages [1-3]. A power grid and linear amplifier were used as a voltage source. The linear amplifier was used to generate sinusoidal and stepwise changing voltage waveforms. A comparison of magnetically nonlinear characteristics determined by the different methods is presented. The magnetically nonlinear dynamic model of power transformer is evaluated through a comparison of measured and calculated currents and voltages under different operating conditions [4], [5].

Description of applied methods

Classical method based on no load test: Magnetically nonlinear characteristic $\psi(i)$of a single-phase transformer is determined at no-load in the steady state, where the voltage u and the current i are periodical functions. Harmonic analysis can be performed on a periodical function. The voltage and current can be expressed in the form of Fourier series (1):

(1)

$$u_h(t) = A_{u0}(t) + \sum_{h=1}^{M} a_{uh}(t)\cos(h\omega t) + b_{uh}(t)\sin(h\omega t)$$

$$i_h(t) = A_{i0}(t) + \sum_{h=1}^{M} a_{ih}(t)\cos(h\omega t) + b_{ih}(t)\sin(h\omega t)$$

where, M is the highest order of harmonic component, h is the order of the harmonic component, T is the period for one cycle of fundamental frequency, $\omega = 2\pi/T$, while A_{u0}, A_{i0}, a_{uh}, a_{ih}, b_{uh} and b_{ih} are the Fourier coefficients of the voltage u and the current i. If we consider only the fundamental harmonic components of the voltage and the current expressions (2) can be written:

(2) $$Z = \frac{U}{I}, \; Z = \frac{P}{I^2}, \qquad L = \frac{1}{\omega}\sqrt{Z^2 - R^2}$$

where Z is the impedance, U and I are the voltage and the current *RMS* values, while P is the active power. L is the inductance while R is the ohmic resistance. The magnetically nonlinear characteristic $\psi(i)$ can be determined by (3) with a set of test.

(3) $$\psi(i) = Li \qquad i = I\sqrt{2}$$

Classical methods based on no load test consider only the fundamental harmonic component of the voltage and the current. This may lead to error in determined magnetically nonlinear characteristic $\psi(i)$, when the current contains higher harmonic components.

Alternative method based on numerical integration and step-wise changing voltage: At no-load the test object is supplied by the stepwise changing voltage. The amplitude of voltage has to achieve such a value that the measured response of current reaches the state of saturation, and at the same time covers the entire range of operation. The stepwise changing voltage and current are measured, as it is shown in Figs. 1 and 2.

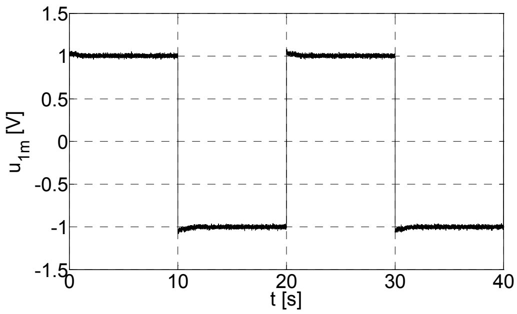

Fig. 1: The stepwise changing voltage.

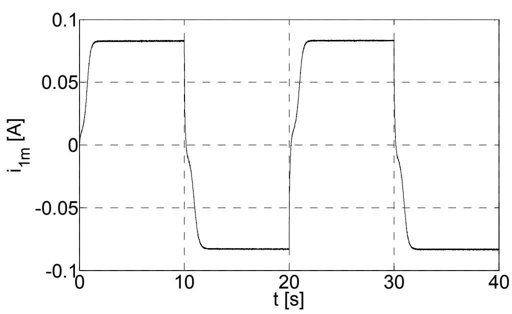

Fig. 2: The stepwise changing current.

The magnetically nonlinear characteristic $\psi(i)$ can be determined after the numerical integration (4).

$$\psi(t) = \psi(0) + \int_0^t \left[u(\tau) - Ri(\tau)\right] d\tau \tag{4}$$

Where $\psi(0)$ is the initial condition due to the remanent flux, while u is the voltage and i is the current. R is the ohmic resistance that can be calculated easily by the steady-state voltage and current at the end of step-change.

Alternative method based on numerical integration and sinusoidal changing voltage: This method is similar to the already described alternative method. The only difference is in the form of the periodically changing voltage. The applied voltage is in this case sinusoidal. This method could be useless if the ohmic resistance R varies during the measurement. When a sinusoidal voltage is applied, there is no stationary state, where the true value of ohmic resistance could be determined.

In this case, two tests were made. Single-phase transformer was supplied by the sinusoidal voltage of the fundamental frequency 50 Hz and by the sinusoidal voltage of the frequency 1 Hz (Figs. 3 and 4).

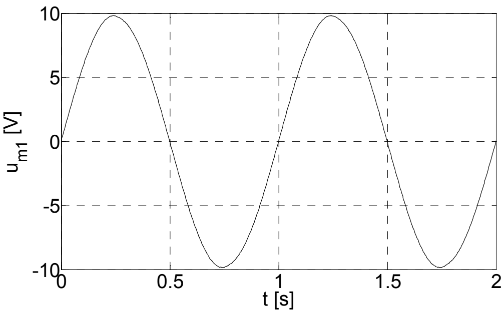

Fig. 3: The sinusoidal changing voltage.

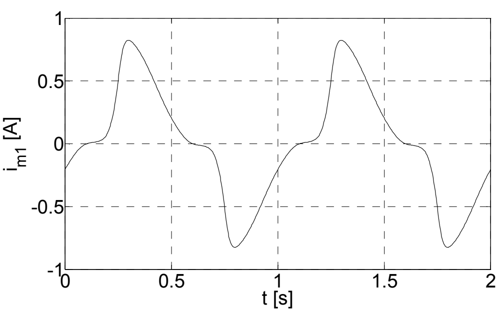

Fig. 4: The sinusoidal changing current.

The magnetically nonlinear iron core characteristics $\psi(i)$ determined by the classical method based on no-load test and by the alternative methods based on the numerical integration are shown in Fig. 5.

If the tops of individual magnetically nonlinear iron core characteristics determined by numerical integration (Fig 5) are connected then the characteristic shown in Fig. 6 is got.

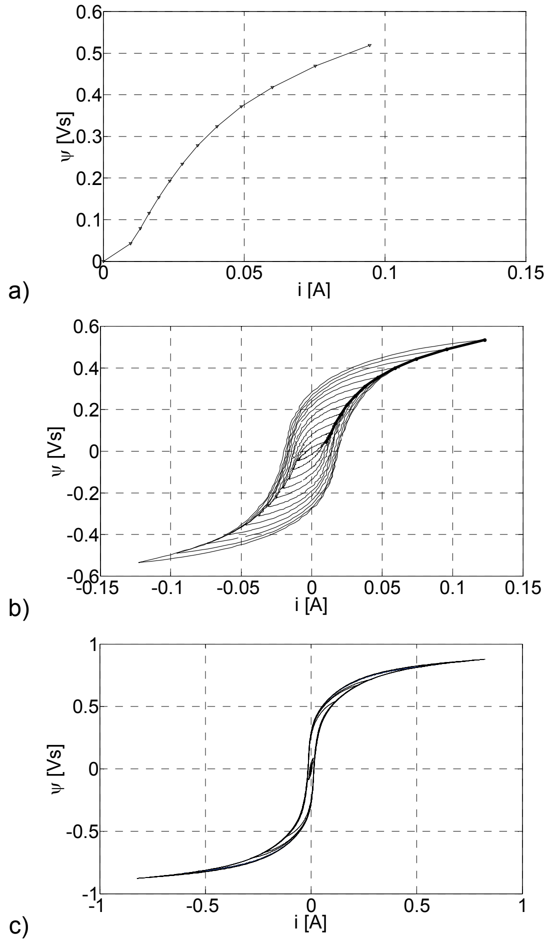

Fig. 5: Magnetically nonlinear iron core characteristic $\psi(i)$ determined by: a) classical method based on no-load test, b) alternative method based on numerical integration supplied by step-wise changing voltage and c) alternative method based on numerical integration supplied by sinusoidal changing voltage

Nonlinear dynamic model of a single-phase transformer

This section deals with the magnetically nonlinear dynamic model of a single-phase transformer. The voltage balance in a single phase transformer, whose iron core behaves magnetically nonlinear, is described by voltage equation (5):

$$\begin{aligned} u_1 &= i_1 R_1 + \frac{d}{dt}\left[\psi_{\sigma 1} + \psi_1\right] \\ u_2 &= i_2 R_2 + \frac{d}{dt}\left[\psi_{\sigma 2} + \psi_2\right] \end{aligned} \tag{5}$$

where u_1, u_2 and i_1, i_2 denote the primary and the secondary voltages and currents, R_1 and R_2 are the primary and secondary resistances, $\psi_{\sigma 1}$ and $\psi_{\sigma 2}$ are the primary and secondary leakage flux linkages, while ψ_1 and ψ_2 are the primary and secondary current-dependent flux linkages. The leakage flux linkages can be expressed by the constant primary and secondary leakage inductances (6).

$$\begin{aligned} \frac{d\psi_{\sigma 1}}{dt} &= L_{\sigma 1}\frac{di_1}{dt} \\ \frac{d\psi_{\sigma 2}}{dt} &= L_{\sigma 2}\frac{di_2}{dt} \end{aligned} \tag{6}$$

The main flux ϕ depends on the magneto motive force θ, which can be expressed by the magnetizing current i_m (7):

$$\theta = N_1 i_1 + N_2 i_2 \qquad i_m = \frac{\theta}{N_1} = i_1 + \frac{N_2}{N_1} i_2 \tag{7}$$

When the magneto motive force is expressed by the magnetizing current $\theta = N_1 i_m$ and a new symbol for the primary flux linkage is introduced as $\psi_0 = N_1\phi$, $d\psi_1/dt$ and $d\psi_2/dt$ can be expressed by (8).

$$\begin{aligned} \frac{d\psi_1}{dt} &= \frac{1}{N_1}\frac{\partial\psi_0}{\partial i_m}\left\{N_1\frac{di_1}{dt} + N_2\frac{di_2}{dt}\right\} \\ \frac{d\psi_2}{dt} &= \frac{N_2}{N_1^2}\frac{\partial\psi_0}{\partial i_m}\left\{N_1\frac{di_1}{dt} + N_2\frac{di_2}{dt}\right\} \end{aligned} \tag{8}$$

By inserting (6) and (8) into (5), the voltage balances in the primary and secondary windings can be expressed by (9):

$$\begin{aligned} u_1 &= i_1 R_1 + L_{\sigma 1}\frac{di_1}{dt} + \frac{\partial\psi_0}{\partial i_m}\left[\frac{di_1}{dt} + \frac{N_2}{N_1}\frac{di_2}{dt}\right] \\ u_2 &= i_2 R_2 + L_{\sigma 2}\frac{di_2}{dt} + \frac{\partial\psi_0}{\partial i_m}\left[\frac{N_2}{N_1}\frac{di_1}{dt} + \frac{N_2^2}{N_1^2}\frac{di_2}{dt}\right] \end{aligned} \tag{9}$$

where $\partial\psi_0 / \partial i_m$ is the dynamic inductance $L_d(i_m)$. In order to simplify (9), the expression (10) can be introduced.

$$\begin{aligned} L_1 &= L_{\sigma 1} + \frac{\partial\psi_0}{\partial i_m} = L_{\sigma 1} + L_d \\ L_{12} &= \frac{N_2}{N_1}\frac{\partial\psi_0}{\partial i_m} = L_d \\ L_2 &= L_{\sigma 2} + \left(\frac{N_2}{N_1}\right)^2\frac{\partial\psi_0}{\partial i_m} = L_{\sigma 2} + \left(\frac{N_2}{N_1}\right)^2 L_d \end{aligned} \tag{10}$$

where L_1 is the self inductance of the primary winding, L_2 is the self inductance of the secondary winding, while L_{12} is the mutual (magnetizing) inductance. If (10) is considered in (9) it yields (11).

$$\begin{aligned} u_1 &= i_1 R_1 + L_1\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \\ u_2 &= i_2 R_2 + L_2\frac{di_2}{dt} + L_{12}\frac{di_1}{dt} \end{aligned} \tag{11}$$

The obtained expression (12) is appropriate to be solved by the explicit integration methods.

$$\begin{aligned} \frac{di_1}{dt} &= \frac{L_2}{L_1 L_2 - L_{12}^2}\left(u_1 - \frac{L_{12}}{L_2}u_2 - i_1 R_1 + \frac{L_{12}}{L_2}i_2 R_2\right) \\ \frac{di_2}{dt} &= \frac{L_1}{L_1 L_2 - L_{12}^2}\left(u_2 - \frac{L_{12}}{L_1}u_1 - i_2 R_2 + \frac{L_{12}}{L_1}i_1 R_1\right) \end{aligned} \tag{12}$$

The formulation (12) gives us the basic equations of the nonlinear model of a single phase transformer. Based on these equations a block scheme in Matlab / Simulink environment was composed for simulation purposes.

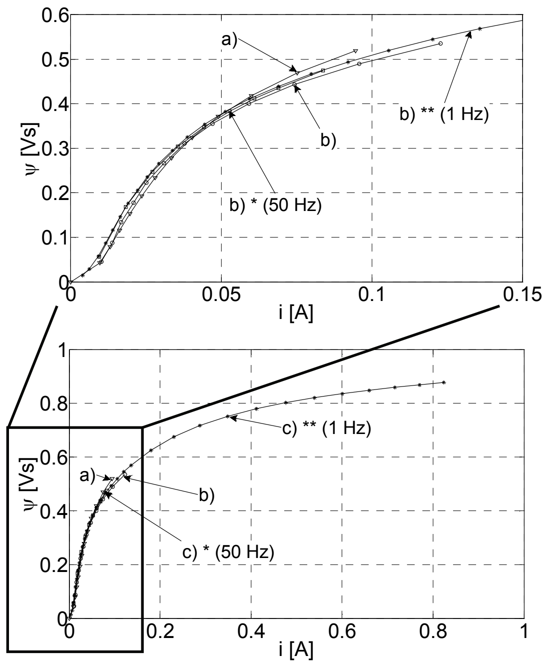

Fig. 6: Magnetically nonlinear iron core characteristic $\psi(i)$ determined by: a) classical method based on no-load test, b) alternative method based on numerical integration supplied by step-wise changing voltage and c) alternative method based on numerical integration supplied by sinusoidal changing voltage.

Results

The test object is a small single phase laboratory transformer. Its data are shown in the Table I. All simulations are performed in the program package Matlab/Simulink using the dynamic model of a single-phase transformer given by equation (12) and the magnetically nonlinear iron core characteristics shown in Fig. 6 c.

Table 1: Testing transformer data

N_1	The number of primary turns	425
N_2	The number of secondary turns	1722
R_1	The primary resistance	11 Ω
R_2	The secondary resistance	141.8 Ω
$L_{\sigma 1}$	The primary leakage inductance	33 mH
$L_{\sigma 2}$	The secondary leakage inductance	33 mH

Fig. 7 shows the measured primary voltage u_1 applied during the no-load test and load test. The same voltage is used in the dynamic model. Its amplitude is 136.7 V at the frequency of 50 Hz.

Fig. 8 shows the comparison of measured and calculated transformer currents in the different operating conditions. In all the figures presented, the measured currents are marked with i_{m1}, while the calculated ones are marked with i_1.

Conclusions

This paper presents a magnetically nonlinear dynamic model of the transformer. The different methods of determining the parameters of that model are given and also evaluated. This model allows the calculation of the dynamic and steady state operating conditions of a transformer in one and two-sided power supply. It is also possible to set the initial value of the magnetic flux (remanence). The presented dynamic model of the transformer was evaluated by the comparing of measured and calculated currents under different operating conditions.

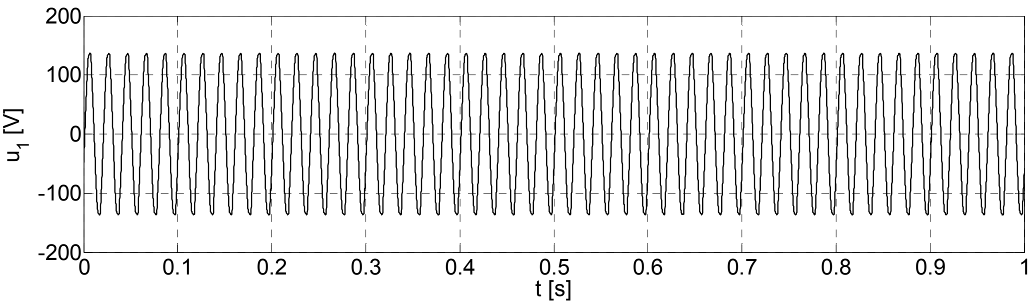

Fig. 7: The primary voltage u_1 measured during the no-load and load test.

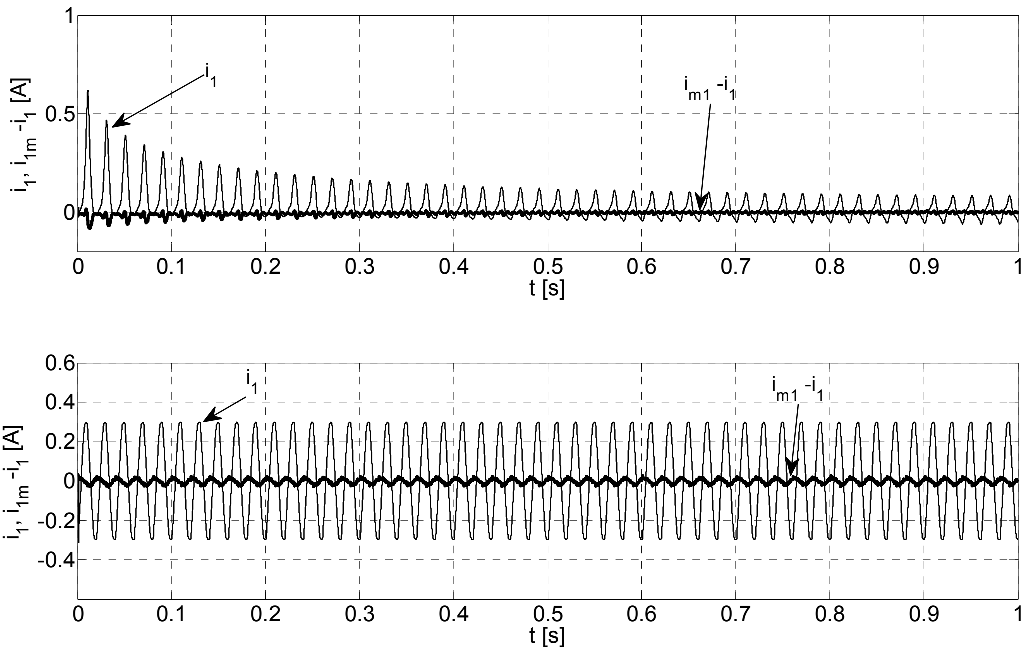

Fig. 8: The comparison of measured and calculated transformer currents during the no-load and load test.

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