Comparison of different experimental methods for determining the magnetically nonlinear iron core characteristics of transformers

Abstract. This paper is divided into two sections. In the first section, different experimental methods for determining magnetically nonlinear iron core characteristics of transformer are described and evaluated. In the second section the obtained characteristic were used in nonlinear dynamic model of a single-phase transformer. The comparison of the measured and by the dynamic model calculated results is given for the case of transformer steady state operation at rated load and for the case of switch-on of unloaded transformer.

Introduction

This paper deals with determining parameters of a magnetically nonlinear dynamic model of power transformer. The paper focuses on determination of magnetically nonlinear iron core characteristic $\psi(i)$. It is determined by the classical methods based on load test and by the alternative methods based on numerical integration of measured currents and voltages [1-3]. A power grid and linear amplifier were used as a voltage source. The linear amplifier was used to generate sinusoidal and stepwise changing voltage waveforms. A comparison of magnetically nonlinear characteristic determined by the different methods is presented. The magnetically nonlinear dynamic model of power transformer is evaluated through a comparison of measured and calculated currents and voltages under different operating conditions [4], [5].

Description of applied methods

Classical method based on no load test: Magnetically nonlinear characteristic $\psi(i)$ of a single-phase transformer is determined at no-load in the steady state, where the voltage $u$ and the current $i$ are periodical functions. Harmonic analysis can be performed on a periodical function. The voltage and current can be expressed in the form of Fourier series (1):

$$u(t) = A_{0u} + \sum_{k=1}^{M} a_k \cos(\omega t) + b_k \sin(\omega t)$$

$$i(t) = A_{0i} + \sum_{k=1}^{M} a_k \cos(\omega t) + b_k \sin(\omega t)$$

where, $M$ is the highest order of harmonic component, $k$ is the order of the harmonic component, $T$ is the period for one cycle of fundamental frequency, $\omega = 2\pi/T$, while $A_{0u}$, $A_{0i}$, $a_k$, $a_k$, $b_k$, and $b_k$ are the Fourier coefficients of the voltage $u$ and the current $i$. If we consider only the fundamental and step-wise changing voltage:

$$Z = \frac{U}{I}, \quad L = \frac{1}{\omega} \sqrt{Z^2 - R^2}$$

where $Z$ is the impedance, $U$ and $I$ are the voltage and the current RMS values, while $P$ is the active power. $L$ is the inductance while $R$ is the ohmic resistance. The magnetically nonlinear characteristic $\psi(i)$ can be determined by (3) with a set of test.

$$\psi(i) = Li \quad i = I\sqrt{2}$$

Classical methods based on load test consider only the fundamental harmonic component of the voltage and the current. This may lead to error in determined magnetically nonlinear characteristic $\psi(i)$, when the current contains higher harmonic components.

Alternative method based on numerical integration and step-wise changing voltage: At no-load the test object is supplied by the stepwise changing voltage. The amplitude of voltage has to achieve such a value that the measured response of current reaches the state of saturation, and at the same time covers the entire range of operation. The stepwise changing voltage and current are measured, as it is shown in Figs. 1 and 2.
The magnetically nonlinear characteristic $\psi(i)$ can be determined after the numerical integration (4).

$$
\psi(i) = \psi(0) + \int_0^i \left[ u(\tau) - R(\tau) \right] d\tau
$$

Where $\psi(0)$ is the initial condition due to the remanent flux, while $u$ is the voltage and $i$ is the current. $R$ is the ohmic resistance that can be calculated easily by the steady-state voltage and current at the end of step-change.

**Alternative method based on numerical integration and sinusoidal changing voltage:** This method is similar to the already described alternative method. The only difference is in the form of the periodically changing voltage. The applied voltage is in this case sinusoidal. This method could be useless if the ohmic resistance $R$ varies during the measurement. When a sinusoidal voltage is applied, there is no stationary state, where the true value of ohmic resistance could be determined.

In this case, two tests were made. Single-phase transformer was supplied by the sinusoidal voltage of the fundamental frequency 50 Hz and by the sinusoidal voltage of the frequency 1 Hz (Figs. 3 and 4).

**Nonlinear dynamic model of a single-phase transformer**

This section deals with the magnetically nonlinear dynamic model of a single-phase transformer. The voltage balance in a single phase transformer, whose iron core behaves magnetically nonlinear, is described by voltage equation (5):

$$
u_1 = i_1 R_1 + \frac{d}{dt} \left[ \psi_{s1} + \psi_1 \right],
$$

$$
u_2 = i_2 R_2 + \frac{d}{dt} \left[ \psi_{s2} + \psi_2 \right],
$$

where $\nu_1$, $\nu_2$, and $i_1$, $i_2$ denote the primary and the secondary voltages and currents, $R_1$ and $R_2$ are the primary and secondary resistances, $\psi_{s1}$ and $\psi_{s2}$ are the primary and secondary leakage flux linkages, while $\psi_1$ and $\psi_2$ are the primary and secondary current-dependent flux linkages. The leakage flux linkages can be expressed by the constant primary and secondary leakage inductances (6).

$$
\frac{d\psi_{s1}}{dt} = L_{s1} \frac{di_1}{dt},
$$

$$
\frac{d\psi_{s2}}{dt} = L_{s2} \frac{di_2}{dt}.
$$
The main flux \( \phi \) depends on the magnetomotive force \( \theta \), which can be expressed by the magnetizing current \( i_m \) (7):

\[
\theta = N_1 i_1 + N_2 i_2 \quad i_m = \frac{\theta}{N_1} = i_1 + \frac{N_2}{N_1} i_2
\]

When the magnetomotive force is expressed by the magnetizing current \( \theta = N_1 i_m \) and a new symbol for the primary flux linkage is introduced as \( \psi_0 = N_1 \phi \), \( d\psi_0/dt \) and \( d\psi_2/dt \) can be expressed by (8).

\[
d\psi_1 = \frac{1}{N_1} \psi_0 \left( \frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) \right)
\]

\[
d\psi_2 = \frac{N_2}{N_1} \psi_0 \left( \frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) \right)
\]

By inserting (6) and (8) into (5), the voltage balances in the primary and secondary windings can be expressed by (9):

\[
u_1 = i_1 R_1 + \frac{d}{dt} \left( \frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) \right)
\]

\[
u_2 = i_2 R_2 + \frac{d}{dt} \left( \frac{N_2}{N_1} \psi_0 \left( \frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) \right) \right)
\]

where \( \partial \psi_0 / \partial i_m \) is the dynamic inductance \( L_d(i_m) \). In order to simplify (9), the expression (10) can be introduced.

\[
L_1 = L_{11} + \frac{\partial \psi_0}{\partial i_m} = L_{11} + L_d
\]

\[
L_2 = \frac{N_2}{N_1} \frac{\partial \psi_0}{\partial i_m} = L_d
\]

\[
L_2 = L_{22} + \left( \frac{N_2}{N_1} \right)^2 \frac{\partial \psi_0}{\partial i_m} = L_{22} + \left( \frac{N_2}{N_1} \right)^2 L_d
\]

where \( L_1 \) is the self inductance of the primary winding, \( L_2 \) is the self inductance of the secondary winding, while \( L_{12} \) is the mutual (magnetizing) inductance. If (10) is considered in (9) it yields (11).

\[
u_1 = i_1 R_1 + L_1 \frac{d}{dt} \left( i_1 + \frac{N_2}{N_1} i_2 \right)
\]

\[
u_2 = i_2 R_2 + L_2 \frac{d}{dt} \left( \frac{N_2}{N_1} \psi_0 \left( \frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) \right) \right)
\]

The obtained expression (12) is appropriate to be solved by the explicit integration methods.

\[
\frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) = \frac{L_2}{L_{14} L_2 - L_{12}^2} \left( u_1 - \frac{L_{12}}{L_2} u_2 - i_1 R_1 + \frac{L_{13}}{L_2} i_2 R_2 \right)
\]

\[
\frac{d}{dt} \left( \frac{N_1 d_i_1}{dt} + \frac{N_2 d_i_2}{dt} \right) = \frac{L_1}{L_{14} L_2 - L_{12}^2} \left( u_2 - \frac{L_{12}}{L_1} u_1 - i_2 R_2 + \frac{L_{13}}{L_1} i_1 R_1 \right)
\]

The formulation (12) gives us the basic equations of the nonlinear model of a single-phase transformer. Based on these equations a block scheme in Matlab / Simulink environment was composed for simulation purposes.
Fig. 7: The primary voltage $u_1$ measured during the no-load and load test.

Fig. 8: The comparison of measured and calculated transformer currents during the no-load and load test.

REFERENCES


Authors: Sebastijan Seme, Institute of Power Engineering, Smetanova 17, 2000 Maribor, E-mail: sebastijan.seme@uni-mb.si; red. prof. dr. Gorazd Štumberger, Institute of Power Engineering, Smetanova 17, 2000 Maribor, E-mail: gorazd.stumberger@uni-mb.si;