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Optimal slot opening in permanent magnet machines for minimum cogging torque

***Abstract**. Based on the 2-d model of a slot and the effective flux density distribution in an equivalent slotless machine, this paper presents an analytical technique for accurately determining the optimal slot opening for minimum cogging torque in permanent magnet machines. It is then used to derive the relationship between slot opening and cogging torque for machines having different combinations of slot number and pole number. In addition, it is applied to slot pairing to effectively reduce the cogging torque. Analytical results are verified by finite element analyses and experiments.*

***Streszczenie.** Wykorzystując dwuwymiarowy model szczeliny i rozkładu indukcji w artykule zaprezentowano technikę określania optymalnej szczeliny dla uzyskania minimalnego momentu zakleszczania się. Analizę wykorzystano do określania momentu w maszynach o różnej liczbie szczelin i biegunów. Wyniki analizy zweryfikowano eksperymentalnie i obliczeniowo (metodą elementu skończonego). (**Optymalna konstrukcja szczeliny w silnikach z magnesami trwałymi warunkująca minimum momentu zakleszczenia**)*

Keywords: Cogging torque, permanent magnet machines, slot opening, slot pairing.
Słowa kluczowe: silniki prądu stałego, zakleszczanie się maszyny.

Introduction

Permanent magnet (PM) brushless machines are being widely used in high performance drives for applications which require accurate velocity and position control, due to easy implementation of field oriented control, high efficiency, high power factor and high torque density compared with other kinds of motors. However, the inherent cogging torque causes torque and speed ripples and results in acoustic noise and vibration, particularly at low speed and light load.

For cogging torque reduction, many researches have been carried out, and numerous methods proposed, such as a fractional number of slots per pole [1-3], slot or magnet skewing [4,5], auxiliary slots or teeth [2], magnet segmentation [6], slots or teeth pairing [4,7], control-based method [8,9], etc.. It is well known that slot opening has significant influence on the cogging torque. For machines in which the number of magnet poles is close to the number of slots, it was shown that it is possible to reduce the cogging torque by adjusting the slot opening [10,11,14]. It was found in [4] and [7] that there existed several slot opening values which could eliminate the cogging torque. However, because of the adoption of simple 1-d airgap permeance model, it was impossible to accurately determine the optimal slot opening and consequently the cogging torque could not be reduced effectively.

This paper presents an analytical technique for accurately determining the optimal slot opening for minimum cogging torque in permanent magnet machines. It is based on the 2-d model of a slot and the effective flux density distribution in an equivalent slotless machine. It is validated by finite element analyses (FEA) and shown to be of significantly improved accuracy. It is then used to derive the relationship between the slot opening and the cogging torque for machines having different combinations of slot number and pole number. In addition, it is applied to slot pairing to effectively reduce the cogging torque. FEA is employed to verify the obtained results and experiments.

Analysis of cogging torque

According to energy method, the cogging torque is caused by the energy deviation within a motor with respect to the rotational angle of rotor. Compared with the energy variation in the airgap, the energy variation in the iron is negligible, while that in the PMs may be neglected without losing the major features of the cogging torque [12] since it may only introduce the error in the predicted peak value of the cogging torque waveform. Then, the cogging torque can be calculated as follows [4]:

$$(1)\quad T_{cog}(\alpha)=\frac{\partial W_{airgap}(\alpha)}{\partial\alpha}=\frac{\partial}{\partial\alpha}\frac{1}{2\mu_0}\int_V G^2(\theta)B^2(\theta,\alpha)dV$$

where: α – rotational angle of the rotor, W_{airgap} – magnetic energy in the airgap, μ_0 – permeability of air, $G(\theta)$ – airgap relative permeance, $B(\theta,\alpha)$ – airgap flux density in an equivalent slotless machine. $G^2(\theta)$ can be expressed by the Fourier series in the interval of $[-\pi/N_S, \pi/N_S]$:

$$(2)\quad G^2(\theta)=G_{a0}+\sum_{k=1}^{\infty}G_{a_{kN_s}}\cos(kN_S\theta)$$

where: N_S – number of stator slots. $B^2(\theta,\alpha)$ can be expressed by the Fourier series in the interval of $[\alpha-\pi/N_P, \alpha+\pi/N_P]$:

$$(3)\quad B^2(\theta,\alpha)=B_{a0}+\sum_{m=1}^{\infty}[B_{a_{mN_P}}\cos(mN_P(\theta-\alpha))]$$

where: N_P – number of permanent magnet poles. Substituting (2) and (3) into (1), the expression for the cogging torque can be obtained by a Fourier series with the fundamental period of $2\pi/N_L$:

$$(4)\quad T_{cog}(\alpha)=-\frac{\pi L_{ef}N_L}{4\mu_0}(R_2^2-R_1^2)\sum_{n=1}^{\infty}nG_{a_{nN_L}}B_{a_{nN_L}}\sin(nN_L\alpha)$$

where: N_L – least common multiple between N_P and N_S, L_{ef} – effective axial length of the machine, R_2 – outer radius of airgap, R_1 – inner radius of airgap.

For $G_{a_{nN_L}}$, [4,7] adopt simplest 1-d airgap permeance model, which assumes that there is no flux under slot. It is mentioned in [4], for $B_{a_{nN_L}}$ the airgap flux density function proposed in [15] can be used. In this paper, in order to effectively and accurately investigate the influence of slot opening on the cogging torque, 2-d slot model and effective flux density distribution are adopted for $G_{a_{nN_L}}$ and $B_{a_{nN_L}}$.

Fig. 1 shows the airgap flux density produced by one magnetic pole in an equivalent slotless machine. In Fig. 1, curve 1 is the actual airgap flux density distribution. By introducing the effective ratio of pole-arc to pole-pitch $\alpha_p^{'}$, the flux density distribution can be equivalent to curve 2, and the Fourier coefficient $B_{a_{nN_L}}$ in (3) can be found as:

(5)
$$B_{a_{nN_L}} = \frac{N_P}{\pi}\int_{-\frac{\alpha'_p\pi}{N_P}}^{\frac{\alpha'_p\pi}{N_P}} B_\delta^2 \cos[(nN_L(\theta-\alpha)]d(\theta-\alpha) = \frac{2N_P}{n\pi N_L}B_\delta^2 \sin(nN_L\frac{\alpha'_p\pi}{N_P})$$

where: B_δ – maximum flux density in the airgap when the machine is slotless. According to [13], the instantaneous airgap field density distribution at the stator surface in an equivalent slotless machine can be gained. For example, for the internal rotor radial magnetized motor at the rotor position α=0, it is:

(6)
$$B(\theta) = \sum_{n=1,3,5,...}^{\infty} \frac{4N_P B_r}{\mu_r \pi} f(n)\cos(\frac{nN_P}{2}\theta)$$

where:

$$f(n) = \sin\left(\frac{n\pi\alpha_p}{2}\right)\frac{1}{(\frac{nN_P}{2})^2-1}\left(\frac{R_m}{R_s}\right)^{\frac{nN_P}{2}+1}$$

(7)
$$\cdot\left\{\frac{(\frac{nN_P}{2}-1)+2\left(\frac{R_r}{R_m}\right)^{\frac{nN_P}{2}+1}-(\frac{nN_P}{2}+1)\left(\frac{R_r}{R_m}\right)^{nN_P}}{\frac{\mu_r+1}{\mu_r}\left[1-\left(\frac{R_r}{R_s}\right)^{nN_P}\right]-\frac{\mu_r-1}{\mu_r}\left[\left(\frac{R_m}{R_s}\right)^{nN_P}-\left(\frac{R_r}{R_m}\right)^{nN_P}\right]}\right\}$$

where: α_p – ratio of pole-arc to pole-pitch, B_r – magnet remanence, μ_r – magnet relative recoil permeability, R_s, R_r, R_m – stator bore radius, rotor bore radius, radius of permanent magnet adjacent to the airgap. Therefore, $R_m=R_1$ and $R_s=R_2$ for internal rotor machines. For external rotor machines, $R_m=R_2$ and $R_s=R_1$, while $f(n)$ can also be obtained according to [13]. Similarly, for parallel magnetized motor, $B(\theta)$ in [13] can be employed. Then, α'_p can be expressed as the ratio of the average flux density to the maximum:

(8)
$$\alpha'_p = \frac{B_{av}}{B_{\max}} = \frac{\frac{N_p\int_0^{\frac{\pi}{N_P}} B(\theta)d\theta}{\pi}}{B(0)} = \frac{\sum_{n=1,3,5,...}^{\infty}\frac{2}{n\pi}f(n)\sin(\frac{n\pi}{2})}{\sum_{n=1,3,5,...}^{\infty} f(n)}$$

For interior permanent magnet machines (IPM), α'_p may be obtained by FEA.

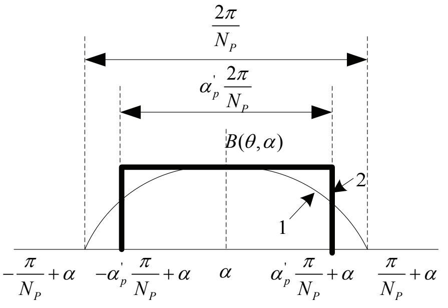

Fig.1. Airgap flux density distribution in an equivalent slotless machine.

From the principle of magnetic circuit, the airgap flux density when slotless is:

(9)
$$B_g = B_r\frac{P_g}{P_m+P_g} = B_r\frac{C_\phi}{1+C_\phi\mu_r g/h_m}$$

where: h_m – thickness of magnets, g – mechanical airgap, P_m – permeance of the magnet, P_g – permeance of the airgap, C_Φ– flux concentration factor For the surface-mounted permanent magnet machine (SPM) with internal rotor C_Φ is given by:

(10)
$$C_\phi = \frac{A_m}{A_g} = \frac{R_2-g-h_m/2}{R_2-g/2}$$

where: A_m, A_g – cross-sectional areas of the magnet and the airgap respectively.

Fig. 2 is the 2-d model for rectangular slots. As the permeability of the iron $\mu_{Fe}>>\mu_0$, it is assumed the magnetic flux crosses the airgap in a straight line under teeth and in a semicircular path with the radii of r_s in slots, as shown in Fig. 2. Based on the assumption, the airgap flux density facing teeth B_{gt} is the same as B_g in (9), that is:

(11)
$$B_{gt} = B_g$$

Because the mechanical airgap extends from g to $g-R_2+R_2\cos(\frac{b_0}{2}-|\theta|)+\frac{\pi r_s}{2}$ due to slotting, the airgap flux density facing slots B_{gs} is:

(12)
$$B_{gs} = B_r\frac{C_\phi}{1+C_\phi\mu_r[g-R_2+R_2\cos(\frac{b_0}{2}-|\theta|)+\frac{\pi r_s}{2}]/h_m}$$

By (9)-(12), the airgap relative permeance $G(\theta)$ is obtained:

(13)
$$G(\theta) = \begin{cases} 1 & \theta\in[-\pi/N_S,-b_0/2]\cup[b_0/2,\pi/N_S] \\ \dfrac{h_m/\mu_r+gC_\phi}{\frac{h_m}{\mu_r}+[g-R_2+R_2\cos(\frac{b_0}{2}-|\theta|)+\frac{\pi R_2}{2}(\frac{b_0}{2}-|\theta|)]C_\phi} & \theta\in[-b_0/2,b_0/2] \end{cases}$$

where: b_0 – slot opening. Thus, the Fourier coefficient of $G^2(\theta)$ is given by:

(14)
$$G_{a_{nN_L}} = \frac{2N_S}{\pi}[\int_{\frac{b_0}{2}}^{\frac{\pi}{N_S}}\cos(nN_L\theta)d\theta + \int_0^{\frac{b_0}{2}}\frac{(\frac{h_m}{\mu_r}+gC_\phi)^2\cos(nN_L\theta)}{[\frac{h_m}{\mu_r}+(g-R_2+R_2\cos(\frac{b_0}{2}-\theta)+\frac{\pi R_2}{2}(\frac{b_0}{2}-\theta))C_\phi]^2}d\theta]$$

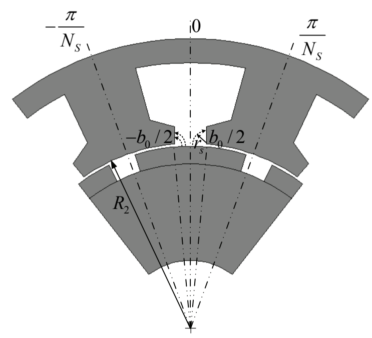

Fig.2. Stator slot.

Influence of slot opening on cogging torque

Substituting (5) into (4), the cogging torque is in direct proportion to:

(15)
$$T_{cog}(\alpha) \propto -\sum_{n=1}^{\infty}[G_{a_{nN_L}}\sin(nN_L\frac{\alpha'_p\pi}{N_P})\sin(nN_L\alpha)]$$

Form (15), as mentioned earlier, the period is $2\pi/N_L$. Therefore, at the rotor position $\alpha=2\pi/N_L$, the amplitude of the cogging torque is in direction proportion to:

(16)
$$T_{cog} \propto \sum_{n=1}^{\infty}T_n = -\sum_{n=1}^{\infty}[G_{a_{nN_L}}\sin(nN_L\frac{\alpha'_p\pi}{N_P})\sin(n\frac{\pi}{2})]$$

and the amplitude of the fundamental order is:

$$T_1 = -G_{a_{1N_L}} \sin(N_L \frac{\alpha_p' \pi}{N_P}) \propto -G_{a_{1N_L}} \quad (17)$$

As expected, it reveals that slot opening greatly affects cogging torque.

In order to validate the analytical expression (16), FEA is adopted. The prototype machines are SPM machines with internal rotors as shown in Fig. 3. The design parameters are given in Table 1.

By (8), the effective airgap flux density and α_p' are obtained as shown in Fig. 4. Then introducing α_p' into (16), the influence of slot opening on cogging torque is derived as shown in Fig. 5 and compared with FEA. The ratio of slot opening to slot pitch is defined as β:

$$\beta = \frac{b_0}{2\pi / N_S} \quad (18)$$

It shows that there exists several slot opening values that make the cogging torque minimal or maximum, the same as FEA. In Fig. 5, the cogging torque waveform undergoes a phase reversal when passing through the optimal slot opening widths for the cogging torque minimization, as shown in Fig. 6.

Although with the method in [4], as mentioned earlier, it can find the optimal slot opening values for cogging torque elimination, but the error is big. Table 2 is the comparison between the FEA results and results derived with the method in [4] and (16).

As for the optimal values of slot opening for minimum cogging torque, [4] has error. Its slot opening for minimum cogging torque is closer to FEA slot opening for maximum cogging torque, rather than that minimum cogging torque. The error of (16) is much smaller and the results are very closer to FEA. In Table 2, it also lists the values of β for minimum and maximum cogging torque with different ratio of pole-arc to pole-pitch. The obtained values of β from [4] and (16) are respectively compared with FEA. It shows for different ratio of pole-arc to pole-pitch, the error of (16) is very low. For different combinations of slot number and pole number, the errors of (16) are also small as shown in Table 3 and Table 4, in which the results are also given for the machines with 18 slots, 24 magnets and 24 slots, 16 magnets, respectively. The error of β changes with the value of N_S/N_P, while the number of optimal β for minimum cogging torque is equal to N_L/N_S including β=0.

Table 1. Machine parameters

Stator outer diameter	120mm	Stator inner diameter	75mm
Stator axial length	65mm	Thickness of magnets	3.5mm
Mechanical airgap	0.5mm	Magnet remanence	0.97T
Ratio of pole-arc to pole-pitch	1	Magnet relative recoil permeability	1.0667
Number of poles	8,18,24	Number of slots	9,24,16

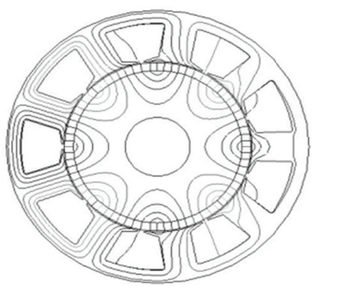

Fig.3. Open-circuit magnetic field distribution (N_S/N_P=9/8).

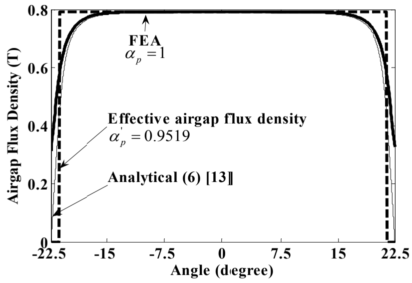

Fig.4. Airgap flux density distribution (N_S/N_P=9/8).

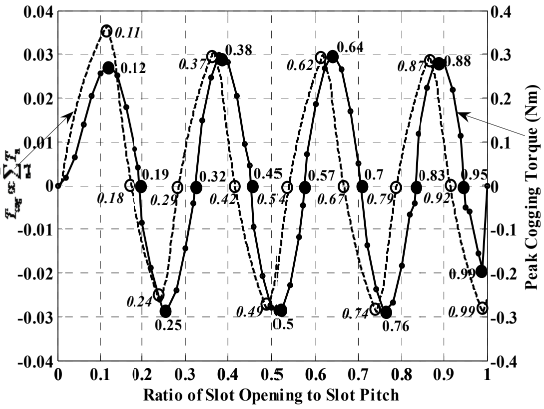

Fig.5. Influence of slot opening on peak cogging torque with (16) and FEA (N_S/N_P=9/8).

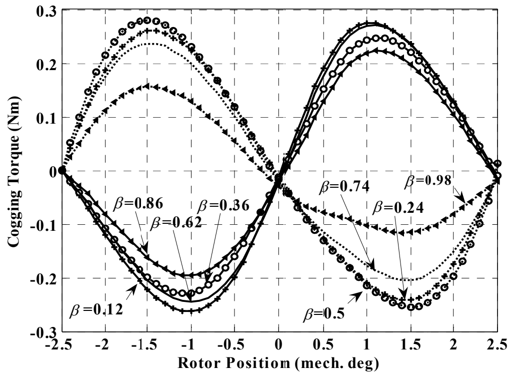

Fig.6. Cogging torque waveform with various slot opening by FEA (N_S/N_P=9/8).

Table 2. Comparison of analytical and FEA results (N_S/N_P=9/8)

		β for minimum cogging torque							
α_p=0~1	[4]	0.125	0.25	0.375	0.5	0.625	0.75	0.875	
α_p=0.6	(16)	0.17	0.29	0.42	0.54	0.66	0.79	0.91	
	FEA	0.15	0.28	0.41	0.53	0.66	0.78	0.9	
	Error	0.02	0.01	0.01	0.01	0	0.01	0.01	
α_p=0.85	(16)	0.17	0.29	0.42	0.54	0.67	0.79	0.92	
	FEA	0.17	0.30	0.43	0.55	0.68	0.8	0.93	
	Error	0	0.01	0.01	0.01	0.01	0.01	0.01	
α_p=1	(16)	0.18	0.29	0.42	0.54	0.67	0.79	0.92	
	FEA	0.19	0.32	0.45	0.57	0.7	0.83	0.95	
	Error	0.01	0.03	0.03	0.03	0.03	0.04	0.03	
		β for maximum cogging torque							
α_p=0~1	[4]	0.06	0.19	0.31	0.44	0.56	0.69	0.81	0.94
α_p=0.6	(16)	0.11	0.24	0.36	0.49	0.61	0.74	0.86	0.99
	FEA	0.1	0.22	0.34	0.47	0.59	0.72	0.85	0.99
	Error	0.01	0.02	0.02	0.02	0.02	0.02	0.01	0
α_p=0.85	(16)	0.11	0.24	0.36	0.49	0.61	0.74	0.86	0.99
	FEA	0.11	0.24	0.37	0.49	0.62	0.74	0.87	0.99
	Error	0	0	0.01	0	0.01	0	0.01	0
α_p=1	(16)	0.11	0.24	0.37	0.49	0.62	0.74	0.87	0.99
	FEA	0.12	0.25	0.38	0.5	0.64	0.76	0.88	0.99
	Error	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0

Table 3. Comparison of analytical and FEA results (N_S/N_P=18/24)

		β for minimum cogging torque			β for maximum cogging torque			
α_p =0~1	[4]	0.25	0.5	0.75	0.125	0.375	0.625	0.875
α_p=0.6	(16)	0.35	0.59	0.85	0.17	0.42	0.68	0.93
	FEA	0.35	0.62	0.87	0.21	0.48	0.74	0.97
	Error	0	0.03	0.02	0.04	0.06	0.06	0.04
α_p=0.85	(16)	0.34	0.57	0.83	0.23	0.48	0.73	0.98
	FEA	0.35	0.6	0.86	0.22	0.48	0.74	0.99
	Error	0.01	0.03	0.03	0.01	0	0.01	0.01
α_p=1	(16)	0.34	0.58	0.83	0.22	0.47	0.72	0.97
	FEA	0.38	0.64	0.89	0.24	0.51	0.77	0.99
	Error	0.04	0.06	0.06	0.02	0.04	0.05	0.02

Table 4. Comparison of analytical and FEA results (N_S/N_P=24/16)

		β for minimum cogging torque	β for maximum cogging torque	
α_p=0~1	[4]	0.5	0.25	0.75
α_p=0.6	(16)	0.69	0.35	0.85
	FEA	0.69	0.41	0.92
	Error	0	0.06	0.07
α_p=0.85	(16)	0.65	0.49	0.99
	FEA	0.71	0.45	0.98
	Error	0.06	0.04	0.04
α_p=1	(16)	0.67	0.39	0.89
	FEA	0.79	0.46	0.96
	Error	0.12	0.07	0.07

Slots (or teeth) pairing

Slots or teeth pairing is proposed by [4,7,11]. By employing two different sets of tooth widths or two different sets of slot opening widths, the cogging torque can be effectively reduced. While because of the error shown in Table 2~4, the slots pairing method using the existing methods in [4] and [7] is not as effective as expected sometimes. In this section, the method for slots pairing is investigated to assure the accuracy of slot pairing.

For the machine with slots pairing, the period of $G^2(\theta)$ is $4\pi/N_S$. $G^2(\theta)$ can be expressed by the Fourier series in the interval of $[-2\pi/N_S, 2\pi/N_S]$:

$$G^2(\theta) = G_{2a0} + \sum_{k=1}^{\infty} G_{a_{2kN_s}} \cos(\frac{1}{2}kN_S\theta) \tag{19}$$

Substituting (19) and (3) into (4), it yields:

$$T_{cog}(\alpha) = -\frac{\pi L_{ef}(R_2^2 - R_1^2)}{4\mu_0}\sum_{n=1}^{\infty} nN_{L2}[G_{a_{nN_{L2}}} B_{a_{nN_{L2}}} \sin(nN_{L2}\alpha)] \tag{20}$$

where:

$$N_{L2} = LCM(N_P, N_S/2) \tag{21}$$

Substituting (6) into (20), the cogging torque with slots pairing is in direct proportion to:

$$T_{cog}(\alpha) \propto -\sum_{n=1}^{\infty}[G_{a_{nN_{L2}}} \sin(nN_{L2}\frac{\alpha'_p\pi}{N_P})\sin(nN_{L2}\alpha)] \tag{22}$$

It shows, at the rotor position $\alpha=\pi/2N_{L2}$, the amplitude of the cogging torque with slot pairing is in direction proportion to:

$$T_{cog}(\alpha) \propto -\sum_{n=1}^{\infty}[G_{a_{nN_{L2}}} \sin(nN_{L2}\frac{\alpha'_p\pi}{N_P})\sin(n\frac{\pi}{2})] \tag{23}$$

where:

$$G_{a_{nN_{L2}}} = G_{a_{nN_{L2}}}(x)\Big|_{x=b_1} + G_{a_{nN_{L2}}}(x)\Big|_{x=b_2} \tag{24}$$

where: b_1, b_2 — two different slot opening for slots pairing, β_1, β_2 — ratio of slot opening to slot-pitch respectively corresponding to b_1 and b_2. The expression of $G_{a_{nN_{L2}}}(x)$ is as follows:

$$G_{a_{nN_{L2}}}(x) = \frac{N_S}{\pi}\Big[\int_{\frac{x}{2}}^{\frac{\pi}{N_S}} \cos(nN_{L2}\theta)d\theta + \int_0^{\frac{x}{2}} \frac{(\frac{h_m}{\mu_r} + gC_\phi)^2 \cos(nN_{L2}\theta)}{[\frac{h_m}{\mu_r} + (g - R_2 + R_2\cos(\frac{x}{2}-\theta)) + \frac{\pi R_S}{2}(\frac{x}{2}-\theta))C_\phi]^2} d\theta\Big] \tag{25}$$

Based on (25), equation (22) can be simplified as:

$$T_{cog} \propto T(x)\big|_{x=b_1} + T(x)\big|_{x=b_2} \tag{26}$$

where:

$$T(x) = -\sum_{n=1}^{\infty}[G_{a_{nN_{L2}}}(x)\sin(nN_{L2}\frac{\alpha'_p\pi}{N_P})\sin(n\frac{\pi}{2})] \tag{27}$$

Equation (27) shows the influence of one set of the slot opening widths on cogging torque when the motor is slots pairing. From (26), if b_1 and b_2 satisfies:

$$T(x)\big|_{x=b_1} = -T(x)\big|_{x=b_2} \tag{28}$$

the cogging torque will be counteracted. For the motor of N_P=24, N_S=18 and α_p=0.85 in Table 1 with slot pairing, the influence of one set of the slot opening on cogging torque, while the alternate set of slot opening is fixed, is shown in Fig. 7, which is obtained by (27) and FEA.

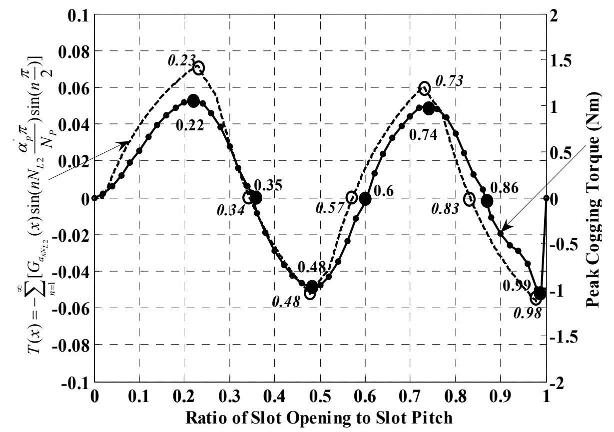

Fig.7. Influence of one set of slot opening widths on cogging torque when slots pairing by (27) and FEA (N_S/N_P=18/24, α_p=0.85).

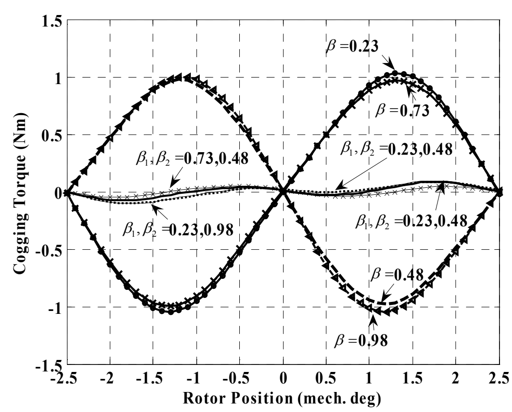

Fig.8. Cogging torque waveform with slots pairing by FEA (N_S/N_P=18/24, α_p=0.85).

As shown in Fig. 7 with (27), the cogging torque reaches negative peak if one set of the slot opening widths β=0.48 or β=0.98, and reaches positive peak if β=0.23 or β=0.73. Thus, by (28), for cogging torque elimination, β_1 and β_2 for slots pairing can be 0.23 and 0.48, 0.73 and 0.98, 0.23 and 0.98, 0.48 and 0.73, as confirmed by the FEA results in Fig. 8.

From Fig. 7, it can directly employ the optimal equal slot widths $\beta_1=\beta_2$=0.34, 0.57 or 0.83 for minimum cogging torque. As shown in Fig. 9, compared with the method of optimal equal slot widths for minimum cogging torque, the slot pairing technique effectively reduces the fundamental order of the cogging torque. Therefore, because of the reduction of the dominant component, the frequency of the cogging torque is somehow doubled, which is also confirmed in [7].

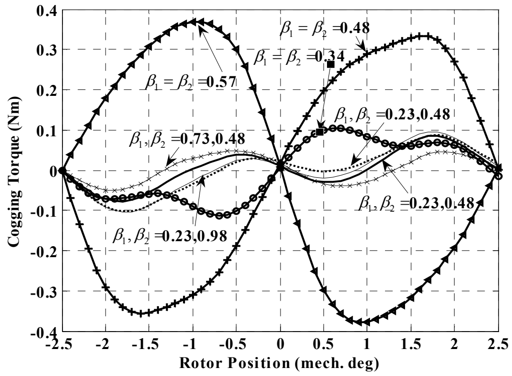

Fig.9. Cogging torque waveform with optimal equal slot opening widths and slots pairing by FEA (N_S/N_P=18/24, α_p=0.85).

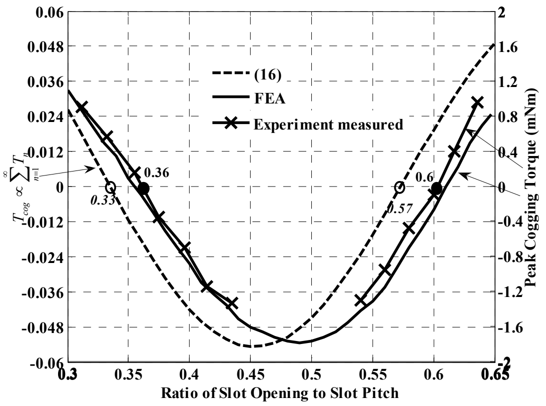

Fig.10. Comparison of variation of analytically predicted, FE calculated and measured cogging torque amplitudes with slot opening.

Experiment validation

The foregoing analyses are further validated by comparing analytically predicted results with the measured results for an external rotor radial-field PM motor [14]. The motor has 8 poles, 6 slots and a bonded ferrite permanent magnet (B_r=0.245, μ_r=1.05) with pole-arc to pole-pitch ratio of 0.8, but a range of stators with different slot openings. The stator outer bore radius is 13.25mm, the magnet thickness is 3mm, the axial length is 8mm, and the airgap length is 0.5mm. Fig. 10 shows the measured results, together with the predicted results obtained by (16) and FEA. As can been seen, the FEA predicted results are very close to the measured results. According to (16), the cogging torque can be minimized if the ratios of slot opening to slot pitch β are 0.33 and 0.57, respectively, which are very close to the measured optimal values of β, which are 0.36 and 0.6. Therefore, although, as already mentioned earlier, the analytical model presented in this paper cannot predict the amplitude of cogging torque accurately, it is very reliable for predicting the optimal slot openings for minimum cogging torque.

Conclusions

Based on the 2-d model of the slot, and with the energy method and Fourier series analysis, the relationship between the slot opening width and the cogging torque is analytically derived for permanent magnet machines. The influence of slot opening on the cogging torque is analyzed with significantly improved accuracy, particularly the optimal slot opening widths for cogging torque minimization. The slots or teeth pairing method is also used to effectively reduce the cogging torque. The developed analytical technique offers acceptable accuracy for determining the optimal slot opening widths, as confirmed by the finite element analysis and experiments.

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