Methodological approach to steady-state and transient investigation of electric circuits using numerical infinite series of two-phase system

Abstract. The paper deals with two-phase system investigation using theory of series for analysis and determination of root-mean-square values and harmonic distortion factor of the periodical non-harmonic. The investigated quantities are output voltages and currents of power electronic converter supplying resistive-inductive load. These voltages are strongly non-harmonic ones, so they must be pulse-modulated due to requested nearly sinusoidal currents with low requested total harmonic distortion. A new method of calculation based on theory of numerical series is presented in paper. Basic idea is based on knowledge of harmonic spectrum of the quantities of sources and/or appliances, respectively.

Streszczenie. Analizowano napięcie wyjściowe i prąd elektryczny dwufazowego konwerteru mocy zasilającego odbiornik rezystancyjno-indukcyjny. Przebiegi napięcia i prądu wyjściowego są niesinusoidalne. Modulacja powinna zapewnić przebieg prądu najbardziej zbliżony do sinusoidalnego. Zaproponowano metodę obliczania współczynników szeregu Fouriera opisujących określone napięcie i prąd wyjściowy. Do obliczenia harmonicznych prądów wykorzystuje się spektrum napięcia i charakterystykę częstotliwościową odbiornika. Wyprowadzono analizy postać współczynnika THD. (Metodologiczne podejście do badania stanów ustalonych i niestabilnych przy pomocynumericznych szeregów nieskończonych systemów dwufazowych)

Keywords: periodical non-harmonic function, Fourier transform, total harmonic distortion, root-mean-square value (effective value)

Inroduction

It is well known that for a 2π-periodic function \( f(x) \) that is integrable on \([-\pi, \pi]\), the Fourier series of \( f(x) \) can be derived together with Fourier coefficients of \( f(x) \) on \([-\pi, \pi]\), [2]. Knowing properties of non-harmonic function it is possible to determine the total harmonic distortion of the investigated function. This is needed for quality evaluation of the electrical quantities. Investigation will be done using Fourier and other numerical infinite and finite series.

Harmonic analysis of output voltage of the voltage sourced inverter in two-phase system

Two-phase systems, Fig. 1 (on the contrary to three-phase transformed αβ-systems) have the same both amplitude- and phase-spectrum of both phases [1], [2]. Such system can be created by two single phase inverters shifted by 90 degrees. Using Fourier analysis ([2], [3]) we obtain the relations for amplitudes of harmonic components \( A_n \) of output voltage for both phases, Figs. 2b,c

\[
\begin{align*}
A_0 &= \frac{4}{\pi} U_{DC} \cos \left( \frac{v \pi}{2} \right) \\
A_n &= \frac{4}{\pi} U_{DC} \sin \left( \frac{v \delta}{2} \right) \quad \text{or} \\
A_{n} &\quad n = 1, 2, 3, \ldots
\end{align*}
\]

where \( v \) - odd number equal to \( 2n + 1, \ n = 1, 2, 3, \ldots \), \( \gamma \) - control angle, \( \delta \) - pulse width, and \( U_{DC} \) - supply voltage.

So, Fourier series of the voltage \( u_c(t) \) in Fig.2b is [3]:

\[
u_n(t) = \sum_{n=1}^{\infty} A_n \sin \left( \frac{v \pi}{2} \right) \cos \left( \frac{v \pi}{2} \right)
\]

RMS value determination of output voltage using numerical infinitive series with numerical solution

Root-mean-square value (RMS) of each harmonic component will be

\[
U_r = \frac{A_n}{\sqrt{2}} = \frac{2^{n-1}}{\sqrt{2}} U_{DC} \left( \frac{n}{\pi} \cos \left( \frac{v \pi}{2} \right) \right)
\]

Fig. 1. Two-phase orthogonal power electronic system

Fig. 2. Circuit diagram of single-phase inverter (a) and output voltage waveforms of the system on Figure 1 b) \( u_c(t) \), c) \( u_\alpha(t) \)
and RMS value of total voltage waveform

\[ U = \sqrt{\sum_{i=1}^{N} U_i^2} = \sqrt{\sum_{i=1}^{N} \left[ \frac{2\sqrt{2}}{\pi} U_{DC} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right) \right]^2} \]

\[ = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right)} \]

For \( \gamma = 0 (\delta = \pi) \) we have

\[ U = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right)} = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\frac{\pi^2}{8}} = U_{DC} \]

where [3] \( \sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right) = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{9}} + \ldots = \frac{\pi^2}{8} \)

For \( \gamma = \frac{\pi}{3} \left( \delta = \frac{2\pi}{3} \right) \) we have

\[ U = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right)} = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\frac{\pi^2}{12}} = U_{DC} \frac{2}{\sqrt{3}} \]

where [2] \( \sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right) = \frac{3}{4} \left( \frac{1}{1} \right) + 0 + \frac{3}{4} \left( \frac{1}{5} \right) + \frac{3}{4} \left( \frac{1}{7} \right) + 0 + \ldots = \frac{3}{4} \left( \frac{1}{1} \right) - \frac{3}{4} \left( \frac{1}{3} \right) - \frac{3}{4} \left( \frac{1}{5} \right) = \frac{3\pi^2}{8} \left( \frac{1 - \frac{1}{3}}{1} \right) = \frac{\pi^2}{12} \)

Similarly, we can continue for other control angles \( \gamma \) [6].

Let us assume the orthogonal voltages with bipolar PWM control depicted in Figs 2a,b [2]. Using Fourier analysis we again obtain for both phases the relation for amplitudes of harmonic components [2], [3].

**Methodology of voltage harmonic distortion determination for PWM pulse train**

Contribution of the voltage pulse pair (Fig. 3) to total root-mean-square value will be

\[ U = \sqrt{\sum_{i=1}^{N} \left[ \frac{2\sqrt{2}}{\pi} U_{DC} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right) \right]^2} + \sum_{i=1}^{N} \left[ \frac{2\sqrt{2}}{\pi} U_{DC} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right) \right]^2 \]

\[ = \frac{2\sqrt{2}}{\pi} U_{DC} \sqrt{\sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right)} \]

Contribution of voltage pulse pair (Fig.3) to total root-mean-square value of fundamental harmonic will be

\[ U_{i} = \frac{2\sqrt{2}}{\pi} U_{DC} \sin \left( \frac{\pi}{2} \right) \]

so, for pulse pair \( U_i = \frac{2}{\pi} U_{DC} \)

For pulse series of PWM control (see Fig. 4) one can generalize, Fig. 5.

For total root-mean-square value of the PWM train

\[ U = \sqrt{\sum_{i=1}^{N} U_i^2} \]

and similarly for root-mean square value of the sum of fundamental harmonics

\[ U = \sqrt{\sum_{i=1}^{N} U_{i}^2} \]

(10) \[ U_1 = \sqrt{2} \sum_{i=1}^{N} U_{i}^2 + U_{1m}^2 + \ldots + U_{1n}^2 \]

Fig.3. Output orthogonal voltages of the half-bridge matrix converter system with bipolar pulse-width-modulation: direct (a) and quadrature voltage (b)

Fig.4. Symmetrical placement of each pulse pair of PWM

Fig.5. Decomposition of PWM series into symmetrical pulse pairs

Another way is to summarize the coherent sine-wave of the each pulse pairs

\[ A_i = \sqrt{\sum_{i=1}^{N} \left( A_{i,j} \right)^2} \]

then \( U_1 = \frac{A_i}{\sqrt{2}} \)

**THD factor determination of output voltage using numerical infinitive series with numerical solution**

The total harmonic distortion factor of the voltage \( THD_U \) will be

\[ THD_U = \sqrt{\frac{U_1^2}{U}} - 1 = \sqrt{\sum_{i=1}^{N} \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{2} \right)} - 1 \]

Example #1 for \( \gamma = 0 (\delta = \pi) \)

\[ THD_U = \frac{\pi^2}{8} - 1 = 0.48 \rightarrow 48\% \]
Example #2 for $\gamma = \frac{\pi}{3}$ and $\delta = \frac{2\pi}{3}$

\[
THD_j = \frac{2\pi}{3} \sqrt{\frac{\pi^2/12}{\sqrt{3/2}}} - 1 = \frac{4}{\sqrt{3}} \pi^2 - 1 = 0.31 \rightarrow 31\% 
\]

Harmonic analysis of output current of the voltage sourced inverter - the case of known steady-state current time waveforms

Supposing R-L load (Fig. 6) and $\gamma = 0$ ($\delta = \pi$) the current $i_{RL}$ during positive half-period will be

\[
i_{RL}(t) = i_{RL\text{max}}(1 - e^{-\delta t}) + i_{RL}(0)e^{\delta t} 
\]

\[
i_{RL}(0) = \frac{U_{DC}}{R} \left(1 - e^{-\delta t}\right) + i_{RL}(0)e^{\delta t} = \frac{U_{DC}}{R} \left[\left(1 - 1 + \tanh\frac{T}{4\tau}\right)e^{-\delta t}\right]
\]

where $\tau = L/R$ and according waveforms on Fig.6

\[
i_{RL}(0) = -i_{RL}(T/2) = \frac{U_{DC\text{max}}}{R} \left(1 - e^{-\delta t/2}\right) + i_{RL}(0)e^{\delta t/2} = -\frac{U_{DC}}{R} \tanh\frac{T}{4\tau}
\]

RMS value and THD factor determination of output current of the voltage sourced inverter - the case of known steady-state current time waveforms

RMS value of total current waveform

\[
I = \sqrt{\frac{2}{T}} \int_0^T i(t) dt = \frac{U_{DC}}{R} \sqrt{\frac{2}{T}} \left[\left(1 - \tanh\frac{T}{4\tau}\right)e^{\delta t}\right] dt = 
\]

\[
= \frac{U_{DC}}{R} \sqrt{\frac{2}{T}} \left[\left(1 - \tanh\frac{T}{4\tau}\right)e^{\delta t} + \left(1 - \tanh\frac{T}{4\tau}\right)e^{\delta t}\right] dt
\]

For simplification let us take $\tau = T/2$. Thus $T/4\tau = 1/2$ and $\tanh 1/2 = 0.462$. Then

\[
I = \frac{U_{DC}}{R} \sqrt{\frac{2}{T}} \left[\left(1 - \tanh\frac{T}{4\tau}\right)e^{\delta t} + \left(1 - \tanh\frac{T}{4\tau}\right)e^{\delta t}\right] dt
\]

and after discretization and using development of exponential function into geometrical series [2]

\[
I_{RL\text{max}} = \frac{U_{DC}}{R} \sqrt{\frac{2}{T}} \left[\sum_{n=1}^{N} \sum_{m=1}^{N} \left(1 - \tanh\frac{T}{4\tau}\right) e^{-\delta t} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} e^{-\delta t}\right]
\]

where

\[
r_{1,2} = e^{-\left(\delta/2\right) \gamma} = e^{-\left(\delta/2\right) 0} = e^{0}
\]

Then for RMS value $I_{RMS}$

\[
I_{RMS} = \frac{U_{DC}}{R} \sqrt{\frac{2}{T}} \left[\sum_{n=1}^{N} \sum_{m=1}^{N} \left(1 - \tanh\frac{T}{4\tau}\right) e^{-\delta t} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} e^{-\delta t}\right]
\]

Based on above equations, THD factor for $N = 360$ is

\[
THD_j = \frac{\frac{2}{N} \sum_{n=1}^{N} (1 - e^{-\delta t})^2 + 1,452^2 \frac{1 - e^{-\delta t}}{1 - e^{-\delta t}}}{8 \frac{1}{\pi}} < 0.22
\]

i.e. $< 22\%$, where $I_{RMS} = \frac{2U_{DC}}{R} \frac{1}{\sqrt{1 + \pi^2}}$

Another cases of loads are shown in [2]. Actual examples are shown in [6]. Note that both currents and voltages of the phases have the same amplitude- and phase-spectra of both phases [1], [2].

Conclusion

A new method for two-phase system investigation using numerical series for determination and calculation has been presented. The investigated quantities have been input and output voltages and currents of power electronic converter supplying resistive-inductive load. These voltages are strongly non-harmonic ones, so they must be pulse-modulated due to requested nearly sinusoidal currents with low requested total harmonic distortion.

The solution given in the paper makes it possible to calculate the root-mean-square values and harmonic distortion factors more effectively and analyze effect of each harmonic component comprised in total waveform on resistive-inductive load more precisely.

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Authors: dr. inż. Mariana Beňová, prof. dr. inż. Branislav Dobrucky, University of Žilina, Faculty of Electrical Engineering, Univerzitná 1, 01 026 Žilina, Slovakia, benova@fel.uniza.sk, branislav.dobrucky@fel.uniza.sk;