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Initial conditions and regularization parameter influence in EIT image reconstruction

***Abstract**. The article describes the testing of new techniques of image reconstruction within the electrical impedance tomography inverse problem. A new technique of image reconstruction was tested for various values of regularization parameter α as well as different values of starting conductivity. The results are presented for the tested parameters, their influence on reconstruction quality, and time exigency of solution.*

***Streszczenie.** W artykule opisano testy nowej metody rekonstrukcji obrazu w elektrycznej tomografii impedancyjnej. Nowa technika została przetestowana dla różnych wartości współczynnika regularyzacji i różnych początkowych rozkładów konduktywności. Zaprezentowano wyniki eksperymentów i przedyskutowano wpływ testowanych parametrów na jakość i szybkość rekonstrukcji. (**Metody rekonstrukcji obrazu w elektrycznej tomografii impedancyjnej**)*

Keywords: Electrical Impedance Tomography, the level set method, inverse problem
Słowa kluczowe: Elektryczna Tomografia impedancyjna, metoda zbiorów poziomicowych, zagadnienia odwrotne

Introduction

Electrical impedance tomography (EIT) is a widely analyzed problem with many applications in physical and biological sciences. In geophysical imaging, which constitutes an important field of EIT utilization, the method is used for searching underground conducting fluid plumes near the surface and obtaining information about rock porosity or fracture formation. Another significant application of electrical impedance tomography consists in non-destructive testing and identification of material defects (such as cracks) or detection of corrosion in production materials. In medicine, EIT imaging can be used primarily for the detection of pulmonary emboli, non-invasive monitoring of heart function and blood flow, and for breast cancer detection. The theoretical background of EIT is given in [1]. The principle of the method is based on back image reconstruction, which is a highly ill-posed inverse problem. The aim is to reconstruct, as accurately and fast as possible, the internal conductivity or permittivity distributions in two- or three-dimensional models.

Description of applied methods

Let us assume the arrangement for EIT back reconstruction as provided in Fig.1. Further, we will only consider conductivity σ for simplicity. The scalar potential U can be therefore introduced; thus, the resulting field is conservative and the continuity equation for the current density can be expressed by the U potential

$$\mathrm{div}(\sigma\,\mathrm{grad}U) = 0 \qquad (1)$$

Equation (1), together with the modified complete electrode model equations, is discretized by the finite element method (FEM) in the usual manner. Using the FEM, we calculate approximate values of electrode voltages for the approximate element conductivity vector σ (NE x 1); NE is the number of finite elements, see Fig. 2. Furthermore, we assume constant approximation of conductivity σ on each of all the elements. The forward EIT calculation yields an estimation of the electric potential field in the interior of the volume under certain Neumann and Dirichlet boundary conditions. The FEM in two or three dimensions is exploited for the forward problem using current sources. Image reconstruction with EIT is an inverse problem, which is usually presented as minimizing the suitable objective function $\Psi(\sigma)$ relative to σ.

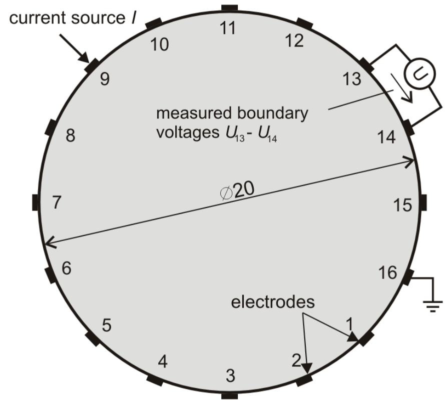

Fig.1. An arrangement for conductivity reconstruction

In order to minimize objective function $\Psi(\sigma)$, we can use a deterministic approach based on the method of least squares. Due to the ill-posed nature of the problem, regularization has to be used. It is possible to use the Total Variation method PD-IPM (TVM) described in [2] as an instrument to solve this inverse EIT problem.

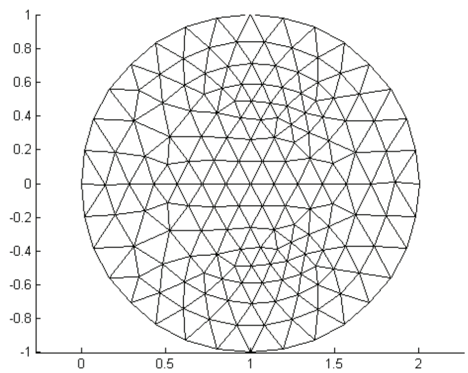

Fig.2. FEM grid with 300 elements and 167 nodes

Then, we have to minimize objective function $\Psi(\sigma)$. Minimized objective function $\Psi(\sigma)$ is described by the equation

$$\Psi(\sigma) = \frac{1}{2}\sum_{ne}\left\|U_M - U_{FEM}(\sigma)\right\|^2 + \alpha TV_\beta, \qquad (2)$$

where $\boldsymbol{\sigma}$ is the vector of unknown volume conductivities, $\boldsymbol{U}_{\mathrm{M}}$ the vector of measured voltages, $\boldsymbol{U}_{\mathrm{FEM}}(\sigma)$ the vector iteratively calculated by using the FEM, α the regularization parameter, and TV the regularization term, which can be described by

(3) $$TV_{\beta} = \sum_{NE} \int \left| grad\sigma \right| d\Omega = \sum \sqrt{\left\| R\sigma \right\|^2 + \beta} \ ,$$

where **R** is a suitable regularization matrix connecting adjacent elements of different conductivity values and β is a small positive parameter, which represents an influence on the smoothing of $\Psi(\sigma)$.

In order to obtain the solution of (2) or (3), we applied the Newton-Raphson method. This iterative procedure is commonly used in the EIT inverse problem for its fast convergence and good reconstruction quality. However, it is likely to be trapped in local minima, and therefore additional regularization must be taken into account if a stable solution is to be obtained. The stability of the TVM algorithm is somewhat sensitive to the setting of the starting value of conductivity and to an optimal choice of parameter α. For this reason, it is necessary to secure balance between the accuracy and the stability of the solution. The value of parameter α can be adaptively changed during this iteration process in both regularization methods. Thus, we can obtain a stable solution with the required higher accuracy of the reconstruction results. The repeated applications of the regularization method are used to find regions with different conductivities and their close surroundings.

The level set method was applied in identifying the location of the regions with different conductivities. The method is used to identify regions with different image or material properties. The level set method (LS) [3 - 5] is based on the deformation of function ϕ. Then, the border of the object is given by the zero level of function ϕ. The evolution equation of level set function ϕ in the general form, described in [4 - 6], is

(4) $$\frac{\partial \phi}{\partial t} + F\left| grad\phi \right| = 0 \ ,$$

where ϕ is the level set function, *F* the speed function, t the time step.

The distribution of the searched unknown conductivity can be described in terms of level set function *F* depending on the position of point ***r*** with respect to boundary *D* between regions with different values of conductivity. During the iteration process based on minimizing objective function $\Psi(\sigma)$, boundary *D* is searched in accordance with the request that $\sigma(r)$ minimize $\Psi(\sigma)$.

(5) $$\sigma(r) = \begin{cases} \sigma_{\text{int}} \{ r : F(r) < 0 \} \\ \sigma_{\text{ext}} \{ r : F(r) > 0 \} \end{cases} \qquad D = \{ r : F(r) = 0 \}$$

In order to improve the stability and the accuracy of EIT image reconstructions, we created a new algorithm based on both the TVM and the level set methods. During the iteration process based on minimizing objective function $\Psi(\sigma)$, boundary D is searched again in accordance with the request that $\sigma(r)$ minimize $\Psi(\sigma)$.

Description of testing

The model for the testing of the parameters (regularization parameter α and the starting value of conductivity) consists of the homogeneous region. The area with different conductivity (the non-homogeneous area) is defined in this region. The parameters were tested on two models. There is one non-homogeneous region in the first model (Fig. 3, top) and two non-homogeneous regions in the second model (Fig. 3, bottom). The values of tissue conductivity were defined with respect to application of the method in biomedicine. The value of homogeneous region conductivity is 0.333 S/m (tissue) and the value of non-homogeneous region conductivity is 0.666 S/m (heart).

There were found final conductivity values identical with the original ones shown in Fig. 3. The objective function value, iterations count, time, and error were monitored in the testing process. The error was computed in equation 6. These values were monitored before the use of the LSM, namely after the TVM had found non-homogeneous regions and their close surroundings (in tables marked *region*). Analogically, the above-mentioned parameters were monitored after the use of the LSM, namely when the TVM was used for finding the final conductivity value (in tables marked *value*). Both these models were tested with parameter α in the range of 10^{-10} to 10^{0}. Parameter α was changed during the reconstruction process by 0.7. The starting conductivity value was set to 0.5 S/m (the average value of both tissue conductivities). The results of testing are in table 1 and the related time can be found in Fig. 4. When parameter α is smaller than 10^{-10}, the iterative solution is not stable.

Both the models were tested on starting conductivity with the initial value of parameter $\alpha = 10^{-8}$. The models were tested for eight conductivities. Conductivity values were set to the maximum of original conductivity 0.666 S/m, the minimum of original conductivity 0.333 S/m, 150% of maximal original conductivity 1 S/m, 50% of minimal original conductivity 0.166 S/m, minimal value for stable solution 0.004 S/m. The initial value was randomly selected on individual elements in the intervals of <0.333; 0.666> S/m, <0.166; 1> S/m. The results are provided in table 2 and the time is shown in Fig. 5.

(6) $$\text{Err} = \sqrt{\frac{\sum_{(i)}^{\text{NE}} \left(\sigma(i) - \sigma_{orig}(i) \right)^2}{\sum_{(i)}^{\text{NE}} \left(\sigma_{orig}(i) \right)^2}} 100\%$$

where σ is the reconstructed conductivity, σ_{orig} the original conductivity, and NE the number of elements.

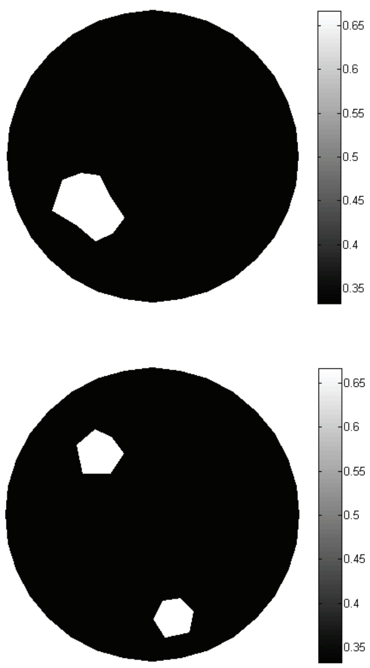

Fig.3. The original and final conductivity distribution in example one (top) and example two (bottom)

Table 1. Parameter α dependence

	one non-homogeneous area						two non-homogeneous areas					
	iterations			$\Psi(\sigma)$			iterations			$\Psi(\sigma)$		
α	region	value	sum	region	value	Err	region	value	sum	region	value	Err
10^{0}	41	37	78	$4.55.10^{-6}$	$1.57.10^{-13}$	$1.22.10^{-3}$	24	44	68	$3.08.10^{-4}$	$3.82.10^{-14}$	$9.14.10^{-4}$
10^{-1}	25	47	72	$1.92.10^{-5}$	$2.59.10^{-14}$	$1.18.10^{-4}$	14	45	59	$6.70.10^{-4}$	$6.79.10^{-14}$	$9.75.10^{-4}$
10^{-2}	19	45	64	$1.27.10^{-5}$	$2.44.10^{-13}$	$4.03.10^{-4}$	16	39	55	$5.39.10^{-4}$	$6.20.10^{-14}$	$8.65.10^{-4}$
10^{-3}	11	22	33	$2.09.10^{-5}$	$6.26.10^{-12}$	$7{,}52.10^{-3}$	13	35	48	$4.99.10^{-5}$	$1.45.10^{-14}$	$7.68.10^{-4}$
10^{-4}	10	21	31	$6.26.10^{-6}$	$1.11.10^{-13}$	$9.63.10^{-4}$	10	34	44	$7.11.10^{-6}$	$3.76.10^{-14}$	$8.93.10^{-4}$
10^{-5}	7	29	38	$6.27.10^{-7}$	$2.35.10^{-15}$	$6.14.10^{-4}$	8	27	35	$1.87.10^{-6}$	$4.21.10^{-14}$	$8.21.10^{-4}$
10^{-6}	7	20	27	$3.05.10^{-7}$	$7.01.10^{-15}$	$8.14.10^{-4}$	5	14	19	$7.31.10^{-6}$	$2.98.10^{-14}$	$7.83.10^{-4}$
10^{-7}	6	33	39	$2.62.10^{-6}$	$5.33.40^{-12}$	$6.89.10^{-4}$	7	37	44	$7.07.10^{-6}$	$2.59.10^{-14}$	$8.75.10^{-4}$
10^{-8}	5	3	8	$4.60.10^{-6}$	$1.66.10^{-15}$	$7.68.10^{-4}$	6	16	26	$1.87.10^{-6}$	$4.12.10^{-14}$	$8.13.10^{-4}$
10^{-9}	6	5	11	$7.51.10^{-5}$	$8.91.10^{-14}$	$7.51.10^{-4}$	5	29	34	$3.72.10^{-6}$	$5.73.10^{-14}$	$7.95.10^{-4}$
10^{-10}	14	4	18	$2.83.10^{-7}$	$1.75.10^{-14}$	$6.27.10^{-4}$	8	4	12	$2.87.10^{-6}$	$8.47.10^{-14}$	$5.89.10^{-4}$

Table 2. Starting conductivity value dependence

	one non-homogeneous area						two non-homogeneous areas					
	iterations			$\Psi(\sigma)$			iterations			$\Psi(\sigma)$		
σ [S/m]	region	value	sum	region	value	$E_{rr}*10^{-4}$	region	value	sum	region	value	$E_{rr}*10^{-4}$
0.333	5	13	18	$1.42.10^{-2}$	$1.66.10^{-15}$	7.60	12	2	14	$5.64.10^{-6}$	$1.49.10^{12}$	9.34
0.666	5	3	8	$8.47.10^{-5}$	$2.03.10^{-13}$	1.25	4	3	7	$3.05.10^{-4}$	$3.37.10^{-13}$	3.05
0.5	5	3	8	$1.09.10^{-5}$	$1.22.10^{-13}$	0.98	7	3	10	$6.62.10^{-4}$	$2.82.10^{-16}$	3.01
1.0	5	3	8	$9.10.10^{-6}$	$1.13.10^{-13}$	0.95	5	3	8	$4.00.10^{-5}$	$1.34.10^{-14}$	0.38
0.1666	6	6	12	$1.43.10^{-2}$	$1.01.10^{-14}$	9.18	7	4	11	$8.06.10^{-4}$	$5.14.10^{-15}$	0.61
0.004	14	17	31	$3.17.10^{-8}$	$5.39.10^{-14}$	5.93	14	18	32	$1.51.10^{-5}$	$1.08.10^{-13}$	0.81
0.3-0.6	11	15	26	$3.98.10^{-7}$	$8.05.10^{-14}$	8.18	11	18	29	$1.88.10^{-6}$	$1.46.10^{-14}$	9.89
0.16-1	13	9	22	$3.43.10^{-6}$	$9.98.10^{-14}$	6.07	12	10	22	$4.15.10^{-6}$	$1.57.10^{-13}$	8.35

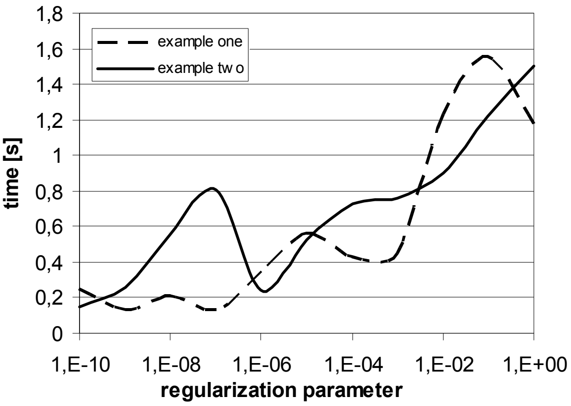

Fig.4. Solution time depending on the regularization parameter

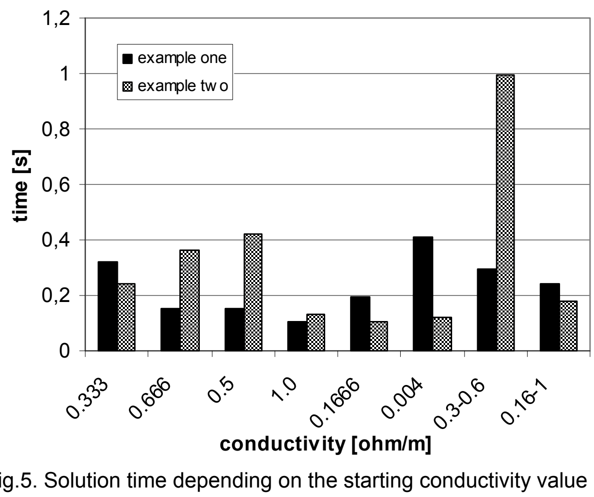

Fig.5. Solution time depending on the starting conductivity value

The effects of parameter α on the count of iterations and $\Psi(\sigma)$ value are shown in Table 1 for both models. We can see from the overview that the optimal parameter α values are within the interval of $<10^{-10}; 10^{-4}>$.

The solution could be instable outside of this interval. The solution is not stable when parameter α is smaller than 10^{-10} for both models. The iterations count increases very quickly with parameter α greater than 10^{-3} in example one and it shows the same rate of progress in example two when α is greater than 10^{-7}. The iterations count is dependent on the location and number of non-homogeneous regions. The final size of objective function $\Psi(\sigma)$ does not depend on parameter α. It depends, however, on the number and location of non-homogeneous regions. The time of solution in relation to regularization parameter α is provided in Fig.4. Non-homogeneous regions for model two is recognized in Fig. 6. The TVM was used for finding the regions. The results were obtained for the value of regularization parameter $\alpha = 10^{-8}$. There was defined zero Level Set boundary, see in Fig. 7. This boundary defines non-homogeneous regions and their close surrounding for the next usage of the TVM and finds the final conductivity results.

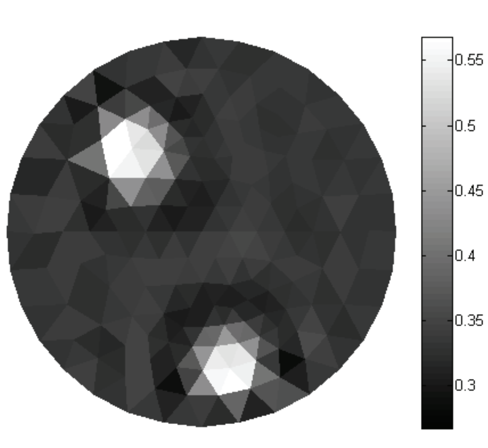

Fig.6. The region with non-homogeneous conductivity found by the TVM in example two and $\alpha = 10^{-8}$

The effects of starting conductivity on iterations count and $\Psi(\sigma)$ value are in Table 2 for both models. If the starting conductivity value was used at the interval <0.004; 1>, the solution is always stable. The iterations count was independent of the starting conductivity value. The final size of objective function $\Psi(\sigma)$ does not depend on the starting conductivity value. The non-homogeneous regions were started to be recognized by TVM after 5 - 9 iterations in the random distribution of starting conductivity on elements. The moment of recognition is identical with the number of iterations related to constant conductivity. It is proper to use starting conductivity near the original value where the iterations count is smaller. The time of solution in relation to starting conductivity is provided in Fig.5.

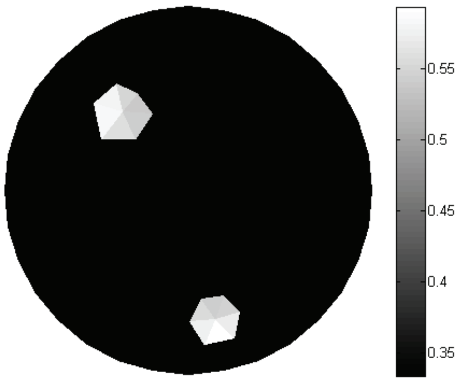

Fig.7. The defined zero level set boundary in example two and $\alpha=10^{-8}$

Conclusions

The paper describes the effect of regularization parameter and initial conductivity on the final reconstruction of conductivity in electrical impedance tomography.

The algorithm was tested on several models showing different distribution of non-homogeneous regions. The optimum size of regularization parameter was found to be $\alpha = 10^{-8}$; the selected starting conductivity for the reconstruction equals to σ = 1 S/m. The retrieved values can be applied in performing fast and efficient conductivity reconstruction in problems with conductivity range of 0.005 – 1 S/m.

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