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Adaptive mesh refinement for magnetic stimulation modeling

***Abstract**. The aim of the paper is discussion of need for adaptive mesh refinement techniques for numerical modeling of magnetic stimulation. Problems of a' posteriori error estimation and automatic mesh refinement algorithms has been addressed. Results present that values in the solution can be changed up to 40% during refinement process. It has been shown that refinement algorithms introduce serious computational cost for realistic 3D meshes.*

Streszczenie.** Celem artykułu jest przedstawienie potrzeby wykorzystania adaptacyjnych algorytmów poprawy sieci elementów skończonych podczas modelowania zagadnień stymulacji magnetycznej. Omówiono problemy estymacji błędu lokalnego, automatycznego zagęszczania sieci bez pogorszenia jej jakości. Wyniki pokazują ponad 40% zmianę wartości rozwiązania podczas poprawiania sieci. Nie można jednak zapominać, że zagęszczanie realistycznej, trójwymiarowej siatki wiąże się z dużymi nakładami obliczeniowymi. **(Adaptacyjne zagęszczanie sieci przy modelowaniu stymulacji magnetycznej)

Keywords: adaptive mesh refinement, magnetic stimulation modeling, eddy currents problem
Słowa kluczowe: adaptacyjne zagęszczanie siatki, modelowanie stymulacji magnetycznej, zagadnienie prądów wirowych

Introduction

The Finite Element Method is a powerful tool for solving problems described by partially differential equations. For given discretization, the method minimizes global norm of error. It is unable to ensure local accuracy. Especially for highly non-homogeneous source fields there is a need to refine mesh according to the solution. The adaptive mesh refinement (AMR) is methodology to address this problem. Despite it is well known, and as old as roots of the FEM, it is current need to adopt the method to new set of numerical challenges. Numerical simulation of biomedical object is excellent example of such challenge.

The magnetic stimulation is term describing therapeutic interaction between magnetic field and human body. There are many applications of low frequency magnetic field in medicine. One of them is therapy called Transcranial Magnetic Stimulation (TMS), where exciting coil is placed with human head (see Fig. 1). High and sharp peak of current in the coil generates impulse of magnetic field. Then brain is stimulated by the eddy currents effect.

Fig.1. Magnetic stimulation of human head. Adaptive mesh refinement increase maximum current density under the excitation coil.

It should be mentioned that refinement techniques have already been successfully applied in two-dimensional problems of bioelectromagnetics. Molinari [8] has used refinement to improve quality of tomography images, Shou [11] has shown that adaptivity can be used in ECG problem. On the other way, adaptivity approach has been investigated for magnetic field problems [5][6]. In authors best opinion, adaptivity aspects for magnetic stimulation of human body has not been discussed yet.

In this paper author exploits mesh refinement techniques to realistic 3D problems. We begin with a mathematical description of magnetic stimulation phenomena. Then, adaptive mesh refinement technique is discussed with special attention given to numerical aspects of local mesh refinement techniques. Finally, method will be validated on two numerical cases.

Mathematical model

Technically magnetic stimulation of human body can be thought as a eddy current problem. Current density vector can be described using electric, scalar potential, and magnetic vector potential:

(1) $$\boldsymbol{J}(t) = \sigma \nabla \varphi(t)\, \sigma \frac{\partial \boldsymbol{A}(t)}{\partial t}$$

Assuming that there are not current sources in the region, we formulate partially differential equation for this phenomena:

(2) $$\nabla \cdot \big(\sigma \nabla \varphi(t) \big) = -\nabla \cdot \Big(\sigma \frac{\partial \boldsymbol{A}(t)}{\partial t}\Big)$$

where σ is material conductivity, and $\frac{\partial \boldsymbol{A}(t)}{\partial t}$ is time derivative of magnetic vector potential **A**. With assumption about low conductivity potential **A** can be treated as a know, and calculated using Biot-Savart Law by coil integration:

(3) $$\boldsymbol{A}(t) = \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J_s}}{r}\, dv$$

Boundary condition is derived from current continuity on the border of the conductor. It is simple Neumann condition where potential derivative is related with normal component of exciting magnetic field:

(4) $$\frac{\partial\, \varphi(t)}{\partial n} = -\frac{\partial A_n}{\partial t}$$

Adaptive mesh refinement

The main aim of the mesh refinement techniques is reduction of maximum solution error. It can be achieved by three main types of algorithms: h-refinement (size of elements are reduced), p-refinement (higher order basis functions are used), r-refinement (mesh is smoothed to increase mesh quality). For magnetic stimulation modeling, author decided to utilize h-refinement, since it can be implemented as a pure mesh transformation, and it doesn't disturb realistic shapes of internal structures of the human body.

The simplest approach is to use a uniform refinement in the entire computational domain. Although it is effective for 2D meshes, in 3D, the uniform mesh refinement quickly leads to an enormous number of unknowns [2]. The adaptive refinement approach can address this problem.

The aim of AMR is to uniformly spread numerical error over the mesh. The error function is related to the solution, so, in a way, AMR forces the mesh to follow the solution. The first step in the construction of the algorithm is to find an appropriate error estimate. Once we have it in all tetrahedra we can develop strategies to improve the solution. Since we strive to keep the error evenly distributed across the mesh, every cell where the error is worse than a threshold fraction of the maximum error is marked for improvement (that means that every cell with the error value higher then threshold will be refined). The threshold value (20%) is chosen experimentally, and is a compromise between large values (a lot of refinement iterations required), and small values (too many unknowns, close to the uniform refinement).

There are many different approaches to deal with posteriori error estimation [3][12], but author of this article deploy estimator based on the fact that there are not current sources in the region. Then value of total current flowing throw each element could be treated as an error:

(5) $$e_i = \sum_{j=0}^{3} s_j (\boldsymbol{J_i} \cdot \boldsymbol{n_j})$$

where s_j is area of the j-th side of tetrahedron and $\boldsymbol{n}_j$ is vector normal to this side. While $\boldsymbol{J}_i$ is current density vector, which is constant in the i cell.

The most popular mesh refinement algorithms are based on Delaunay triangulation [11]. Unfortunately this is not the case for magnetic stimulation, where boundaries between body tissues have to be preserved. In this paper recursive Rivara algorithm [4] for bisection has been successfully applied. The heart of the algorithm is the longest-edge bisection repeated as many times as needed to obtain conforming mesh. Only using the longest-edge splitting guarantees that the mesh quality will not deteriorate during the refining process. The algorithm does not have a mathematical proof, but many numerical experiments confirm its robustness. The main challenge that has to be addressed by a refinement algorithm is how to maintain the mesh conformity. Conformity, in this context, means that every facet (except facets on the model external surface) is shared by exactly two tetrahedral cells. So, every edge bisection, must be followed by a sequence of bisections in neighboring cells [9].

Algorithm 1 – Recursive Rivara LEPP

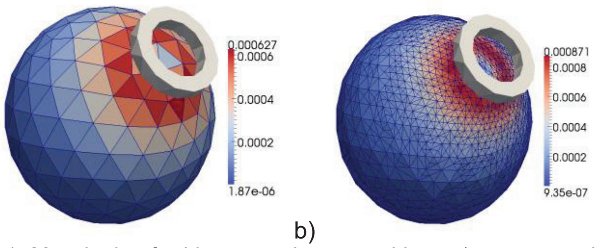
```
for all cell marked for refinement:
  if cell has not been bisected:
    bisect( cell, null )

function bisect( cell, nonconforming edge ):
  find the longest edge
  if nonconforming edge is the longest edge :
    generate two conforming cells
  else :
    perform bisection on the longest edge
    for all cell sharing the edge :
      bisect( cell, edge )
```

The Rivara longest-edge propagation path (LEPP) can be implemented in various ways. We found that for complicated 3D meshes the best efficiency is obtained by utilizing recursion (see Algorithm 1). Function bisect() is called recursively in each cell. The nonconforming edge is passed as an argument to the function, that way the algorithm avoids the global edge storage problem, a potentially serious issue in an iterative implementation. As a result we have an excellent, refined mesh obtained in a very efficient way.

It should be mentioned than despite the general idea of LEPP is the same, there is serious difference in complexity of implementation between algorithm in 2D and 3D. Moreover realistic models can easily contain millions of cells, what easily lead problem to be too large because of memory or time limitations. We deal this challenges by implementing above algorithms in Dolfin library [1].

Simulation results

To verify the method, simple, homogeneous sphere was generated. The original mesh contained less that 1000 elements which should be understood as a coarse mesh. Small, ring shape coil was placed close to the surface of the sphere. That way we produce non-homogeneous excitation field, which highlights need of adaptive refinement.



a) b)

Fig.1. Magnitude of eddy current in test problem. a) coarse mesh, b) mesh after 7 iterations of refinement.

After 7 iterations of refinement final mesh consists of over 50000 of cells (compare Fig 1a), and 1b)). One can easily observe that mesh was refined mainly under the coil, in the region of high current density. Detailed analysis of maximum current magnitude value (see dashed line on Fig. 2.) shows that it increased by more than 40%.

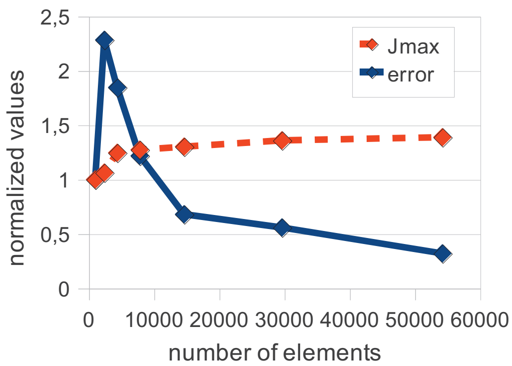


Fig.2. Maximum current density value (Jmax) and maximum error (error) during adaptive mesh refinement.

Unexpected shape of error convergence line from Fig. 2 requires some comment. The AMR suppose to reduce maximum error, but actually in our simple sphere model first iteration doubled error value. Explanation of this strange behavior lays in very high quality of original mesh. Analytically generated elements are so proportional, that first bisection just make them worse. Fortunately, after 4th iteration error was below the original level.

Head stimulation

The final challenge for the developed software was model of Transcranial Magnetic Stimulation. Near the right temple of human head model exciting coil has been placed (see Fig. 1). Mesh consists of 620000 tetrahedral cells, which define five different tissues of head [7]. Electrical parameters of the body, as well as parameters of excitation coil has been taken from literature [2]. Obviously, they play very important role for realistic calculations, but we won't discuss them, since solution can be scaled linearly. For that reason we normalize calculated values.

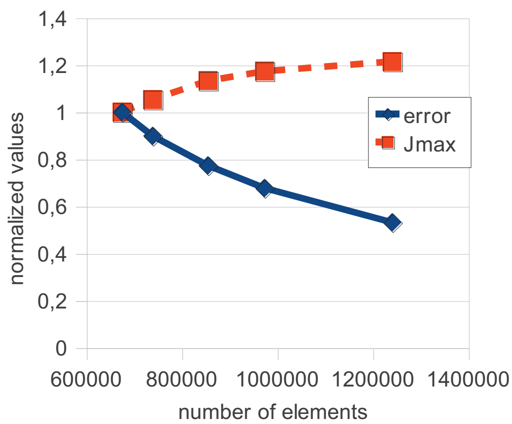


Fig.3. Convergence of AMR algorithm for head model

Five cycles of AMR has been performed for head stimulation model. Initial mesh with 670000 elements was refined to over 1200000 elements. As presented on Fig. 3 average estimated error was significantly (by 50%) reduced during refinement, at the same time maximum value of the current density increased by 20%.

Generation of mesh for biological objects is much more complicated than for simple shapes (e.g. sphere), and from that reason original mesh is lower quality than sphere mesh, so we don't observe any error jumps at the first iterations.

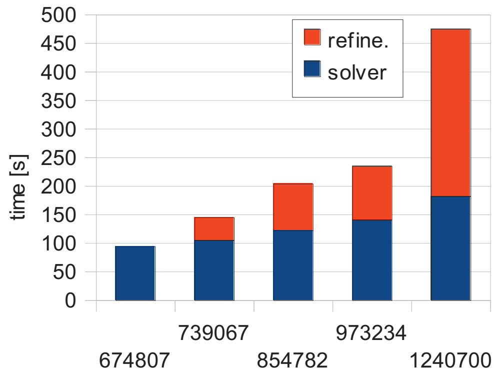


Fig.4. Calculation time for 5 iterations of AMR.

Computations was performed using novel desktop computer (CPU: 3GHz, RAM: 4GB). Algorithm is sequential, so only one core of processor was utilized.

Fig. 4 shows calculation time for every iteration of refinement. Total time of each iteration consists of solver time (needed to formulate and solve FEM problem), and refinement time (consumed by 3D LEPP refinement algorithm). One can notice that solver time scales linearly. This very nice feature is provided by Algebraic Multigrid algorithms (AMG), which has been deployed in preconditioner for linear solver [10]. Unfortunately refinement algorithm scales much worse. In our case for meshes above 1mln. of elements LEPP algorithm took more time that actual solution of FEM problem.

The mesh was properly refined under the stimulation coil, where field values are the highest. Fig. 5 visualize mesh density inside the model, before and after refinement.

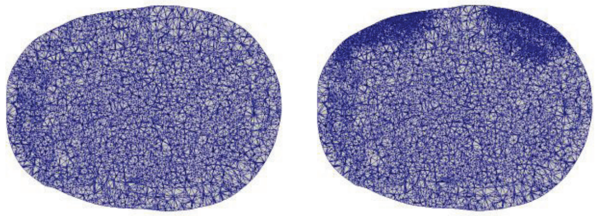


Fig.5. Cross-section of the mesh used for head stimulation, (left) without refinement, (right) after 5 iterations of refinement.

Conclusions

Presented results proves that refinement is important factor for magnetic stimulation modeling. Proper element size adjustment can increase maximum current density on about 20%-40%.

We experienced serious computational costs introduced by the mesh refinement algorithm. This is serious impediment for detailed, biomedical models, however global refinement is definitely less advisable solution. Further efforts should be placed on utilization of modern computer multicolored architecture by developing parallel version of algorithms.

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