Determination of dynamic characteristics of the single-phase capacitor induction motor

Abstract. The paper presents a mathematical model and simulation results of dynamic characteristics of the single-phase capacitor induction motor for different values of the capacitor capacitance and moment of inertia at no-load and nominal load conditions. The model has been used to study the effect of some machine parameters on the start-up and load performance of the motor. Computed dynamic characteristics were compared with measured ones to confirm correctness of the mathematical model and parameters.

Streszczenie. Artykuł przedstawia model matematyczny oraz wyniki symulacji stanów dynamicznych jednofazowego silnika indukcyjnego z pomocniczym uzwojeniem kondensatorowym dla różnych pojemności kondensatora pracy oraz momentu bezwładności. Obliczone charakterystyki dynamiczne porównano z wynikami eksperymentalnymi dla sprawdzenia poprawności modelu matematycznego i przyjętych parametrów.

Introduction

Single-phase capacitor induction motors (SPCIMs) are commonly used as a drive for fans, pumps and compressors. Generally, they are induction machines of the symmetrical rotor cage and non-symmetrical two stator windings (the main winding and auxiliary winding with starting or running capacitor) supplied with the same sinusoidal voltage source. In modelling performance characteristics of the induction motors circuit models of lumped parameters are still often used due to their simplicity and fast computation. At first approximation of the mathematical models linearity of magnetic circuit is assumed and slotting of the stator and rotor are neglected [1-3]. Majority of SPCIM models for analysis of performance of the motor with variety variants of winding-capacitor arrangements referred in literature have used two-axis models or based on rotating magnetic field theory models that include higher mmf harmonics. Authors in the paper [4] presented a general model the permanent split capacitor induction machine of asymmetrical windings and arbitrary tapping of the main winding with the auxiliary winding. The model was validated by experimental results obtained at steady-state performance of the tested motor. In the paper [5] a simplified equivalent circuit model was proposed which have been used for calculating steady-state characteristics for various values of capacitor capacitance and phase angle difference. Experimental study of an influence of stator winding harmonics on torque-speed characteristics were presented in [6] which shown considerable impact of the harmonics on the starting performance of the motor. Authors in [7] presented the concept of equivalent circuit which parameters were determined by finite element method. Dynamic analysis of the SPCIM were also studied in [8]. General equations of a single-phase induction motor in dq system and their application for simulation at continuously varied capacitance to improve performance of the motor were presented. In the paper [9] only simulation of currents, electromagnetic torque and voltage across capacitor of the motor using PSPICE and EMTP packages were presented without velocity waveforms and comparison of the electromagnetic torque with an experiment. These models may be completely useful for preliminary or initial searching of better structural solutions in design optimization.

In the paper the mathematical model of the capacitor induction motor in the stationary dq system is described and developed for simulation start-up of the motor at no-load and application of nominal load torque after starting from no-load operation. The mathematical model and parameters of the capacitor motor in the dq system for steady-state operation is described in details in [10, 11, 12]. The simulations include waveforms and steady-state characteristics of the tested capacitor induction motor for different values of capacitor capacitance placed in the auxiliary stator winding.

Description of mathematical model of SPCIM

The single-phase 2 poles capacitor induction motor which cross-section and the dq model are shown in Fig. 1 was used for computational analysis. The main (M) and auxiliary (A) stator windings are distributed in stator slots so that they form the dq axes fields. The capacitor capacitance is placed in the stator auxiliary winding.

![Fig. 1. Cross-section view and the d-q model of tested capacitor induction motor](image-url)
The mathematical model of the capacitor induction motor can be described by electrical and mechanical differential equations in the arbitrary reference frames as follows:

\[
\frac{di_{ax}}{dt} = \frac{1}{L_z} \left[ i_{ax} L_z R_s + i_{ax} \left( -L_z \alpha \cdot \omega \right) + L_z i_{ax} \right] - i_{ax} R_s i_{ax} + i_{ax} L_z \omega - i_{ax} u_{ax} + u_{ax} L_z \omega
\]

\[
\frac{di_{ay}}{dt} = \frac{1}{L_z} \left[ i_{ay} L_z R_s + i_{ay} \left( -L_z \alpha \cdot \omega \right) + L_z i_{ay} \right] - i_{ay} R_s i_{ay} + i_{ay} L_z \omega - i_{ay} u_{ay} + u_{ay} L_z \omega
\]

\[
\frac{di_{az}}{dt} = \frac{1}{L_z} \left[ i_{az} L_z R_s + i_{az} \left( -L_z \alpha \cdot \omega \right) + L_z i_{az} \right] - i_{az} R_s i_{az} + i_{az} L_z \omega - i_{az} u_{az} + u_{az} L_z \omega
\]

\[
\frac{dw_{ax}}{dt} = \frac{1}{C} i_{ax}
\]

\[
\frac{dw_{ay}}{dt} = \frac{1}{C} i_{ay}
\]

\[
\frac{dw_{az}}{dt} = \frac{1}{C} i_{az}
\]

\[
\theta = \frac{1}{s} \omega_t
\]

where:

\[
L_z = L_{ax}^2 - L_{ax} L_{ay} - L_{ay}^2 - R_s = R_{ax}, L_{ax} = L_{ay} = L_{ax}
\]

and \(i_{ax}, i_{ay}, i_{az}\) are the direct- and quadrature- components of stator currents, \(i_{ax}, i_{ay}, i_{az}\) are the direct- and quadrature-components of rotor currents, \(u_{ax}, u_{ay}, u_{az}\) are the direct- and quadrature-components of stator voltages, \(u_{ox}, u_{oy}, u_{oz}\) are the voltage across the capacitor, \(R_s\) is the resistance of the rotor windings, \(L_{ax}\) is the self inductance of the stator windings, \(L_p\) is the self inductance of the rotor windings, \(L_{ms}\) is the stator magnetizing inductance, \(p\) is the number of pole pairs, \(D_f\) is the viscous friction coefficient, \(J\) is the moment of inertia, \(T_e\) is the electromagnetic torque, \(T_L\) is the load torque, \(\omega_r\) is the electrical angular velocity of the rotor, \(\theta_t\) is the electrical angular displacement of the rotor.

The dynamic behaviour of the capacitor motor supplied from sinusoidal voltage source of 230V and 50 Hz at different capacitor capacitance was studied for two cases:

- the start-up of the motor with no-load (\(T_L=0\)) until steady-state is reached (free acceleration)
- the starting of the motor at no-load and after 0.7s applying the rated load (\(T_L=0\)) until steady-state is reached.

Dynamic operation of the tested motor

The dynamic operation of the tested motor at no-load and after 0.7s applying the rated load (\(T_L=0\)) until steady-state is reached.

Table 1. Ratings and data of the tested capacitor induction motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>0.09 kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>230 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>0.9 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>2840 rev/min</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.55</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.9</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Number of stator windings</td>
<td>2</td>
</tr>
<tr>
<td>Running capacitor capacitance</td>
<td>3μF</td>
</tr>
<tr>
<td>Connection of stator windings</td>
<td>parallel</td>
</tr>
<tr>
<td>Number of poles: main/auxiliary winding</td>
<td>2/2</td>
</tr>
<tr>
<td>Windig layout: main/auxiliary winding</td>
<td>single layer</td>
</tr>
<tr>
<td>Rotor winding</td>
<td>squirrel cage</td>
</tr>
<tr>
<td>Number of slots: stator/rotor</td>
<td>18/11</td>
</tr>
<tr>
<td>Lamination material - generator sheet</td>
<td>M 600-50A</td>
</tr>
<tr>
<td>Laminated core length</td>
<td>32 mm</td>
</tr>
</tbody>
</table>

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All simulations were performed at the nominal moment of inertia of the motor ($J=0.00007 \text{ kgm}^2$) and the viscous friction coefficient $D_{f}=0.00005 \text{ Nms}$.

Motor starting waveforms obtained at no-load operation are given in Figs. 2-4.

After the motor is reaching no-load steady-state operation, it can be observed in torque waveform oscillations at twice line frequency which influence on non-uniform running of the rotor (pulsations of the angular velocity). This is the reason of elliptical rotational field in the air-gap due to asymmetry of the stator windings. Phase shift between currents and voltages of the stator windings for no-load and the capacitor capacitance of $3 \mu \text{F}$ are illustrated by the relative current and voltage runs in Figs. 5-6.

The waveforms of the motor starting from no-load operation and after application of rated load are shown in Figs. 7-11.
Influence of capacitor capacitance

Influence of the capacitor capacitance placed within the auxiliary stator winding on amplitude of the torque, start-up time and pulsation of rotor velocity are shown in Figs. 12-15.

It is seen that increasing capacitor capacitance up to 4 μF causes decreasing electromagnetic torque pulsation and simultaneously increasing start-up time of the motor. Further increase of the capacitance leads to increase of pulsation of the torque. It is evident that the greater capacitor capacitance brings about phase shift increasing between the current and voltage in the auxiliary winding (Figs. 14, 15).

It follows that the tested motor has the shortest start-up time and the lowest pulsations of torque and angular velocity at steady-state for capacitor capacitance of 4 μF.
Influence of moment of inertia

An effect of the moment of inertia of the rotor on torque, start-up time and velocity during free acceleration of the motor and after application of rated load are shown in Figs. 16-18 and 21-23.

Fig. 16. The electromagnetic torque $T_e(t)$ at rated load: $C=3\mu F$, $J=0.00014$ kgm$^2$

Fig. 17. The angular velocity of rotor $\omega_r(t)$ at rated load: $C=3\mu F$, $J=0.00014$ kgm$^2$

Fig. 18. Torque-speed characteristic at rated load for $C=3\mu F$, $J=0.00014$ kgm$^2$

Fig. 19. The current of stator main winding at rated load for $C=3\mu F$, $J=0.00014$ kgm$^2$

Fig. 20. The current of stator auxiliary winding at rated load for $C=3\mu F$, $J=0.00014$ kgm$^2$

Fig. 21. The electromagnetic torque $T_e(t)$ at rated load: $C=3\mu F$, $J=0.00021$ kgm$^2$

Fig. 22. The angular velocity of rotor $\omega_r(t)$ at rated load: $C=3\mu F$, $J=0.00021$ kgm$^2$

Fig. 23. Torque-speed characteristic at rated load for $C=3\mu F$, $J=0.00021$ kgm$^2$

It is evident that the greater moment of inertia causes decreasing pulsations of the electromagnetic torque, rotor velocity and at the same time increasing start-up time of the
motor. After application of load pulsations of the rotor velocity are visibly decreased.

The stator currents waveforms indicate some asymmetry in the main and auxiliary windings for the capacitor capacitance of 3 μF (Figs. 19-20, 24-25).

It is shown by simulation that the lowest pulsations of the torque and velocity at starting occur for the capacitor capacitance of 4 μF and further increase of the capacitance leads to increase of the pulsations. On the ground of dynamic simulation it is possible to find an optimal value of the capacitor capacitance which assures desired performance parameters (the start-up time, pulsation of torque and angular velocity) of the capacitor induction motor in transient and steady-state operations.

Experimental results

A general scheme of a test bench which was used for measurement of transient characteristics of the tested motor is shown in Fig. 25.

It allows measuring the following electrical and mechanical quantities:

- voltage and intake current of the motor
- rotational speed and torque
- voltage across capacitor

Using the Labview virtual instrument all data for transient waveforms are recorded and then processed by algorithm developed in the Matlab software.

Experimental tests were performed at the same load conditions as simulation which results are shown in Figs. 26-29.

Simulating waveforms of torque, velocity and intake current presented in Figs. 21-24 show satisfactory degree of convergence with the ones obtained by measurements (Figs. 26-29). Discrepancies between simulation and measurements in the case of pulsation of torque and current are mainly caused by non-sinusoidal shape of the voltage supply which had been used in laboratory test (4% total harmonic distortion) and due to neglecting non-linearity of magnetic circuit in the simulating method.
Conclusions

Application of the stationary dq model of the single-phase capacitor induction motor makes possible simulation of dynamic performance characteristics and selection of the capacitor capacitance to obtain desired starting transients at minimal pulsation of the torque and angular velocity. More accurate simulation of performance characteristics require taking into account of non-linearity of magnetic circuit of the motor and skin-effect in the rotor bars. It needs in turn combining the above circuit method with numerical calculation of the magnetic field in the motor in order to consider changes of inductance and resistance parameters especially at starting of the motor. Despite of its some inaccuracy in simulating dynamic behaviour of the motor the above presented mathematical model is very effective as regards time of computation.

REFERENCES


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