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Design of an optimal actuation signal for identification of a torsional spring system

***Abstract**. The choice of an input signal used for perturbation of the system is critical in the task of model building and parameter identification. System identification, in practice is carried out by perturbing processes or plants in operation. In the paper the optimal excitation signal was generated for a torsional spring model. The objective of this kind of experiment design is to minimise the variance of the parameters to be estimated. In this case, the objective function was formulated through maximisation of the Fisher information matrix determinant (D-optimality) in the form of a conventional integral criterion with amplitude constraints. It was shown that the optimal input signal used for system excitation minimises the volume of the ellipsoidal confidence region of parameters estimates.*

***Streszczenie.** Dokładność uzyskiwanych estymat parametrów identyfikowanego modelu zależy przede wszystkim od doboru odpowiedniego sygnału wejściowego, który wzbudza wybrane wejście obiektu regulacji. W wielu praktycznych zastosowaniach identyfikacja jest przeprowadzana w czasie rzeczywistym, podczas normalnej pracy obiektu (procesu technologicznego). W pracy przedstawiono wyniki doboru optymalnego sygnału pobudzającego układem skrętnym. Celem takiego eksperymentu jest minimalizacja wariancji uzyskiwanych estymat parametrów. Maksymalizowano funkcjonał celu określony jako wyznacznik macierzy informacyjnej Fishera uwzględniając nałożone ograniczenia na amplitudę sygnału wejściowego. Stwierdzono, że optymalne pobudzenie identyfikowanego obiektu minimalizuje elipsoidalne obszary ufności estymowanych parametrów. (**Dobór optymalnego sygnału pobudzającego w zadaniu identyfikacji układu skrętnego**).*

Keywords: optimal excitation signal, *D*-optimality, system identification, ellipsoidal confidence region.
Słowa kluczowe: optymalny sygnał pobudzający, *D*-optymalność, identyfikacja systemu, elipsoidalny obszar ufności.

Introduction

The choice of an input signal used for actuation of the system is critical in the task of model building and parameter identification. System identification is the process of constructing an accurate and reliable dynamic mathematical model of the system from observed data and available knowledge. It is a common practice to perturb the system of interest and use the resulting data to build the model [1, 2, 3]. The accuracy of parameter estimates is increased by the use of optimal excitation signals [4, 5].

Particular industries, such as petrochemical and refining industries, rely almost exclusively on system identification as the principal means for obtaining dynamic models for advanced control purposes. The input design problem with respect to the intended model application, which is often control, has received considerable attention recently [6, 7, 8, 9, 10]. It was reported that model development absorbs about 75% of the costs associated with advanced control projects [11]. That is why the input signal used for perturbing the system should by carefully selected.

System identification, in practice is carried out by perturbing processes or plants in operation. In many industrial applications a plant-friendly input signal would be preferred for system identification. Plant-friendly identification experiments are those that satisfy plant or operator constraints on experiment duration, input and output amplitudes or input rate [12, 13, 14]. Techniques for synthesising multi-harmonic signals with low crest factors, which are attractive from a plant-friendly perspective, have been reported in [1]. It was demonstrated that plant friendliness demands are often in conflict with requirements for good identification [15]. Hence, plant-friendly input design is inherently multi-objective in nature.

There have been some recent reports on multi-objective optimisation based methods, applied to identification and control [16]. However, such an approach to optimal input design has not been attempted. In the design of optimal inputs for system identification the sensitivity of the state variable to the unknown parameter has been maximised so far. We present a Mayer's canonical formulation of the performance index for optimal input design of a torsional spring system. The results of optimal input design utilising Mayer's canonical formulation of the performance index for the inertial case study were presented in [17, 18, 19].

Multiparameter optimal inputs

In this section the design of optimal inputs for systems with multiple unknown parameters are considered. In the design of optimal excitation signals for estimating more than one parameter, a suitable scalar function of the Fisher information matrix $\mathbf{M}$ must be selected as the performance criterion. A criterion, which is often used, is the trace of the matrix $\mathbf{M}$, wherein the sum of diagonal elements of the Fisher information matrix is maximised. Other measures of performance are as follows [20]:

- *A*-optimality: $tr(\mathbf{M}^{-1})$, minimises the average variance of the parameters.
- *E*-optimality: $\lambda_{max}(\mathbf{M}^{-1})$, minimises the maximum eigenvalue of $\mathbf{M}^{-1}$.
- *D*-optimality: minimises the volume of the ellipsoidal confidence region.

However, the choice of the experiment criterion is important, as it is possible that inputs designed based on some criteria may not be persistently exciting [1].

We consider the problem of synthesising an optimal input in the time domain for a system described by the following state space model

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{v}(t)\end{aligned} \qquad (1)$$

where $\mathbf{x}$ is an $n \times 1$ state vector, $\mathbf{u}$ is a $m \times 1$ input vector, $\mathbf{y}$ is a $r \times 1$ output vector, $\mathbf{v}$ is a $r \times 1$ measurement noise vector, $\mathbf{A}$ $(n \times n)$, $\mathbf{B}$ $(n \times m)$, $\mathbf{C}$ $(r \times n)$ are, respectively: state, input and output matrices of the system. The vector $\mathbf{v}$ is a zero-mean Gaussian white noise process

$$E[\mathbf{v}(t)] = 0 \qquad (2)$$

$$E\left[\mathbf{v}(t)\mathbf{v}^{\mathrm{T}}(\tau)\right] = \mathbf{R}\delta(t-\tau) \qquad (3)$$

where $\mathbf{R}$ $(r \times r)$ is the covariance matrix of the measurement noise.

Let θ denote a $k \times 1$ vector of unknown parameters. The Fisher information matrix (FIM) is defined as

$$\mathbf{M} = \int_0^T \mathbf{X}_\theta^{\mathrm{T}} \mathbf{C}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{C} \mathbf{X}_\theta dt \tag{4}$$

where $\mathbf{X}_\theta$ is a $n \times k$ parameter influence coefficient matrix with ij-th component $\partial x_i/\partial\theta_j$. For input design purposes, it is convenient to assume an unbiased estimator, so that the covariance of the parameter estimates is given by the Cramer-Rao lower bound, viz., the inverse of the FIM. The covariance of the estimation error is

$$\operatorname{cov}\left[\widehat{\boldsymbol{\theta}}\right] = E\left[\left(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)\left(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)^{\mathrm{T}}\right] \geq \mathbf{M}^{-1} \tag{5}$$

where $\widehat{\boldsymbol{\theta}}$ is a vector of parameters to be estimated. The optimal input is to be determined in such a way that the trace of the $k \times k$ Fisher information matrix (*A*-optimality) [4]

$$\operatorname{tr}(\mathbf{M}) = \int_0^T \operatorname{tr}\left[\mathbf{X}_\theta^{\mathrm{T}} \mathbf{C}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{C} \mathbf{X}_\theta\right] dt \tag{6}$$

is maximised subject to the input energy constraint

$$\int_0^T \mathbf{u}^{\mathrm{T}} \mathbf{u} dt \leq E \tag{7}$$

In order to maximise $\operatorname{tr}(\mathbf{M})$, let us define the augmented $[(k+1)n \times 1]$ state vector [4]

$$\mathbf{x}_a = \left[\mathbf{x}^{\mathrm{T}}, \quad \mathbf{x}^{\mathrm{T}}{}_{\theta 1}, \quad \cdots, \quad \mathbf{x}^{\mathrm{T}}{}_{\theta k}\right]^{\mathrm{T}} \tag{8}$$

where $\mathbf{x}_{\theta i} = \partial \mathbf{x}/\partial\theta_i$. The state equation can be written as

$$\dot{\mathbf{x}}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{B}_a \mathbf{u} \tag{9}$$

where the $(k+1)n \times (k+1)n$ matrix $\mathbf{A}_a$ and $(k+1)n \times m$ matrix $\mathbf{B}_a$ are given by

$$\mathbf{A}_a = \left[\begin{array}{c|c|c|c} \mathbf{A} & 0 & \cdots & 0 \\ \hline \mathbf{A}_{\theta 1} & \mathbf{A} & \ddots & \\ \hline \vdots & & \ddots & \\ \hline \mathbf{A}_{\theta k} & & & \mathbf{A} \end{array}\right] \tag{10}$$

$$\mathbf{B}_a = \left[\mathbf{B}, \quad \mathbf{B}_{\theta 1}, \quad \cdots, \quad \mathbf{B}_{\theta k}\right]^{\mathrm{T}} \tag{11}$$

where $\mathbf{A}_\theta = \partial \mathbf{A}/\partial\theta_i$ and $\mathbf{B}_\theta = \partial \mathbf{B}/\partial\theta_i$. In addition we define $\mathbf{C}_a$ $(rk \times (k+1)n)$ and $\mathbf{R}_a^{-1}$ $(rk \times rk)$ as follows

$$\mathbf{C}_a = \begin{bmatrix} 0 & \mathbf{C} & 0 & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \mathbf{C} \end{bmatrix} \tag{12}$$

$$\mathbf{R}_a^{-1} = \operatorname{diag}\left(\mathbf{R}^{-1}\right) \tag{13}$$

Then the expression for $\operatorname{tr}(\mathbf{M})$, given by equation (6), can be written in the form

$$\operatorname{tr}(\mathbf{M}) = \int_0^T \mathbf{x}_a^{\mathrm{T}} \mathbf{C}_a^{\mathrm{T}} \mathbf{R}_a^{-1} \mathbf{C}_a \mathbf{x}_a dt \tag{14}$$

The objectives are:
– maximise $\operatorname{tr}(\mathbf{M})$,
– minimise integral of the square input signal,
– minimise time T.

The cost function, containing the input energy constraint, can be expressed as

$$J = \max_{\mathbf{u}} \frac{1}{2} \int_0^T \left[\mathbf{x}_a^{\mathrm{T}} \mathbf{C}_a^{\mathrm{T}} \mathbf{R}_a^{-1} \mathbf{C}_a \mathbf{x}_a - q \mathbf{u}^{\mathrm{T}} \mathbf{u}\right] dt \tag{15}$$

where q is a constant chosen so that the integral equation is satisfied. It must be noted that the time is not explicitly incorporated in the above cost function. Instead, the optimisation problem is solved for different values of termination time. Utilising Pontryagin's maximum principle, the Hamiltonian function is

$$\Re = \frac{1}{2}\left[-\mathbf{x}_a^{\mathrm{T}} \mathbf{C}_a^{\mathrm{T}} \mathbf{R}_a^{-1} \mathbf{C}_a \mathbf{x}_a + q \mathbf{u}^{\mathrm{T}} \mathbf{u}\right] + \boldsymbol{\lambda}^{\mathrm{T}}\left[\mathbf{A}_a \mathbf{x}_a + \mathbf{B}_a \mathbf{u}\right] \tag{16}$$

The co-state vector $\lambda(t)$ is the solution of the vector differential equation

$$\dot{\boldsymbol{\lambda}} = -\left[\frac{\partial \Re}{\partial \mathbf{x}_a}\right]^{\mathrm{T}} = \mathbf{C}_a^{\mathrm{T}} \mathbf{R}_a^{-1} \mathbf{C}_a \mathbf{x}_a - \mathbf{A}_a^{\mathrm{T}} \boldsymbol{\lambda} \tag{17}$$

The input vector $\mathbf{u}(t)$ that maximises $\Re$ is

$$\frac{\partial \Re}{\partial \mathbf{u}} = q\mathbf{u} + \mathbf{B}_a^{\mathrm{T}} \boldsymbol{\lambda} = 0, \quad \mathbf{u} = -\frac{1}{q} \mathbf{B}_a^{\mathrm{T}} \boldsymbol{\lambda} \tag{18}$$

The two-point boundary-value problem is then

$$\begin{bmatrix} \dot{\mathbf{x}}_a \\ \dot{\boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & -(1/q)\mathbf{B}_a \mathbf{B}_a^{\mathrm{T}} \\ \mathbf{C}_a^{\mathrm{T}} \mathbf{R}_a^{-1} \mathbf{C}_a & -\mathbf{A}_a^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_a \\ \boldsymbol{\lambda} \end{bmatrix} \tag{19}$$

with boundary conditions

$$\mathbf{x}_a(0) = \left[\mathbf{x}^{\mathrm{T}}(0), \quad 0, \quad \cdots, \quad 0\right]^{\mathrm{T}}, \ \boldsymbol{\lambda}(T) = 0 \tag{20}$$

The above differential equations are integrated to give $\mathbf{u}(t)$ using the method of complementary functions. However, calculation of the optimal excitation signal requires the knowledge of the system parameters $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and noise characteristics $\mathbf{R}$. A common technique is to design the optimal inputs based on an initial estimate of the model parameters and improving the choice of the parameters by iterative identification and experiment design [7].

The purpose of the current work is to formulate the multi-objective optimisation problem using Mayer's canonical formulation of the performance index for optimal input design.

Optimal input design for identification of a torsional spring system

The torsional spring model represents many similar physical plants including rigid bodies, flexibility in drive shafts, gearing and belts and coupled discrete vibration with actuator at the drive input. Thus the plant model order may be as high as six with either four, two, or no zeros [21].

In this section the second-order linear dynamic system is considered. The dynamic model for the one degree of freedom (1 DOF) plant is shown in below figure.

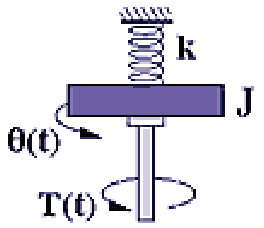

Fig. 1. The dynamic model for the one degree of freedom plant (free-clamped)

The equation of motion is as follows

(21) $$J_1\ddot{\theta}+c_1\dot{\theta}_1+k_1\theta_1=T(t)$$

where J_1, k_1, c_1, $T(t)$, θ_1 are, respectively: disk inertia, spring coefficient, damping ratio, force signal, position of the first disk of the plant. For notational convenience, let us introduce $x_1=\theta_1$ and $x_2=\dot{\theta}_1=\dot{x}_1$. Then, the problem of synthesising an optimal input in the time domain for a torsional spring plant (1 DOF) can be described by the following single input, single output state space model

(22) $$\begin{aligned} &\dot{x}_1=x_2; && x_1(0)=x_{10}\\ &\dot{x}_2=ax_1+bx_2+cu; && x_2(0)=0\\ &y(t)=x_1(t) \end{aligned}$$

where a = $-k_1/J_1$, b = $-c_1/J_1$, c = $1/J_1$. In order to design an optimal input signal, it was necessary to scale the model of the system as follows

(23) $$\begin{aligned} &\xi_1=px_1 \;\rightarrow\; \dot{x}_1=\frac{1}{p}\dot{\xi}_1\\ &\xi_2=gx_2 \;\rightarrow\; \dot{x}_2=\frac{1}{g}\dot{\xi}_2 \end{aligned}$$

Utilising equation (23), the state space model (22) can be expressed as

(24) $$\begin{aligned} &\dot{\xi}_1=\frac{p}{g}\xi_2\\ &\dot{\xi}_2=a\frac{g}{p}\xi_1+b\xi_2+cgu \end{aligned}$$

Assuming that the parameter p = 1, from equation (23) we obtain ξ_1 = x_1. Then the above problem can be suitably modified by defining the state space model as

(25) $$\begin{aligned} &\dot{x}_1=\frac{1}{g}x_2\\ &\dot{x}_2=agx_1+bx_2+cgu\\ &y(t)=x_1(t) \end{aligned}$$

where x_1 = $x_1(t;\ a,\ b,\ c)$, x_2 = $x_2(t;\ a,\ b,\ c)$ and model parameters a, b, c are constant.

The Fisher information matrix for the torsional spring (1 DOF) model (25) can be expressed as

(26) $$\mathbf{M}(T)\cong\int_0^T\begin{bmatrix}x_{1a}\\x_{1b}\\x_{1c}\\x_{2a}\\x_{2b}\\x_{2c}\end{bmatrix}\begin{bmatrix}x_{1a} & x_{1b} & x_{1c} & x_{2a} & x_{2b} & x_{2c}\end{bmatrix}dt$$

where: x_{ia} = $\partial x_i/\partial a$, x_{ib} = $\partial x_i/\partial b$, x_{ic} = $\partial x_i/\partial c$, i = 1, 2. The problem can be suitably modified by defining the augmented state vector as

(27) $$\begin{array}{lll}
x_1=x_1, & \dot{x}_1=\frac{1}{g}x_2, & x_1(0)=x_{10}\\
x_2=x_2, & \dot{x}_2=agx_1+bx_2+cgu, & x_2(0)=0\\
x_3=x_{1a}, & \dot{x}_3=\frac{1}{g}x_6, & x_3(0)=0\\
x_4=x_{1b}, & \dot{x}_4=\frac{1}{g}x_7, & x_4(0)=0\\
x_5=x_{1c}, & \dot{x}_5=\frac{1}{g}x_8, & x_5(0)=0\\
x_6=x_{2a}, & \dot{x}_6=g(x_1+ax_3)+bx_6, & x_6(0)=0\\
x_7=x_{2b}, & \dot{x}_7=g(ax_4)+x_2+bx_7, & x_7(0)=0\\
x_8=x_{2c}, & \dot{x}_8=gax_5+bx_8+gu, & x_8(0)=0\\
x_9=m_{11}, & \dot{x}_9=x_3^2, & x_9(0)=0\\
x_{10}=m_{12}=m_{21}, & \dot{x}_{10}=x_3x_4, & x_{10}(0)=0\\
x_{11}=m_{13}=m_{31}, & \dot{x}_{11}=x_3x_5, & x_{11}(0)=0\\
x_{12}=m_{14}=m_{41}, & \dot{x}_{12}=x_3x_6, & x_{12}(0)=0\\
x_{13}=m_{15}=m_{51}, & \dot{x}_{13}=x_3x_7, & x_{13}(0)=0\\
x_{14}=m_{16}=m_{61}, & \dot{x}_{14}=x_3x_8, & x_{14}(0)=0\\
x_{15}=m_{22}, & \dot{x}_{15}=x_4^2, & x_{15}(0)=0\\
x_{16}=m_{23}=m_{32}, & \dot{x}_{16}=x_4x_5, & x_{16}(0)=0\\
x_{17}=m_{24}=m_{42}, & \dot{x}_{17}=x_4x_6, & x_{17}(0)=0\\
x_{18}=m_{25}=m_{52}, & \dot{x}_{18}=x_4x_7, & x_{18}(0)=0\\
x_{19}=m_{26}=m_{62}, & \dot{x}_{19}=x_4x_8, & x_{19}(0)=0\\
x_{20}=m_{33}, & \dot{x}_{20}=x_5^2, & x_{20}(0)=0\\
x_{21}=m_{34}=m_{43}, & \dot{x}_{21}=x_5x_6, & x_{21}(0)=0\\
x_{22}=m_{35}=m_{53}, & \dot{x}_{22}=x_5x_7, & x_{22}(0)=0\\
x_{23}=m_{36}=m_{63}, & \dot{x}_{23}=x_5x_8, & x_{23}(0)=0\\
x_{24}=m_{44}, & \dot{x}_{24}=x_6^2, & x_{24}(0)=0\\
x_{25}=m_{45}=m_{54}, & \dot{x}_{25}=x_6x_7, & x_{25}(0)=0\\
x_{26}=m_{46}=m_{64}, & \dot{x}_{26}=x_6x_8, & x_{26}(0)=0\\
x_{27}=m_{55}, & \dot{x}_{27}=x_7^2, & x_{27}(0)=0\\
x_{28}=m_{56}=m_{65}, & \dot{x}_{28}=x_7x_8, & x_{28}(0)=0\\
x_{29}=m_{66}, & \dot{x}_{29}=x_8^2, & x_{29}(0)=0
\end{array}$$

In the design of optimal inputs for system identification the sensitivity of the state variable to the unknown parameter or the sensitivity of the observation to the unknown parameter is maximised. The justification for this approach is the Cramer-Rao lower bound, which provides a lower bound for the estimation error covariance. The parameter estimate or observation sensitivity tends to be lowered for an optimal input

(28) $$\operatorname{cov}([a,b]) \geq \mathbf{M}^{-1}$$

An optimal input for exciting the torsional spring (1 DOF) system is formulated through maximisation of the Fisher information matrix determinant (*D*-optimality) in the form of a conventional integral-criterion optimal control problem. The optimal input is to be determined such that the determinant of FIM is maximised, subject to the input energy constraint [17]

(29) $$\mathbf{M}(t) = \begin{bmatrix} m_{11}(t) & \cdots & m_{16}(t) \\ \vdots & \ddots & \vdots \\ m_{61}(t) & \cdots & m_{66}(t) \end{bmatrix}$$

where $m_{ij} = m_{ji}$. The equivalent optimal control problem utilising Mayer's canonical formulation which maximises the performance index, subject to the input energy constraint, is

(30) $$J = \det[\mathbf{M}(T_f)] - q \int_0^{T_f} u^{\mathrm{T}}(t) u(t) dt$$

where q is the input energy factor.

The above problem can be solved using one of the existing packages for solving dynamic optimisation problems, such as RIOTS_95 [22], DIRCOL [23] or MISER [24]. All computations were performed using low-cost PC (Pentium 4, 3,00 GHz, 512 MB RAM) running Windows XP and Matlab 7 (R14). Optimal and sub-optimal signals are computed for nominal values of parameters a = -88,95, b = -0,42, c = 52,02 assumed termination time T_f = 10 seconds and the scaling factor $g = 10^5$ utilising SQP algorithm [19]. The system is assumed to be at an initial state $x_1(0) = 0{,}393$, the initial value of the input signal is $u(0) = 1$ and $-5 \leq u(t) \leq 5$. The system dynamics was integrated using the fixed step-size fourth-order Runge-Kutta method with grid intervals of 0,2 seconds. The *D*-optimal input signals obtained for a different factors q are shown in below figures.

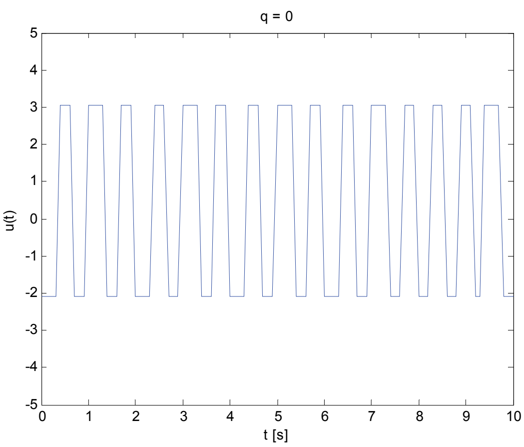

Fig. 2. Optimal input signal for the coefficient $q = 0$

In order to avoid getting stuck in a local minimum, all computations were repeated several times from different initial conditions. Each run took about sixty seconds. The optimal excitation signal obtained when there was no constraint on the input energy (i.e., for $q = 0$) is shown in Figure 2. The input energy factor was increased (Fig. 4) to obtain the critical value at the level of $q = 5 \times 10^{13}$. For comparison, Figure 3 shows the sub-optimal input signal, which corresponds to the value $q = 2 \times 10^{12}$. Figure 5 contains the graphical display of the non-optimal signal obtained for $q = 6 \times 10^{13}$, where the FIM determinant component was dominated by the integral of the squared input signal component of the maximised performance index.

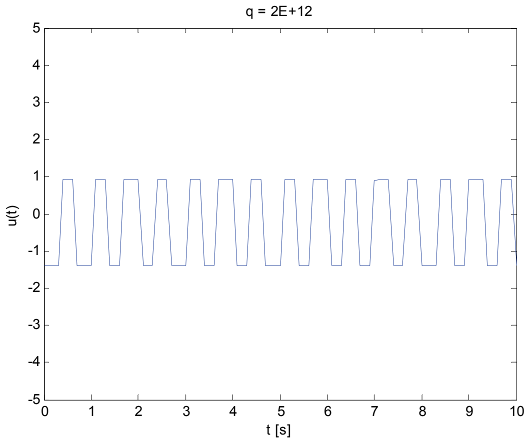

Fig. 3. Sub-optimal input signal for the coefficient $q = 2 \times 10^{12}$

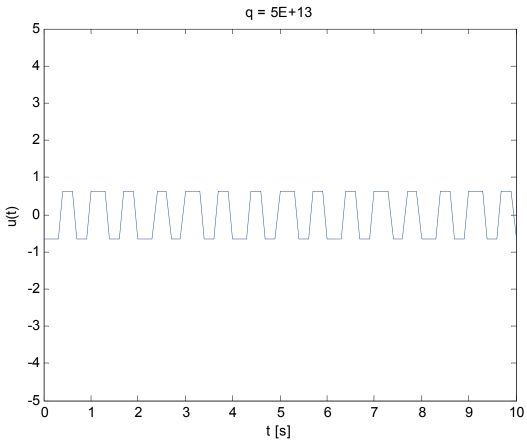

Fig. 4. Sub-optimal input signal for the coefficient $q = 5 \times 10^{13}$

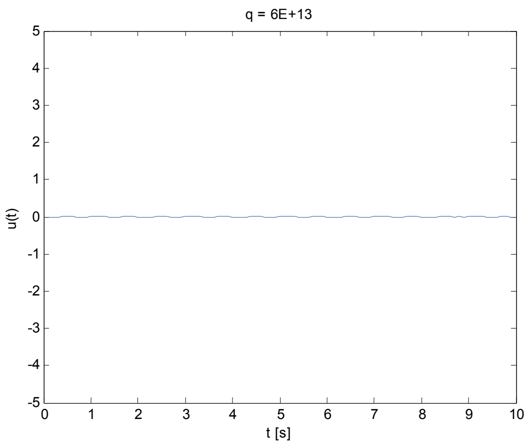

Fig. 5. Sub-optimal input signal for the coefficient $q = 6 \times 10^{13}$

The expression for the cost function, given by equation (30), can be written in the following form

(31) $$J = J_1 - qJ_2$$

where J_1 denote the FIM determinant and J_2 denote integral of the squared input signal. The numerical results for different selections of the input energy factor q are displayed in Table 1.

Table 1. Comparison of the performance index components

q	J	$q \times J_2$
0,00	$4{,}14 \times 10^{32}$	0,00
$2{,}00 \times 10^{12}$	$2{,}89 \times 10^{28}$	$5{,}64 \times 10^{14}$
$5{,}00 \times 10^{13}$	$8{,}11 \times 10^{25}$	$4{,}26 \times 10^{15}$
$5{,}60 \times 10^{13}$	$5{,}10 \times 10^{25}$	$4{,}50 \times 10^{15}$
$6{,}00 \times 10^{13}$	$1{,}66 \times 10^{-11}$	$3{,}46 \times 10^{-10}$

As it had been expected, the constraint (7) was active in the optimal solutions. We obtained $J = 1{,}66 \times 10^{-11}$, which means that the corresponding FIM is practically singular (i.e., for $q = 6{,}00 \times 10^{13}$).

Ellipsoidal confidence regions of parameter estimates of the torsional spring (1 DOF) model (25) for different selections of the input signal are shown in Figures 7, 8 and 9. The flow of information in system identification can be summarised as in Figure 6. We act on the physical system through the input $u(t)$ and collect information through the observations of its output $y(t)$. The presence of the white noise with variance from the interval $0 \le \sigma^2 \le 0{,}7$ makes the observations random variables. The model corresponds to theoretical vision of the system (25) which depends on a vector of unknown parameters θ.

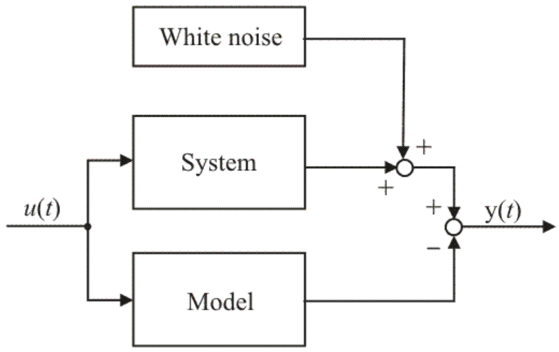

Fig. 6. Flow of information in identification

The objective of the system identification is to find the best value of θ in terms of the performance criterion. The two hundred runs have been made for minimisation of the output $y(t)$ with the torsional spring (1 DOF) model initial state from the interval $-0{,}4 \le x_1(0) \le 0{,}4$ [rad] and angular velocity from the interval $0 \le x_2(0) \le 4$ [rad/s] [19]. The optimisation was performed using Nelder-Mead method. The uncertainty ellipsoids of estimated torsional spring model parameters, obtained using the optimal input signal, are shown in Figures 7, 8 and 9(a). For comparison, Figures 7, 8 and 9(b) show the confidence regions obtained utilising the step input signal, and Figures 7, 8 and 9(c, d) contain the graphical display of the uncertainty regions obtained using sine wave input signals. Thus, it is clear that the optimal input signal used for system excitation minimises the volume of the ellipsoidal confidence region of parameter estimates.

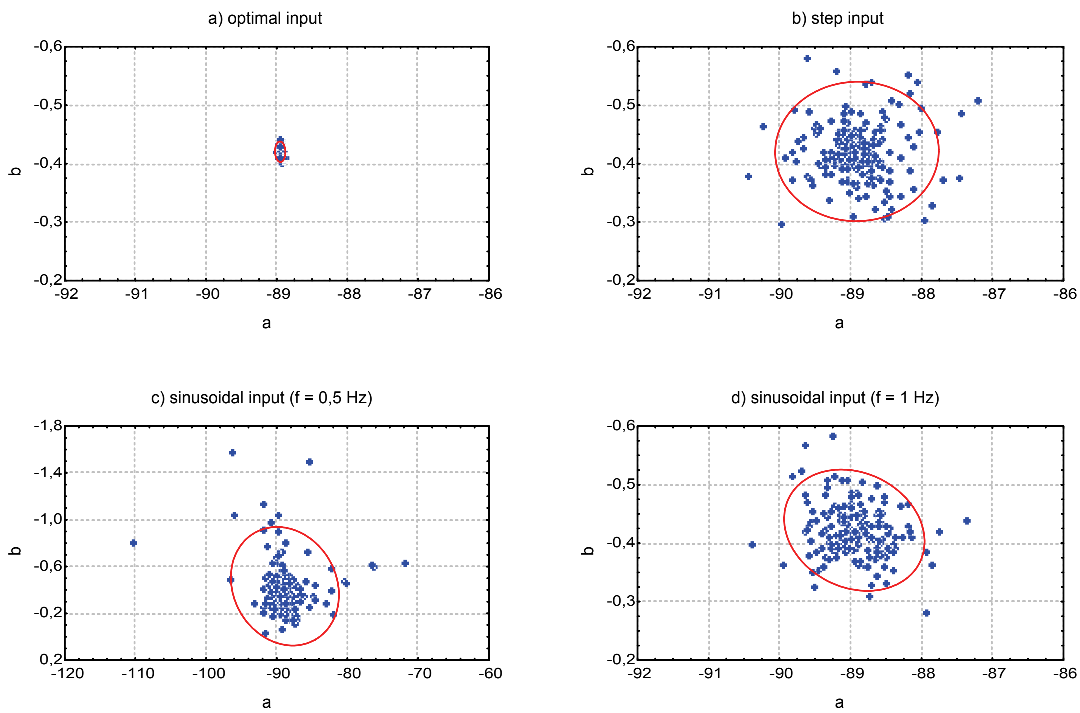

Fig. 7. Ellipsoidal confidence regions of the torsional spring model parameters estimates which was perturbed using: a) optimal input signal, b) non-optimal step input signal, c) non-optimal sine wave input signal with frequency $f = 0{,}5$ Hz, d) non-optimal sine wave input signal with frequency $f = 1$ Hz

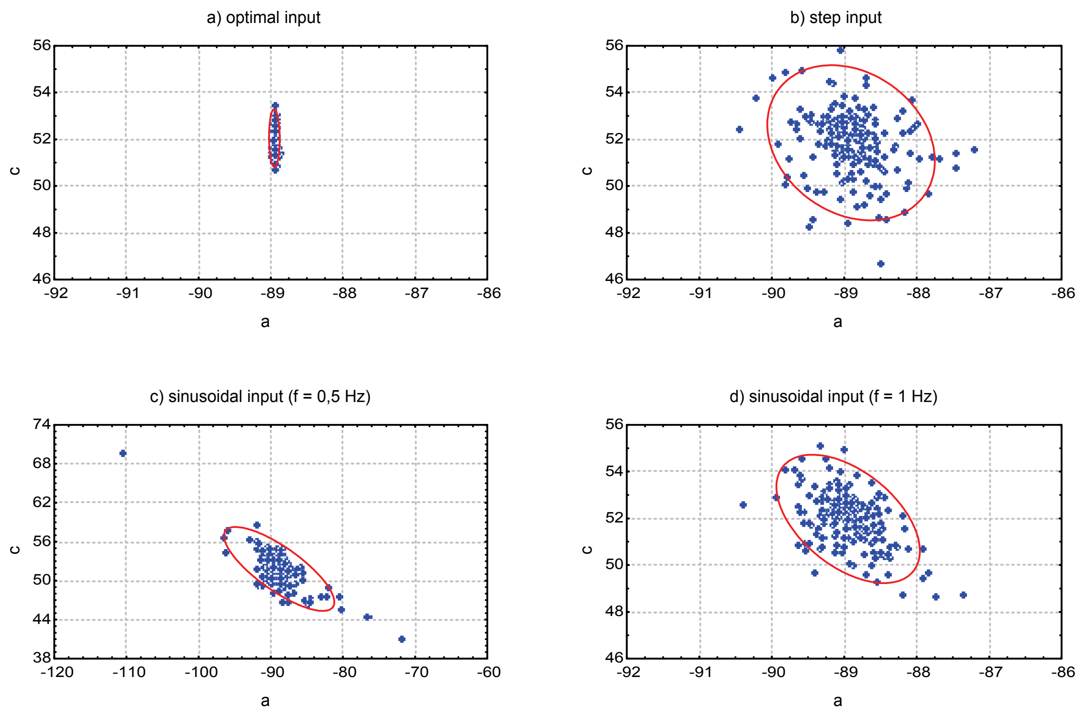

Fig. 8. Ellipsoidal confidence regions of the torsional spring model parameters estimates which was perturbed using: a) optimal input signal, b) non-optimal step input signal, c) non-optimal sine wave input signal with frequency $f = 0{,}5$ Hz, d) non-optimal sine wave input signal with frequency $f = 1$ Hz

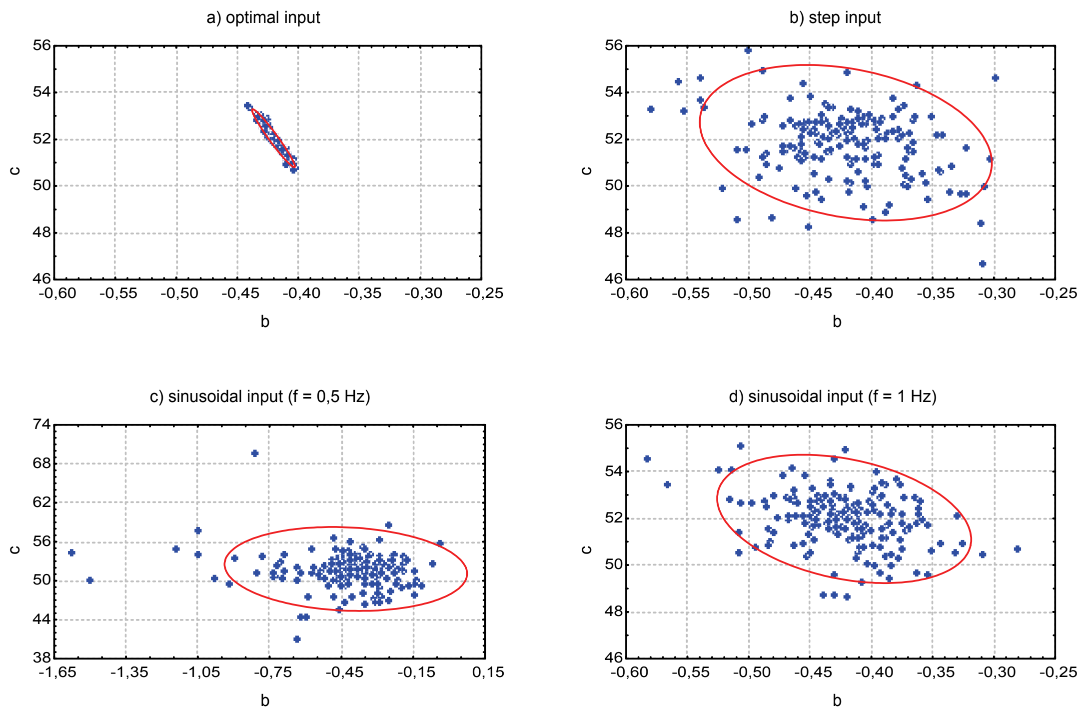

Fig. 9. Ellipsoidal confidence regions of the torsional spring model parameters estimates which was perturbed using: a) optimal input signal, b) non-optimal step input signal, c) non-optimal sine wave input signal with frequency $f = 0{,}5$ Hz, d) non-optimal sine wave input signal with frequency $f = 1$ Hz

The optimal input signal (Fig. 2) was verified utilising the second-order torsional spring (1 DOF) system [21]. The system was perturbed using the optimal input signal from the interval $-432 \leq u(t) \leq 421$ encoder counts. Such an excitation (i.e., deflection for about 22,5 degrees) causes that the system wouldn't be damaged. The output signal of the torsional spring system is shown in Figure 10.

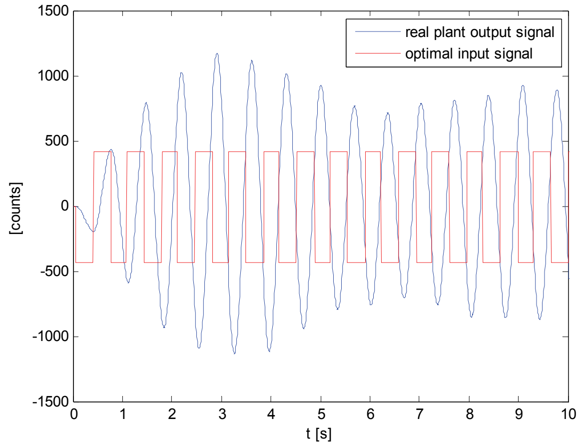

Fig. 10. Optimal excitation signal verification

As it had been expected, oscillatory system identification should be performed utilising quick changing signals.

Conclusions and future work

A multi-objective input signal design for system identification was formulated and the methods of the problem solution were outlined. The results of optimal input signal design for the torsional spring (1 DOF) case study were presented. Of significant importance is the fact that the proposed formulation can be transcribed into an equivalent optimal control problem in the Mayer form, and that it can be then solved using one of the existing packages for solving dynamic optimisation problems. Optimal input signal design for identification of other dynamical systems would be presented.

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