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Chaotic oscillators with single polynomial nonlinearity and digital sampled dynamics

***Abstract**. In this brief paper the possibility to realize third-order chaotic system by using mixed analog-digital circuit synthesis is described and experimentally verified. A single third-order differential equation with quadratic and cubic polynomial function is utilized as original mathematical model. It is suggested that the proposed approach can be generalized for almost any dynamical system as long as its global behavior is not affected by the sampling procedure.*

***Streszczenie.** Artykuł opisuje możliwość realizacji system chaotycznego przez zastosowanie syntezy analogowo-cyfrowej. Równanie różniczkowe trzeciego rzędu jest użyte jako model matematyczny. Proponowane rozwiązanie może być stosowane w różnych systemach dynamicznych. (**Generowanie funkcji chaotycznej z wielomianem nieliniowym i próbkowaniem cyfrowym**)*

Keywords: analog oscillator, chaotic motion, nonlinear dynamics, mixed-mode design.
Słowa kluczowe: funkcja chaotyczna, dynamika nieliniowa.

Introduction

Plenty of the methods for synthesis of the linear, piece-wise linear and nonlinear (polynomial type) analog circuits have been already published. The corresponding results are summarized in the well written paper [1] and study [2]. The main drawback of such approaches is in that it is quite difficult to implement strongly nonlinear vector fields. This is problem associated especially with the dynamical systems having cyclically symmetrical vector fields [3], [4]. Another example of the troubles with the nonlinear transfer function synthesis is the power row approximation [5] of some goniometrical function. To avoid creating the complex analog networks susceptible on the noise presence the better way is to use n-port with internal digital signal processing.

It eventually turns out that the well-known Nyquist sampling theorem is generally not strict enough in the case of the chaotic dynamics. The sampling frequency should be at least ten times higher than the largest distinguishable frequency component associated to the desired state space attractor. The existence of the chaotic attractor should be proved by the numerical integration process as well as by the precise calculation of the largest Ljapunov exponent [6]. Both these analysis are provided in the next chapter. Third chapter brings more details covering the concrete circuitry implementation. Then the experimental measurements are discussed and compared to the expected results. To this end the future perspectives and ideas about mixed analog-digital circuit synthesis are given.

Overall numerical analysis

Assume the third-order differential equation known from the Newtonian dynamics

$$\dddot{x} + a\ddot{x} + b\dot{x} = f(x) \tag{1}$$

where state variable x can be interpreted as position, its first derivative as velocity and second derivative as acceleration. As mentioned above the numerical integration is a first step to quantify dynamical behaviour of any system. There are many programs capable to do this. In our case Mathcad and build-in fourth-order Runge-Kutta method has been utilized with final time 600, 10000 integration steps and the initial conditions equal $(0{,}1 \quad 0 \quad 0)^T$. The corresponding results are demonstrated in Fig.1 and Fig.2 for the quadratic and cubic nonlinearity respectively. System with quadratic nonlinearity generates the so-called single-scroll attractor for the set of parameters a=0.8, b=1.1 and $f(x)=\pm 0.8(x^2-1)$.

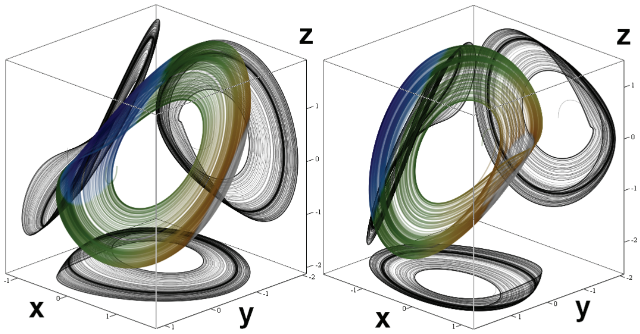

Fig.1. Integration of the typical quadratic chaotic attractors

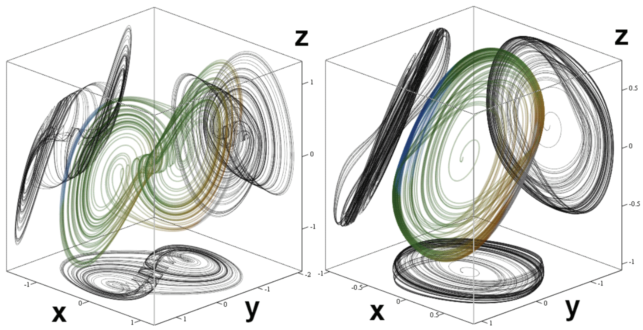

Fig.2. Integration of the typical cubic chaotic attractors

Note that there are two mirrored attractors for the different sign in the polynomial function. Similarly system with cubic polynomial is capable to produce the double-scroll attractor if a=0.7, b=0.9 and $f(x)=-0.8x(x^2-1)$ or the dual double-scroll attractor for the set a=0.3, b=0.8 and $f(x)=0.8x(x^2-1)$. It has been verified that at least 7 bits are necessary for vertical sampling to preserve a global behaviour.

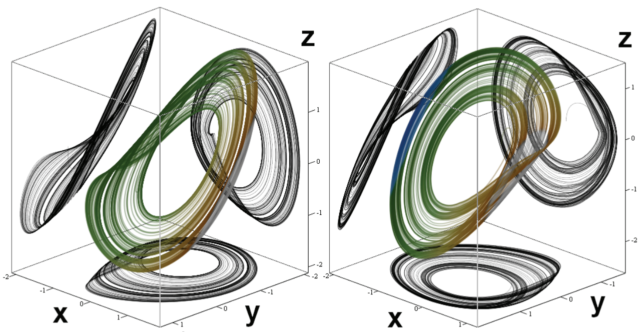

Fig.3. Integration of the sampled quadratic chaotic attractors

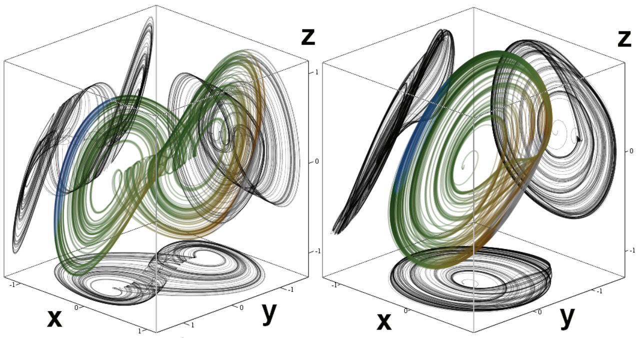

Fig.4. Integration of the sampled cubic chaotic attractors

For horizontal sampling it turns out that each chaotic state space attractor is bounded in the volume x∈(-3, 3) and 7 bits are again sufficient. The sampled chaotic dynamics is visible in Fig.3 and Fig.4. From the viewpoint of practical implementation the parameters of the nonlinear function will be fixed. To observe some routing to chaos scenario the bifurcation parameter is a and b, represented by variable resistors in the real circuit. To specify the regions of the system solution sensitive to the small changes of the initial conditions the surface-contour plot of the largest Ljapunov exponent (LE) has been created, see Fig.5 and Fig.6. The individual routine parameters are exactly the same as for the numerical integration discussed above.

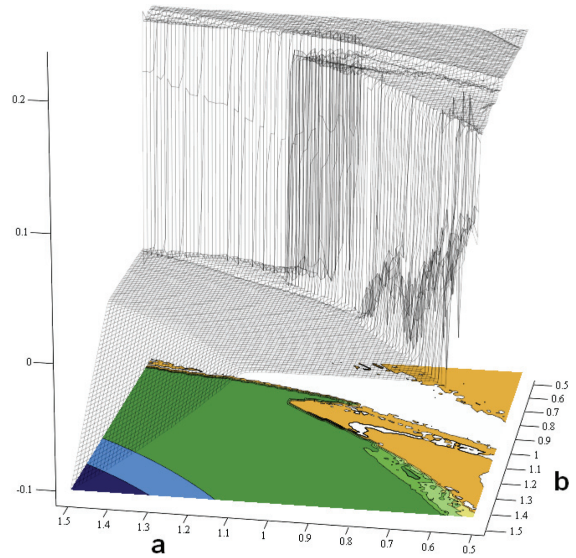

Fig.5. The largest LE for system with quadratic nonlinearity

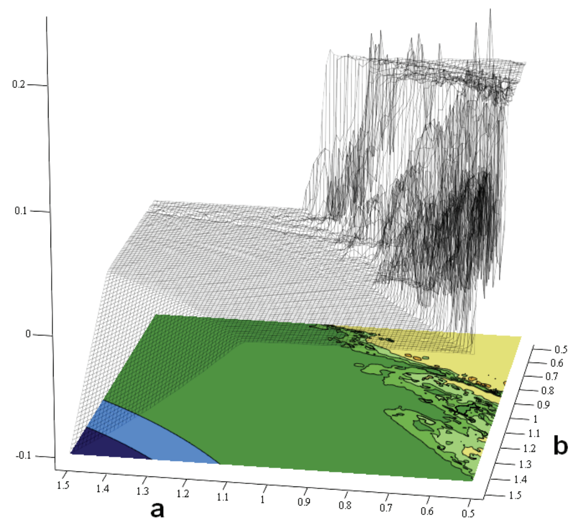

Fig.6. The largest LE for system with cubic nonlinearity

Mixed-mode circuitry realization

To simplify the final circuitry implementation and verify the conception of the sampled dynamics the digital two-port network has been designed. The core engine is processor STM32F107 with 516kB memory. This device cooperates with input analog to digital converter TLC2574 and output DAC8734 using SPI bus. KEIL uVision V3.90 has been used as C/C++ software environment. There are several esential parameters which must be taken into account in the digital network design. First of all the dynamical range of the input and output voltages must be large enough since it defines the maximum size of the state space attractor. Fortunately the expected attractors are bounded in volume cube with size 2V×2V×2V. The concrete configuration is provided in Fig.7. Assume that the chaotic oscillator with high frequency natural component is needed. Thus FPGA based development board will be a good choice to do this. The suggested improvement is illustrated by means of Fig.8. The linear part of the vector field is implemented as a cascade connection of the non-inverting integrators with input current summation process. For the correct function of the whole circuit it is required for conveyors to operate in the linear region. The individual mathematical operations are realized by means of the positive second generation current conveyors available commercially available under notion AD844. This analog building block is described by the following set of the nodal equations

(2) $V_X = V_Y \qquad I_Y = 0 \qquad I_X = I_Z \qquad V_O = V_Z$

The corresponding circuitry is given in Fig.9. It is evident that natural frequency component is uniquely determined by the time constant

(3) $$\tau = R \cdot C = 10^3 \cdot 10^{-7} = 100 \mu s$$

The experimental study of chaos evolution can be done via changing the value of the variable resistors in the ranges Ra∈(500, 1500)Ω and Rb∈(500, 1500)Ω.

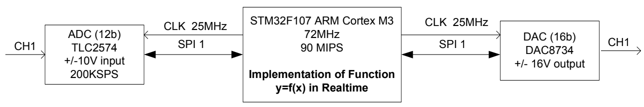

Fig.7. Principal configuration of polynomial digital two-port

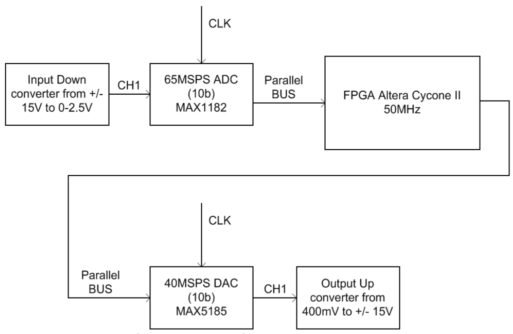

Fig.8. Principal configuration of improved digital two-port

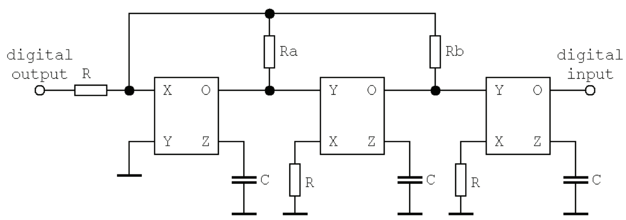

Fig.9. Detailed implementation of the analog part using AD844

Experimental results

The typical state space trajectories measured by means of the digital oscilloscope Agilent Infiniium are displayed in Fig.10 and Fig.11 for the quadratic and cubic nonlinearity respectively. These measured results are in a very good accordance with theoretical expectations, i.e. numerical integration of the given mathematical model. It has been verified that the time constant can not be much lower than τ=10μs.

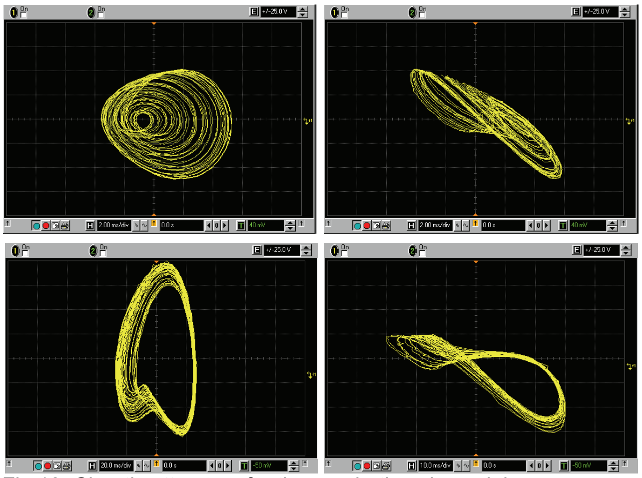

Fig.10. Chaotic attractors for the quadratic polynomial

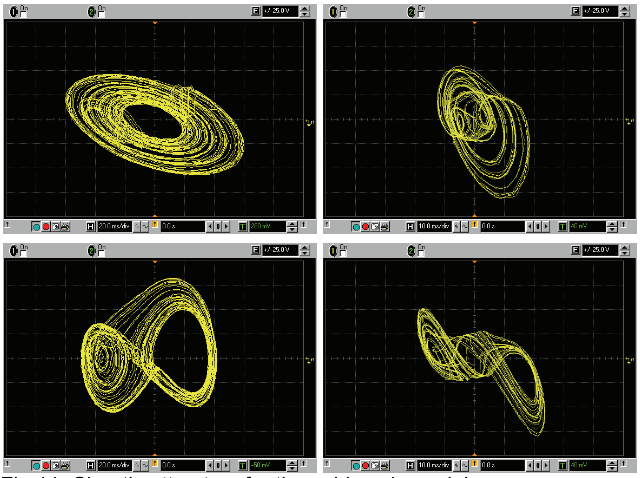

Fig.11. Chaotic attractors for the cubic polynomial

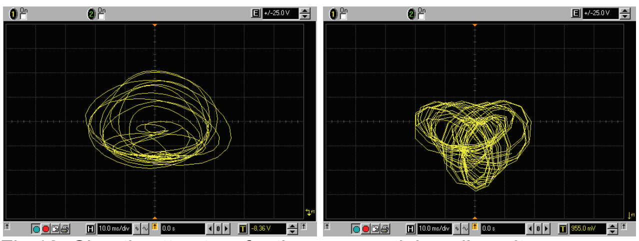

Fig.12. Chaotic attractors for the exponential nonlinearity

The proposed mixed mode oscillator generates a full scale of the dynamical motion, basically any attractor observable in the original system with smooth vector field. This means that also periodic and quasiperiodic orbits can be observed. A perfect agreement between numerical integration and experimental measurement has been proved also in the particular cases of the exponential nonlinearities provided in the paper [7]. These plane projections are shown in Fig.12.

Conclussion

It is demonstrated by means of the several examples that it is effective to use mixed analog-digital synthesis even in the case of the chaotic dynamics. There exist some dynamical systems (for example with cyclically symmetrical vector field) with the multiple complex nonlinear functions. In such a case truly analog implementation becomes very difficult. Recently the generators of the multi-scroll attractors attract increasing interest of many researchers [8], [9] especially from the viewpoint of producing as many spirals as possible. Circuitry realization of such systems using only analog elements is bounded by dynamical ranges and represents almost impossible task. The presented approach can be utilized to solve such problems. In fact, the whole circuitry can be fully realized in the language of a digital signal processing. It is worth nothing that the individual network configurations can be tested using Pspice circuit simulator since the sampling procedure can be also taken into account. Because the same results the corresponding figures are not provided.

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