Second-order statistics of dual SC macrodiversity system over channels affected by Nakagami-\(m\) fading and correlated gamma shadowing

Abstract. This paper considers second-order statistics of wireless communication system with micro- and macrodiversity reception in correlated gamma shadowed Nakagami-\(m\) fading channels. Macrolevel is of selection combining (SC) type and consists of two base stations (dual diversity) while \(N\)-branch receiver employing maximal ratio combining (MRC) is implemented on microlevel. Level crossing rate (LCR) and average fade duration (AFD) represent the system’s second-order statistics. They are important design criteria and performance measure for wireless communication systems. Rapidly converging infinite-series expressions for LCR and AFD are derived. Numerical results are presented graphically to illustrate the proposed mathematical analysis and to examine the effects of system’s parameters on the concerned quantities.

Streszczenie. W artykule analizowano statystykę drugiego rzędu w systemach komunikacji bezprzewodowej ze zróżnicowanym odbiorem. Statystyka ta jest reprezentowana przez parametry LCR (level crossing ratio) i AFD (average fade duration). (Statystyka drugiego rzędu w systemie komunikacji bezprzewodowej zakłóconej przez tłumienie: Nakagami-\(m\) i skorelowane gamma)

Keywords: Microdiversity, macrodiversity, average fade duration, level crossing rate, Nakagami-\(m\) fading, gamma shadowing. Słowa kluczowe: komunikacja bezprzewodowa, tłumienie i zanik sygnału.

Introduction

The performance of wireless communication system is degraded by various factors. Short-term fading, caused by multipath propagation, and long-term fading (shading), caused by large obstacles and large deviations in terrain profile between transmitter and receiver, are major source of signal corruption [1]. The problem concerning fading and shadowing as well as their deleterious effects on the wireless system performance has been in focus for a long time. The system performance degraded by severe fading and shadowing can be improved using diversity combining. The diversity combining, in which two or more copies of the same information-bearing signal are combined skilfully, still offers one of the greatest potential for system performance improvement to many of the current and future wireless technologies [2]. There are several diversity techniques: time diversity, frequency diversity and space diversity. All these diversity techniques require some redundancy in time, frequency and/or spatial domain, which consume resources in power and/or bandwidth [3]. Compared with other diversity techniques, space diversity is power- and bandwidth-efficient therefore it is the most common form of diversity [4]. Space diversity reception is based on using multiple antennas at the reception. There are several principal types of space diversity combining techniques which essentially depend on the complexity restrictions put on the communication system and the amount of channel state information available at the receiver. The most popular are selection combining (SC), equal-gain combining (EGC) and maximal-ratio combining (MRC). MRC is optimal combining technique in the sense that it gives the best performance regardless of the fading statistics on the diversity branches. However, MRC requires the knowledge of the channel fading amplitudes and phases of each diversity branch which must be continuously estimated by the receiver. These estimations require separate receiver chain for each branch of the diversity system increasing its complexity. EGC provides performance comparable to MRC, but with simpler implementation complexity. EGC does not require the estimation of the channel fading amplitudes since it combines signals from all branches with the same weighting factor. In opposition to MRC and EGC, SC receiver processes only one of the diversity branches, and is much simpler and cheaper for practical realization. SC selects the antenna branch with the highest signal.

While the use of diversity techniques at the single base station (microdiversity) mitigates short-term fading, it is not sufficient to mitigate the overall channel degradation when long-term fading is also concurrently present. Macrodiversity technique, which employs the processing of signals from multiple base stations, can reduce deterioration caused by shadowing. Having in mind that short- and long-term fading conditions coexist in wireless channels, in order to improve wireless system performance, the use of both micro- and macrodiversity is essential. Rayleigh, Rician, Nakagami-\(m\), Hoyt and Weibull are distributions that can be used for short-term variation modelling of fading envelope. In open technical literature, the most frequently applied is Nakagami-\(m\) model. It often gives the best fit to land-mobile [5]-[7] and indoor-mobile multipath propagation [8]. The average power of the received signal is also random due to shadowing. Long-term fading channels are usually modelled as lognormal and gamma. Unfortunately, the use of lognormal distribution to account for shadowing does not lead to a closed-form solution for the probability density function (PDF) of signal after micro- and macrodiversity combining [9]-[12]. This makes the analysis of system in shadowed fading environment very ponderous. Papers [13, 14] show that, based on theoretical results and measured data, gamma distribution does the job as well as lognormal. A compound fading model [14]-[19] uses a gamma distribution to model the average power to account for shadowing. Such approach leads to a closed-form expression for the PDF facilitating further analysis.

Outage probability, as an important and widely accepted performance measure, can be used to describe diversity systems [17, 19-24]. However, in certain applications such as in adaptive transmission, outage probability does not provide enough information for the overall system design and configuration. In that case, in addition to outage probability, the level crossing rate (LCR) and the average fade duration (AFD) should be obtained to reflect the correlation properties of the fading channels and to provide a dynamic representation of the system’s outage performance [25-29]. LCR and AFD represent the system’s second-order statistics and they can be used as important performance measures for a proper selection of the adaptive symbol rates, interleaver depth, packet length and time slot duration in wireless communication system.
In this paper, motivated by the results of propagation measurements in land-mobile and indoor-mobile systems, and by the fact that gamma distribution can describe shadowing reliably, shadowed fading channels are modelled as Nakagami-gamma. MRC algorithm, which is optimal combining algorithm, is implemented at the microlevel. SC diversity, which is basically a fast response hand-off mechanism that instantaneously or with minimal delay chooses the best base station to serve mobile based on the signal power received [24], is implemented at the macrolevel. Exact and rapidly converging infinite-series expressions for LCR and AFD of received signal are provided. Numerical results are graphically presented to show the effects of fading severity, shadowing severity, number of diversity branches at the microlevel and correlation between base stations on the concerned system performance.

System and channel model

We consider system consisting of two N-branch MRC receivers at the microlevel and dual SC receiver at the macrolevel (Fig.1). In paper [29], the correlation between diversity branches at the microlevel has been considered while base stations have been treated to have nonzero correlation. Independent fading can be ensured when the separation between antennas is on the order of one half of a wavelength [30]. In this paper, we consider correlation between base stations, which is more realistic case because shadowing has a larger correlation distance (base stations are likely to be shadowed by the same obstacles) and it is difficult to ensure that base stations operate independently, especially in microcellular systems [31].

![Fig.1. System model](image)

The PDF of the signal received by the \( i \)th antenna at the \( j \)th base station in the presence of Nakagami-\( m \) fading is

\[
p_{x_j}(r_j) = \frac{2m^m r_j^{2m-1}}{\Gamma(m) \Omega_j^m} \exp\left(-\frac{m r_j^2}{\Omega_j}\right),
\]

where \( \Gamma(m) \) is gamma function, \( \Omega_j \) is the average power of the signal at the \( j \)th base station and \( m \) is Nakagami fading parameter which describes fading severity (\( m \geq 5 \)). As parameter \( m \) increases, the fading severity decreases.

After transformation \( x_j = r_j^2 \), (1) becomes

\[
p_{x_j}(x_j) = \frac{m^m x_j^{m-1}}{\Gamma(m) \Omega_j^m} \exp\left(-\frac{m x_j}{\Omega_j}\right), \quad i = 1, N, \quad j = 1, 2
\]

The result signal at the output of the base station which employ MRC diversity algorithm is the sum of squared envelopes of Nakagami-\( m \) faded signals, \( x_j = \sum_{i=1}^{N} x_{ij}^2 \), or equivalently, \( x_j = \sum_{i=1}^{N} x_{ij} \) with PDF given by [32]

\[
p_{x_j}(x_j | y_j) = \frac{x_j^{M-1} M^M}{\Gamma(M)} y_j^M \exp\left(-\frac{M y_j}{x_j}\right), \quad j = 1, 2
\]

where \( y_j \) is the total input power (\( y_j = N \Omega_j \)) and \( M = Nm \).

The joint PDF of signal and its time derivative is

\[
p_{x_j, \dot{x}_j}(x_j, \dot{x}_j | y_j) = p_{x_j}(x_j | y_j) p_{\dot{x}_j}(\dot{x}_j)
\]

where \( p_{\dot{x}_j}(\dot{x}_j) \) represents the PDF of time derivative of \( x_j \) given by

\[
p_{\dot{x}_j}(\dot{x}_j) = \frac{1}{\sqrt{2\pi} \sigma_{\dot{x}_j}} \exp\left(-\frac{\dot{x}_j^2}{2\sigma_{\dot{x}_j}^2}\right),
\]

\[
\sigma_{\dot{x}_j}^2 = \frac{4x_j \pi^2 f_w y_j}{M}, \quad j = 1, 2
\]

The conditional nature of the PDF in (3) and (4) reflects the existence of shadowing with \( y_j \) being random variable. The joint PDF of \( x_1 \) and \( x_2 \) follows the correlated gamma distribution [33, 34]

\[
p_{x_1, x_2}(y_1, y_2) = \rho e^{\frac{-c}{2} y_1} \frac{\Gamma(c)(1-\rho)}{y_2^{\frac{c}{2}}} \exp\left(-\frac{y_1 + y_2}{y_2(1-\rho)}\right) I_{c-1}\left[\frac{2\sqrt{\rho y_1 y_2}}{y_2(1-\rho)}\right]
\]

where \( I_c(\cdot) \) is the first kind and \( n \)th order modified Bessel function, \( \rho \) is the correlation between \( y_1 \) and \( y_2 \), \( c \) is the order of gamma distribution and \( y_0 \) is related to the average power of \( y_1 \) and \( y_2 \). The severity of gamma shadowing is measured in terms of \( c \). The lower value of \( c \) means the higher shadowing, while the value of \( c = \infty \) corresponds to a pure short-term fading channel. The relationship between the parameter \( c \) and standard deviation \( \sigma \) of shadowing in dB in the lognormal shadowing is \( \sigma (dB) = 4.3429 \sqrt{\psi'} (\cdot) \), where \( \psi' (\cdot) \) is the trigamma function. The typical values of \( c \) are between 2-12 dB.

Selection diversity is applied at the macrolevel.Namely, the base station with the larger average power is selected to provide service to the user. Using the concepts of probability, the joint PDF of the \( x \) and its derivative after diversity combining at the micro- and macrolevel can be derived as

\[
p_{x, \dot{x}}(x, \dot{x}) = \int_{0}^{x} p_{x_{y_1}}(x, \dot{x} | y_1) dy_1 \int_{0}^{y_1} p_{y_2}(y_1, y_2) dy_2
\]

\[
+ \int_{0}^{x} p_{x_{y_2}}(x, \dot{x} | y_2) dy_2 \int_{0}^{y_2} p_{y_1}(y_1, y_2) dy_1
\]

\[
= 2 \int_{0}^{x} p_{x_{y_1}}(x, \dot{x} | y_1) dy_1 \int_{0}^{y_1} p_{y_2}(y_1, y_2) dy_2
\]

which, after substitution appropriate expressions and after integrations, becomes
p_m (x, y) = 2 \frac{M^{-x} \sum_{n=0}^{\infty} \frac{\rho^n}{n!} \Gamma(n + 1) \Gamma(n + y_0) \Gamma(n + 1 + y)}{(n + 1 + y_0)(n + 1 + y_0 + 1)} \end{22.1}
\end{equation}

(8)

Second-order statistics

LCR and AFD taken together represent second order statistics and give a useful means of characterising the severity of the fading over time. LCR is the measure of the rapidity of the fading. It quantifies how often the fading process crosses some threshold in a positive or negative going direction. Output LCR can be calculated as following.

(9) \quad N_s (x) = \int_0^x \rho \, dx

After substituting (8) into (9) and some numerical transformations, final expression for LCR is

\begin{equation}
N_s (x) = \frac{2x^{-1/2}}{\Gamma(M/2)(M/2)} \sum_{n=0}^{\infty} \frac{\rho}{n!} \Gamma(n + 1) \Gamma(n + 1 + y_0) \Gamma(n + 1 + y_0 + 1) \end{equation}

(10)

AFD quantifies how long the signal spends below the threshold. Output AFD is given by

(11) \quad AFD = \frac{F_s (x)}{N_s (x)}

where \( F_s (x) \) is the cumulative distribution function (CDF) of \( x \) derived in [19]

\begin{equation}
F_s (x) = \frac{4}{\Gamma(M) \Gamma(c)} \sum_{n=0}^{\infty} \frac{\rho^n}{n!} \frac{1}{n! \Gamma(n + 1) \Gamma(n + c + j) \Gamma(n + 1 + c + 1 + y_0 \Gamma(n + 1 + c + 1 + y_0 + 1))} \end{equation}

(12)

To the best of the authors' knowledge, the above presented expressions for LCR and AFD are novel in the open technical literature.

Numerical results and discussion

In this section, the behaviour of the second-order statistical measures at the output of dual SC macrodiversity is illustrated for different values of system's parameters.

Fig. 2 depicts the normalized average LCR of signal envelope after micro- and macrodiversity processing for several values of correlation coefficient and shadowing severity.

Fig. 3. Normalized average LCR of signal envelope after micro- and macrodiversity processing for several values of correlation coefficient and shadowing severity.

In Fig. 3, the normalized average LCR for different \( c = 0.3 \) (dB) versus normalized envelope level \( x_n = x/y_0 \) for several values of \( M \). As it was expected, as \( M \) increases, the number of diversity branches increases or fading severity decreases, the normalized average LCR decreases. This means that fades occur less frequently. The influence of \( M \) on the average LCR is more evident for lower envelope levels. Increase of \( M \) will not bring much improvement in terms of LCR for higher normalized envelope levels.

In Fig. 3, the normalized average LCR for different correlation coefficient and severity of shadowing is plotted. For lower shadowing severity, the signal crosses lower levels more frequently. For values of \( x_n \) less than level for which LCR reaches its maximum, the average LCR increases when correlation coefficient increases. Otherwise, the number of crossings slightly decreases as separation between base stations decreases.

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In Fig. 4, the influence of average power is in focus. For lower signal levels, the average LCR becomes smaller with increase of $y_0$. Higher signal levels are crossed more frequently for higher values of $y_0$. Also, for higher values of $y_0$, envelope level, for which LCR reaches its maximum, increases.

In Fig. 5, the normalized AFD is plotted as a function of the normalized envelope level for several values of $M$. For $x_i < x_{d0}$, (where $x_{d0}$ is 5 dB for $y_0 = 0$ dB, 10 dB for $y_0 = 5$ dB and 15 dB for $y_0 = 10$ dB), when the signal has faded below this value, it remains below for a slightly less amount of time for higher $M$. The value of $x_i$ increases as $y_0$ increases.

Fig. 6 shows normalized AFD as a function of the normalized envelope level for several values of correlation coefficient and severity of shadowing. As it was expected, the AFD decreases with the decrease of shadowing severity and/or correlation coefficient (the influence of correlation coefficient is minimal).

In Fig. 7, the normalized AFD is lower for higher values of $y_0$. Also, for lower values of envelope level, $x < x_{d0}$, (where $x_{d0}$ is 5 dB for $y_0 = 0$ dB, 10 dB for $y_0 = 5$ dB and 15 dB for $y_0 = 10$ dB), when the signal has faded below this value, it remains below for a slightly less amount of time for higher $M$. The value of $x_i$ increases as $y_0$ increases.

The main problem in the infinite-series expressions can be their convergence. But, expressions in the paper converge rapidly, and thus, they can be efficiently used in performance analysis. As an indicative example, Table 1 shows the number of terms needed to be summed in the expression for the average LCR to achieve accuracy at the 3rd and 4th significant digit after the truncation of the infinite series. It can be noticed that the number of required terms depends strongly on the correlation coefficient. An increase of $\rho$ leads to an increase of number of terms.

Table 1. Number of terms of (10) required to achieve accuracy at the significant digit presented in the brackets ($M=1.2, c=5.2$)

<table>
<thead>
<tr>
<th>$y_0$ (dB)</th>
<th>$x=10$ dB</th>
<th>$x=0$ dB</th>
<th>$x=10$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho=0.2$</td>
<td>15 19 13</td>
<td>17 12 15</td>
<td></td>
</tr>
<tr>
<td>$\rho=0.4$</td>
<td>16 21 15</td>
<td>21 15 18</td>
<td></td>
</tr>
<tr>
<td>$\rho=0.6$</td>
<td>25 30 23</td>
<td>27 25 27</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

In this paper, the infinite-series expressions for the average LCR and AFD of dual selection based macrodiversity system, which involves $N$-branch MRC receivers at the microlevel and operates over gamma shadowed Nakagami-$m$ fading channels, were presented. LCR and AFD represent the system’s second-order statistics and reflect the correlation properties of the fading channels and provide a dynamic representation of the microdiversity order increases and/or fading severity decreases. For $x_{d0} > 12$ dB, AFD increases with increase of parameter $M$, i.e., when output signal has faded below determined value, it spends more time below that value for higher $M$. This property is caused by the higher correlation of the fading envelope for a larger Nakagami parameter. Similar behavior is also observed in [35] for Rician fading channels.
system's outage performance. Numerical results were presented graphically to illustrate the effect of number of diversity branches, severity of fading and shadowing, average power and correlation between base stations on the system performance. Expressions obtained in the paper can be employed in the parameter optimization of diversity system in different propagation conditions.

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