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Field distribution in separator's working space for various winding configuration

Abstract. *Superconductor separators are a new generation of magnetic separators; as a source of magnetic field a dc superconductor electromagnet is used. It creates, between other things, a chance to reject weak-magnetic particles as well as extra small ones (the basic problem for beneficiation of useful minerals). The subject of this paper is field distribution in a magnetic separator containing ferromagnetic matrix where the magnetic field is generated by a dc superconducting magnet. To develop invariable conditions for the extraction of particles from the slurry in a matrix separator, it is necessary to create a homogenous magnetic field within the working space of the device. The source of the field is usually a solenoidal coil winding with superconducting wire and, in order to achieve the design objective of field uniformity, various configurations have been considered. Some results are presented and the most promising solutions are highlighted.*

Streszczenie. *Separatory nadprzewodnikowe są nową generacją separatorów magnetycznych; do wzbudzania pola magnetycznego wykorzystywany jest elektromagnes nadprzewodnikowy. Ich zastosowanie stwarza, między innymi, szansę wydzielania cząstek o mikronowych rozmiarach a także słabo magnetycznych (jest to podstawowy problem przeróbki kopalin). Przedmiotem artykułu jest analiza rozkładu pola magnetycznego w separatorze zawierającym matrycę ferromagnetyczną, w którym pole wzbudzane jest przez stałoprądowy elektromagnes nadprzewodnikowy. Dla zapewnienia takich samych warunków ekstrakcji cząstek z przepływającej przez matrycę zawiesiny, konieczne jest w ukształtowanie jednorodnego pola magnetycznego w przestrzeni roboczej separatora. Źródłem pola w nadprzewodnikowym separatorze matrycowym jest zwykle solenoid nawinięty nadprzewodnikiem. Dla uzyskania jednorodnego pola Autor rozważa kilka konfiguracji uzwojeń. Prezentowane są rezultaty obliczeń.* ***Rozkład pola magnetycznego w przestrzeni roboczej separatora dla różnych konfiguracji uzwojenia***

Keywords: magnetic field distribution, magnetic separation, shape of winding, superconducting magnet.
Słowa kluczowe: rozkład pola magnetycznego, separacja magnetyczna, kształt uzwojenia, magnes nadprzewodnikowy.

High Gradient Magnetic Separator

When fine particles are dispersed in air, water, sea water, oil, organic solvents, etc., their separation or filtration by using a magnetic force is called magnetic separation.

There are two kinds of separators for separation of mixtures with High Gradient Magnetic Separator (HGMS) using superconducting magnets: deflecting and capturing (matrix) separators. The latter type is widely used in industry [1], [2].

In the magnetic field, generated by the superconductor winding of an axial-symmetric construction (solenoid), there is a matrix in which particles from slurry flowing through a separator get extracted. The matrix is a canister filled with gradient-generating elements such as chips or ferromagnetic wool, to which particles of specific magnetic features are attracted (Fig. 1). The matrix should be placed in a homogenous magnetic field to develop similar conditions during the technological process. So, the shape of the superconducting winding that generates a magnetic field of an assumed homogeneity must be carefully chosen.

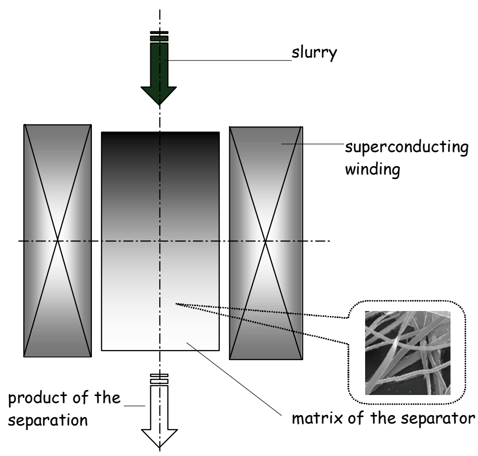

Fig. 1. High Gradient Magnetic Separator -matrix of the separator in magnetic field

Magnetic field in working space of the separator

The distribution of magnetic fields is a very important issue in many cases of electrotechnical machines application. It also concerns the distribution of the field in working space of the separator. Similar conditions of separation are aimed at. Therefore, it is necessary to create a proper shape of magnetic field and electromagnet winding. In many separators the magnetic field is induced by solenoidal winding. In this case there are some problems to be solved.

a) Problems of domain optimization

The field distribution that has to satisfy a specific requirement within a region is defined as a domain optimization. For example, the design of the magnets may require:

a) a constant air gap flux density,
b) uniform air gap field,
c) prescribed field distribution in the air gap,
d) prescribed field distribution in a specific region.

These problems are regarded as the domain optimization. The predetermined targets are met with suitable dimensions of the magnet and/or by using a suitable existing coil.

A predetermined distribution of the magnetic strength along the axis of a solenoid or in a plane perpendicular to the axis of the solenoid or in a volume within the solenoid are commonly required. All these problems are solvable by using Fredholm's integral equation of the first kind.

Let's assume that the solenoid is composed of a number of similar sections shown in Fig. 2 (a). Each of these sections may have different current densities [3].

The vector potential of a filamentary circular loop with radius r' (Fig. 2(b).) is

(1) $$\vec{A} = A(r,z)\vec{n}_0$$

where:

(2) $$A(r,z) = \frac{\mu_0 I}{2\pi}\left(\frac{r}{r'}\right)^{1/2} f(k)$$

(r,z) is the location of the observation point and I is the current in the loop.

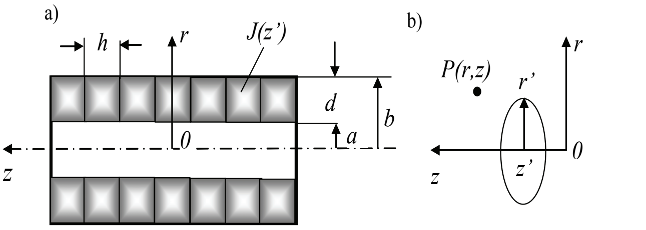

Fig. 2. A solenoid composed of segments (a); a filamentary circular loop (b)

In Eq. (2):

(3) $$f(k)=\left(\frac{2}{k}-k\right)K(k)-\frac{2}{k}E(k)$$

where:

(4) $$k^2=\frac{4rr'}{(r+r')^2+(z-z')^2}$$

$K(k)$, $E(k)$ are complete elliptic integrals of the first and second kind.

The curl of vector potential $\vec{A}$ is taken to obtain the magnetic field strenght $\vec{H}$:

(5) $$\vec{H}=\frac{1}{\mu_0}\nabla\times\vec{A}$$

The component of the magnetic field strength along z direction is:

(6) $$H_z(r,z)=\frac{1}{\mu_0 r}\frac{\partial}{\partial r}\left[rA(r,z)\right]==\frac{I}{2\pi}g(r,r',z,z')$$

where:

(7) $$g(r,r',z,z')=\frac{1}{\left[(r+r')^2+(z-z')^2\right]^{1/2}}\times$$
$$\times\left[K(k)+\frac{r'^2-r^2+(z-z')^2}{\left[(r+r')^2+(z-z')^2\right]^{1/2}}E(k)\right]$$

Assuming that the current density is uniform in each small section of length h and width d, the field strength is:

(8) $$H(r,z)=\frac{J_i}{2\pi}\int_a^b\int_{z'-h/2}^{z'+h/2}g(r,r',z,z')$$

Considering the symmetry of the solenoid, the magnetic strength produced by whole solenoid is:

(9) $$H(r,z)=\frac{1}{2\pi}\sum_{i=1}^{N}J_i\cdot$$
$$\cdot\int_a^b\int_{z'-h/2}^{z'+h/2}\left[g(r,r',z,z')+g(r,r',z,-z')\right]dr'\,dz'$$

where N is half of the total number of sections.

If $r=0$, the integrand of Eq. (9) is:

(10) $$g(r,r',z,z')=r'^2\left[r'^2+(z-z')^2\right]^{-3/2}$$

The discredited form of Eq. (9) is:

(11) $$H(r_j,z_j)=\frac{1}{2\pi}\sum_{i=1}^{n}a_{ij}J_i$$

This is a set of algebraic equations. It can be written in a general form:

(12) $$\mathbf{AX}=\mathbf{B}$$

The $\mathbf{X}$ is the unknown current density $\{\mathbf{J}\}$ in each section of the solenoid, $\mathbf{B}$ is the desired field distribution. The elements of matrix $\mathbf{A}$ are:

(13) $$a_{ij}=\int_a^b\int_{z'-h/2}^{z'+h/2}\left[g(r,r',z,z')+g(r,r',z,-z')\right]dr'\,dz'$$

The solution of Eq. (12) yields the current distribution. [3]

b) Distribution of magnetic field in HGMS

In [4] the author analyses magnetic field distribution in the working space of the matrix separator.

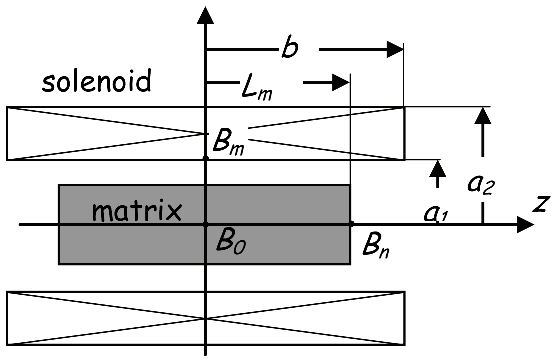

Fig. 3. Configuration of the matrix and solenoid of the separator

A circular solenoid of rectangular cross-section (Fig. 3) is the most common coil shape used in magnetic separation. The shaded region is where a cylindrical HGMS matrix is located. The coil is characterized by the parameters, collected in Table 1.

Table 1. Parameters Characterizing Coil Windings

Geometric parameters
$2a_1$ – inside diameter of solenoid, $2a_2$ – outside diameter of solenoid, $2b$ – length of solenoid, L_m – length of matrix. The above parameters are interconnected through the following relationships: $\frac{a_2}{a_1}=\alpha$; $\frac{b}{a_1}=\beta$; $\frac{L_m}{a_1}=\beta_e$; $\frac{b-L_m}{a_1}=\Delta\beta$.
Electric parameters
J – averaged density of current in solenoid, B_0 – magnetic flux density in the geometric centre of the solenoid, B_m – maximum value of flux density on the Winding external surface, in its middle plane, B_n – value of flux density at the end of matrix.

The basic relationships that may be applied to the solenoid winding are [4]:

(14) $$B_0=\mu_0 a_1 J K_0(\alpha,\beta)$$

(15) $K_0(\alpha,\beta)=\beta\ln\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}}$, $V=2\pi a_1^3\beta(\alpha^2-1)$

To determine the shape of winding, the relationship (16) should be used:

(16) $$v(\alpha,\beta)=\frac{V}{2\pi a_1^3}=\beta(\alpha^2-1)$$

where V is the volume of the winding.

In equations (15) and (16) four variables occur: v, K_0, α and β ; two of them are independent variables.

An equation that allows for the optimization of the winding shape is expressed as follows [4]:

(17) $$(\alpha^2-1)\frac{\partial K_0(\alpha,\beta)}{\partial\alpha}=2\alpha\beta\frac{\partial K_0(\alpha,\beta)}{\partial\beta}$$

With the assumed volume and inside diameter of the winding, it is possible to determine optimal values of α and β coefficients from the equation (17), and additionally from (18) below:

(18) $$\beta(\alpha^2-1)=\frac{V}{2\pi a_1^3}\ \textit{(given value)}$$

With the assumed volume B_0 induction in the geometric centre of the solenoid and for its assumed inside diameter, the equation (17) and also (19) appear obligatory:

(19) $$\beta\ln\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}}=\frac{B_0}{\mu_0 a_1 J}\ \textit{(given value)}$$

c) Superconductor winding

In a winding made of a multifibre superconductor, this superconductor should not remain under the influence of a magnetic field with an induction value exceeding by much the B_0 value. So, the influence of α and β on the magnetic field homogeneity is a factor defining the superconductor's current condition power (i.e. the ability of this superconductor to conduct the current), as well as the range of the magnet's general application. B_m - the highest induction value usually occurs in a spot lying near the winding's internal surface, in this middle plan. The admissible value of the B_m conduction value is set on the basis of the current density given by the manufacturer.

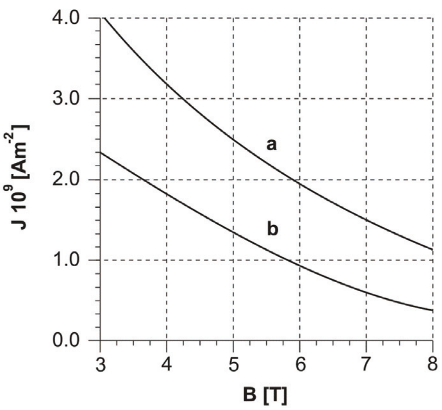

Fig.4. Critical characteristics of the NbTi superconductor: a – curve of a short section, b – curve – a characteristic that includes a degradation effect

In Fig. 4, the a – curve demonstrates such a characteristic for a multifibre conductor containing 60 fibres of the NbTi superconductor (the superconductor type F60(0.6) Vacuumschmelze made). The b – curve represents the characteristic of the pivotal density of the described superconductor at the temperature T = 4.2 K, reduced by coefficient which makes allowance for a stabilizing metal, insulation layers separating the winding layers and the material filling free spaces between turn of windings, and the degradation effect.

It is necessary to implement an analytical expression of the relationship $J=f(B)$ to analyze field distribution in the working space of the separator. This expression has the form as given in [4]:

(20) $$J=f\left(\frac{B}{\mu_0 a_1 J}\right)=\frac{p}{q+\frac{B}{\mu_0 a_1 J}}$$

where: p and q are adapting coefficients determined with regard to the actual characteristic of the superconductor (they take impact of factors into consideration that decide the degradation effect).

For the purpose of setting the winding geometry with regard to the α and β impact on both the field distribution and the maximum value of induction, the analytical expression of B_m induction in the following form is taken [4]:

(21) $$B_m=\mu_0 a_1 J K_m(\alpha,\beta)$$

where:

(22) $$K_m(\alpha,\beta)=K_0(\alpha,\beta)-\frac{1}{2}K_2(\alpha,\beta)+\\+\frac{3}{8}K_4(\alpha,\beta)-\frac{5}{16}K_6(\alpha,\beta)\ldots$$

(23) $$K_2(\alpha,\beta)=\frac{1}{2\beta}\left(c_1^{3/2}-c_3^{3/2}\right)$$

(24) $$K_4(\alpha,\beta)=\frac{1}{24\beta^3}\cdot\\\cdot\left[c_1^{3/2}\left(2+3c_2+15c_2^2\right)-c_3^{3/2}\left(2+c_4+15c_4^2\right)\right]$$

(25) $$K_6(\alpha,\beta)=\frac{1}{240\beta^5}\cdot\\\cdot\left[\begin{array}{l}c_1^{3/2}\left(8+12c_2+15c_2^2-70c_2^3+315c_2^4\right)-\\-c_3^{3/2}\left(8+12c_4+15c_4^2-70c_4^3+315c_4^4\right)\end{array}\right]$$

(26) $$c_1=\frac{1}{1+\beta^2},\quad c_2=\frac{\beta^2}{1+\beta^2},\\c_3=\frac{\alpha^2}{\alpha^2+\beta^2},\quad c_4=\frac{\beta^2}{\alpha^2+\beta^2}$$

Merging the relationship (21) and (22), it can be written that:

(27) $$J=f[K_m(\alpha,\beta)]$$

Next, substituting (27) into the equation (14), a relationship is set that enables to select geometric coefficients of a solenoid superconductor winding:

$$\nu(\alpha,\beta)=\beta\left(\alpha^2-1\right) \tag{16}$$

$$B_0^*(\alpha,\beta)=\frac{B_0}{\mu_0 a_1}=K_0(\alpha,\beta)f\left[K_m(\alpha,\beta)\right] \tag{28}$$

Those two approaches to the optimization of classical winding overall dimensions as described above have been expressed now, for the superconductor winding, in the following manner:

$$\begin{aligned}&\left(\alpha^2-1\right)\frac{\partial}{\partial\alpha}\left\{K_0((\alpha,\beta)f\left[K_m(\alpha,\beta)\right])\right\}=\\&=2\alpha\beta\frac{\partial}{\partial\beta}\left\{K_0((\alpha,\beta)f\left[K_m(\alpha,\beta)\right])\right\}\end{aligned} \tag{29}$$

$$\beta\left(\alpha^2-1\right)=\frac{V}{2\pi a_1^3}\ \textit{(given value)} \tag{18}$$

and:

$$\begin{aligned}&\left(\alpha^2-1\right)\frac{\partial}{\partial\alpha}\left\{K_0((\alpha,\beta)f\left[K_m(\alpha,\beta)\right])\right\}=\\&=2\alpha\beta\frac{\partial}{\partial\beta}\left\{K_0((\alpha,\beta)f\left[K_m(\alpha,\beta)\right])\right\}\end{aligned} \tag{30}$$

$$K_0(\alpha,\beta)f\left[K_m(\alpha,\beta)\right]=\frac{B_0}{\mu_0 a_1^3}\ \textit{(given value)} \tag{31}$$

In [4] the results of optimal dimensions calculations for the classical and superconductor windings are presented, with regard to various assumed values of B_0 induction.

The Obtained Computing Results

To determine the shape of the winding, the relationships (15) and (16) should be applied. In equations (15) and (17), four variables occur: ν, K_0, α, and β; two of them being independent ones. An analytical expression for the magnetic field distribution on the symmetry axis of the solenoid can be achieved:

$$H=\frac{I}{2}\left(\begin{aligned}&(b-z)\ln\frac{a_2+\sqrt{a_2^2+(b-z)^2}}{a_1+\sqrt{a_1^2+(b-z)^2}}+\\&+(b+z)\ln\frac{a_2+\sqrt{a_2^2+(b+z)^2}}{a_1+\sqrt{a_1^2+(b+z)^2}}\end{aligned}\right) \tag{32}$$

Fig. 5 presents graphs of distributions of the relative value of the z component of magnetic flux density on the symmetry axis of the solenoid as shown in Fig 3. The analysed magnetic field distribution is for the value of $z = \pm$ 150 mm and is related to the length of the matrix ($2\ L_m$ = 300 mm). It is worth noticing, that the considered solenoid can provide the homogenous field distribution (required for the efficient separation process) only when its length significantly exceeds the length of the matrix. That situation requires big quantity of expensive superconducting wire.

The author [4] proposes considering other possibilities of windings for magnetic field excitation in HGMS, which promises improved homogeneity of the magnetic field in the working space of the separator. Two of the possible improved designs are presented in Fig. 6. Field distributions for the designs of Fig. 6 have been predicted numerically using finite elements modelling.

Fig. 7 and Fig. 8 present distribution graphs of a relative value of *z* component of magnetic flux density on the symmetry axis for $z = \pm$ 150 mm for the two proposed new designs (Fig. 7 and Fig. 8 should be compared with Fig. 5).

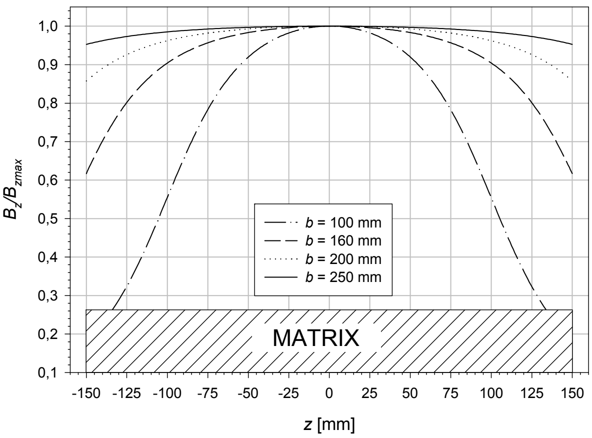

Fig. 5. Distribution of B_z (*z* component of magnetic flux density) for $r = 0$, for the coil of the HGMS

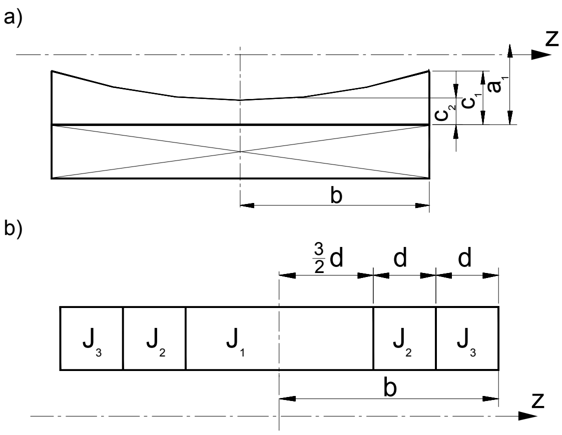

Fig. 6. Proposed designs: a) coil with additional turns of the winding, b) coil with variable current density in the cross-section (J_1, J_2, J_3); $\beta_1=L_m/(a_1-c_1)$, $\beta_2=L_m/(a_1-c_2)$

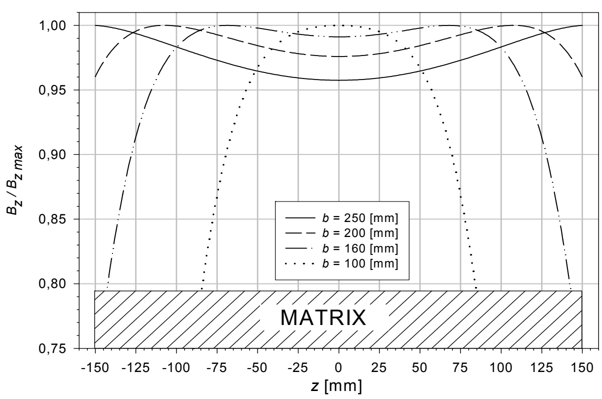

Fig. 7. Distribution of B_z (*z* component of the magnetic flux density) for $r = 0$ for the design from Fig. 6a;
(β_1 = 5 and β_2 = 3,75).

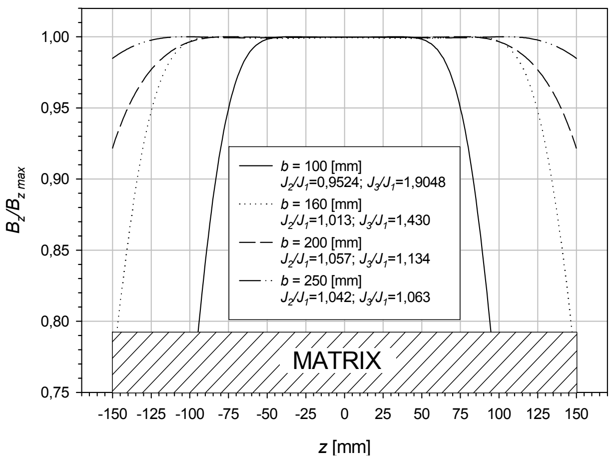

Fig. 8. Distribution of B_z (*z* component of the magnetic flux density) for $r = 0$ for the design from Fig. 6b.

The author proposes the shaping of the magnetic field using the construction from Fig. 6b, where the coil is divided into small sections with different current density. This solution leads to both technical and economical considerations. The technical aspect consists in a good usage of the superconductor. From the economical point of view, the variable cross-section method makes it possible to minimize the volume of the superconductor used. However, there may be a problem of supplying different currents to different sections, as in the case of using the superconductor this would require using several current leads. This could cause an increase in liquid helium evaporation from the cryostat.

Conclusions

The high force separation capabilities of superconducting magnets and their application for the most difficult separation problems of paramagnetic or low susceptibility materials are now recognized.

The magnetic separation is effective providing there are the same conditions of particle's extraction in the whole working space of the separator. Homogenity of the magnetic field distribution in the separator is necessary. Thus, it is a problem concerning optimization of electric devices.

In the paper a solenoid of rectangular shape was considered. Its dimensions were chosen in a such a way that the separator matrix was placed in a homogenous field. However, in that case a very long solenoid (in comparison to the matrix length) should be constructed, which would result in the use of a big quantity of an expensive superconductor.

The author has proposed other solutions, with the noteworthy one consisting in distribution of the winding into some sections powered by the current of different density. The obtained computing results show that the above solution gives a very profitable distribution of the field. However, the disadvantage of the proposed idea lies mainly in the necessity of using current leads to supply current to the particular sections of the solenoid. That, in turn, results in the increased evaporation of liquid helium.

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