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Passivity Control with sliding mode observer of induction motor

***Abstract**. In this paper we will present a control based on the passivity of the induction motor - the difficulty of the measurement of flux rotoric is surmounted by a sliding mode observer. A variety (surface) on which the error in estimation of the output is null (stable) is defined. There are established the conditions of sliding (calculation of the observer gains) for which all trajectories of system go towards surface (attractivity) and there remain (invariance). The synthesis of the passivity control with the estimate of flux rotoric by the means of this observer is illustrated by a simulation under Matlab-Simulink. Total stability is guaranteed and the tracking of flux and the torque is reached.*

***Streszczenie.** Zaprezentowano system sterowania silnikiem indukcyjnym przy wykorzystaniu obserwatora ślizgowego. Poszukiwane są warunki ślizgowe dla których wszystkie trajektorie są skierowane do powierzchni i pozostają niezmienne. Symulacje potwierdziły stabilność systemu. (**Sterowanie ślizgowe silnikiem indukcyjnym)**.*

Keywords: induction motor, passivity control, sliding mode observer.
Słowa kluczowe: silnik indukcyjny, sterowanie ślizgowe.

Introduction

The problem of global torque tracking and rotor flux norm regulation of induction motors perturbed by an unknown constant load torque was solved with an observer based controller in [5]. In this paper the result from that work is extended to treat the practically important case when the rotor flux norm is required to follow a time-varying reference [1] [2]. The controller design follows the passivity based approach and proceeds in two steps: First, a target closed loop dynamics compatible with the physical model of the motor that delivers the desired rotor flux and torque is designed. Second, a nonlinear dynamic output feedback controller is proposed, which ensures that this behaviour is asymptotically achieved. A proof of global tracking is given under the assumption of known motor parameters. The controller consists of a globally convergent nonlinear observer and observed state feedback control law [3]. From a theoretical perspective it puts this passivity-based approach on equal footing (with respect to the achievable control objectives) with linearizing based controllers. It must be pointed out that the observer used in literature is simple, with only updating from current error terms in the rotor equations. It would in general be advantageous to have updating also in stator equations for making the observer more robust. However, the gain of the proposed observer is obtained from some differential equations which are not usually desirable for implementation purposes. In [7], a constant gain observer has been proposed for general single output systems that are uniformly infinitesimally observable under some regularity assumptions on the vector field. However, the main shortcoming of the proposed observer is that its practical construction is difficult to realize because the computation of the observer gain is not direct. Despite these approaches, for some particular classes of nonlinear systems, observers have been proposed irrespective of the inputs. In this paper, we present an sliding mode observer. The proposed design does not use any kind of transformation to update the gain of the observer. As a result, its implementation is greatly facilitated.

Mathematical model and problem formulation

Mathematical model

For analytical purposes it is common to substitute the squirrel (single) cage rotor, which has a uniform conductor distribution, with an equivalent fictitious rotor with the same number of phases as the stator, and sinusoid ally distributed conductors. This implies that in the analysis, only the first order harmonic of the rotor MMF is accounted for. Experimental results indicate that analysis and controller designs based on this simplified model will also be valid for the real machine. In cases of deep bar or double cage rotors, care should however be taken when modelling these with sinusoid ally distributed windings [4]. The dynamic equations are derived by direct application of Euler-Lagrange equations [5]. This results in:

(1) $$D_e(q_m)\ddot{q}_e+W_1(q_m)\dot{q}_m\dot{q}_e+R_e\dot{q}_e=M_e u$$

(2) $$D_m\ddot{q}_m+R_m\dot{q}_m=\tau-\tau_L=\tfrac{1}{2}\dot{q}_e^T W_1(q_m)\dot{q}_e-\tau_L$$

The standard two phase model (In this model the axes for the stator have a fixed position while those corresponding to the rotor are rotating at the electrical angular speed) of an pole pair squirrel cage induction motor with uniform air gap has electrical parameters

$$D_e(q_m)=\begin{bmatrix} L_s I_2 & L_m e^{Jpq_m} \\ L_m e^{-Jpq_m} & L_r I_2 \end{bmatrix} \quad M_e=\begin{bmatrix} I_2 \\ 0 \end{bmatrix}$$

$$R_e=\begin{bmatrix} R_s I_2 & 0 \\ 0 & R_r I_2 \end{bmatrix},\quad J=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}=-J^T$$

$$e^{Jpq_m}=\begin{bmatrix} \cos(pq_m) & -\sin(pq_m) \\ \sin(pq_m) & \cos(pq_m) \end{bmatrix},\quad e^{-Jpq_m}=\left(e^{Jpq_m}\right)^T$$

with

$$W_1(q_m)=\frac{dD_e(q_m)}{dq_m}=\begin{bmatrix} 0 & pL_m J e^{Jpq_m} \\ -pL_m J e^{-Jpq_m} & 0 \end{bmatrix}$$

where $L_s, L_r, L_m>0$ are the stator, rotor and mutual inductances, R_s, R_r are stator and rotor resistances. $\dot{q}_e:=\left[\dot{q}_s^T,\dot{q}_r^T\right]^T$ is current vector, $\dot{q}_m$ is the rotor angular velocity. $D_m>0$ is the rotor inertia. The control signals $u=\left[u_1,u_2\right]^T$ are the stator voltages τ_L is the external load torque, and $R_m>0$ is the mechanical viscous damping constant. The flux vector $\lambda:=\left[\lambda_s^T,\lambda_r^T\right]^T$ is related to the current vector $\dot{q}_e$ via $\lambda=D_e(q_m)\dot{q}_e$.

To carry out the controller design, a suitable linear factorization of these workless forces into a form

(3) $$\begin{bmatrix} W_1(q_m)\dot{q}_m\dot{q}_e \\ -\frac{1}{2}\dot{q}_e^T W_1(q_m)\dot{q}_e \end{bmatrix} = C(q,\dot{q})\dot{q}$$

$C(q,\dot{q})$ will required to be such that $\dot{D}(q)-2C(q,\dot{q})=C^T(q,\dot{q})$ is skew-symmetric and the third and fourth rows of are independent of $\dot{q}_e$.

These conditions are needed for the following stability analysis. Using the transpose expression of $W_1(q_m)$ it is clear that the objectives can be achieved with the choice

(4) $$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 f(q,\dot{q}) \\ -pL_m J e^{-Jpq_m}\dot{q}_m 0 & 0 \\ f^T(q,\dot{q}) & 0 \quad 0 \end{bmatrix}$$

with

(5) $f(q,\dot{q}) = pL_m J e^{Jpq_m}\dot{q}_r$

This factorization leads to following compact model presentation:

(6) $$D(q)\dot{q}+C(q,\dot{q})\dot{q}+R\dot{q}=Mu+\xi$$

where

$$D(q)=diag\{D_e(q_m), D_m\}, \quad R=diag\{R_e, R_m\}$$
$$\dot{q}=[\dot{q}_e^T \ \dot{q}_m]^T, \ \xi=[0 \ -\tau_L]^T$$

Problem formulation

Consider the induction motor model [6] with outputs torque τ and rotor flux norm $\|\lambda_r\|$ to be controlled. Assume:

A.1 The load torque τ_L is an unknown constant.

A.2 Stator currents $\dot{q}_{s1}, \dot{q}_{s2}$ rotor speed and position are available for measurement.

A.3 All motor parameters are exactly known, and the viscous mechanical damping constant is nonzero $R_m>0$.

Let the desired torque $\tau_d(t)$ be a bounded and differentiable function with known bounded first order derivative, and the desired rotor flux norm be a strictly positive bounded and twice differentiable function β with known bounded first and second order derivatives. Under these conditions, design a control law that will ensure internal stability and asymptotic torque and rotor flux norm tracking, that is, the closed loop system must give

(7) $\lim_{t\to\infty}|\tau-\tau_d|=0, \quad \lim_{t\to\infty}\big|\|\lambda_r\|-\beta\big|=0$

from all initial conditions and with all signals uniformly bounded.

Ideal Case: Full State Feedback

For the sake of presentation, the problem will first be solved under the temporary assumption of full state measurement and known load torque. This is referred to as the ideal case. It will then be explained in the text section how the controller can be modified to remove these assumptions.

Following the approach used in [5], it can be shown that the control problem can be recast in terms of tracking of the motor currents. To this end, let a vector of desired currents and an internal desired rotor speed be defined as $\dot{q}_d := [\dot{q}_{sd}^T \ \dot{q}_{rd}^T \ \dot{q}_{md}]^T$, and define the error as $\dot{\tilde{q}} := \dot{q}-\dot{q}_d$. Equation [6] can then be rewritten as

$$D(q)\ddot{\tilde{q}}+C(q,\dot{q})\dot{\tilde{q}}+[R+K]\dot{\tilde{q}}=\psi$$

where K is a positive semi-definite matrix (to be defined below) that injects the required damping in the output feedback case. It is set to zero when the state is measurable. The right-hand side in the equation above is defined as

$$\psi := -D(q)\ddot{q}_d - C(q,\dot{q})\dot{q}_d - R\dot{q}_d + K\dot{\tilde{q}} + Mu + \xi$$

This is a perturbation term that should be set to zero with a suitable choice of u and $\dot{q}_d$. If this is possible that $\dot{\tilde{q}}\to 0$ as $t\to\infty$. The problem is henceforth solved if u and $\dot{q}_d$ can be chosen to ensure

a. $\psi\equiv 0$

b. $\lim_{t\to\infty}\dot{\tilde{q}}=0 \Rightarrow \lim_{t\to\infty}|\tau-\tau_d|=0, \ \lim_{t\to\infty}\big|\|\varphi_r\|-\beta(t)\big|=0$

The problem of setting $\psi\equiv 0$ is first solved. It is clear from [6] and the definition of M that, for any given $q, \dot{q}, \dot{q}_d, \ddot{q}_d$, the first two equations of $\psi\equiv 0$ can be satisfied with

(8) $$u = L_s\ddot{q}_{sd} + L_m e^{Jpq_m}\ddot{q}_{rd} + f(q,\dot{q})\dot{q}_{md} + R_s\dot{q}_{sd}$$

It follows from [5] that this control law requires the measurement of the rotor currents $\dot{q}_r$. Also, the fifth equation $\psi\equiv 0$ is satisfied if $\dot{q}_{md}$ is defined as the solution to

(9) $D_m\ddot{q}_{md} - f^T(q,\dot{q})\dot{q}_{sd} + R\dot{q}_{md} = -\tau_L$

The next problem is to solve the third and fourth equations of $\psi\equiv 0$ which for convenience can be rewritten explicitly as

(10) $$\dot{\lambda}_{rd} + R_r\dot{q}_{rd} = 0$$

where λ_{rd} has been defined as

(11) $$\lambda_{rd} = L_m e^{-Jpq_m}\dot{q}_{sd} + L_r\dot{q}_{rd}$$

From the two equations above it can be seen that the problem is now reduced to the definition of a desired rotor flux (with known derivative) whose norm is $\beta(t)$ and such that the machine delivers the desired torque τ_d . Towards this end, we choose [6]:

$$\lambda_{rd} = \beta(t)\begin{bmatrix} cos(\rho_d) \\ sin(\rho_d) \end{bmatrix} = e^{J\rho_d}\begin{bmatrix} \beta(t) \\ 0 \end{bmatrix}$$

(12) $$\dot{\rho}_d = \frac{R_r}{p\beta^2}\tau_d, \ \rho_d(0)=0$$

With these choices of u and $\dot{q}_d$ it can be concluded that $\lim_{t\to\infty}\dot{\tilde{q}}=0$, and torque and flux tracking follows.

Proposition.1 (Ideal case): Consider the motor model [6] in closed loop with [8], [9], [5] where $\dot{q}_{sd}, \ddot{q}_{sd}, \dot{q}_{rd}, \ddot{q}_{rd}$ are calculated from [11] and [10] using [12]. Then, for all initial conditions equation [7] holds with all signals uniformly bounded.

Ouput feedback controller

The main result if this section, a nonlinear observer-based controller, is presented. Consider the induction motor [6] with outputs torque τ and rotor flux norm $\|\lambda_r\|$ to be controlled, and assumptions A1-A3. Let the control law be defined as

$$u = L_s\ddot{q}_{sd} + L_m e^{Jpq_m}\ddot{q}_{rd} + pL_m J\dot{\hat{q}}_r\dot{q}_{md} + R_s\dot{q}_{sd} - K_1\dot{\tilde{q}}_s \quad (13)$$

where $\dot{\hat{q}}_r$ is estimation of rotor currents, and

$$\dot{q}_{rd} = -e^{J\rho_d}\begin{bmatrix} \dfrac{\dot{\beta}}{R_r} \\ \dfrac{\tau_d}{p\beta} \end{bmatrix} \qquad \dot{q}_{sd} = \frac{1}{L_m}e^{J(pq_m+\rho_d)}\begin{bmatrix} \beta + \dfrac{L_r}{R_r}\dot{\beta} \\ \dfrac{L_r}{p\beta}\tau_d \end{bmatrix}$$

and with controller dynamics

$$\dot{\rho}_d = \frac{R_r}{p\beta^2}\tau_d, \quad \rho(0) = 0 \quad (14)$$

$$\ddot{q}_{md} = \frac{1}{D_m}\left(-pL_m\dot{\hat{q}}_r^T J e^{-Jpq_m}\dot{q}_{sd} - R_m\dot{q}_{md} - \hat{\tau}_L + K_2\dot{\tilde{q}}_m\right) \quad (15)$$

where $\dot{q}_{md}(0) = \dot{q}_m(0)$ and K_1, K_2 are the gains. The load adaptation law is $\dot{\hat{\tau}} = -\gamma_L\dot{\tilde{q}},\ \gamma_L > 0$.

Sliding mode observer

It is well-known that passivity based control strategies required the knowledge of the rotor flux. Since these quantities are not easily accessible, many research efforts have been focused on their estimation in the past few years. In this section, we present construct of a reduced flux observer for an induction motor written in the α,β Park's frame. The proposed observer uses the measurements of the stator voltage and current, and the rotor speed. More precisely, the observer is designed up to an injection of the speed measurements so that only the electrical equations are considered. Consider (α,β) Park's model for induction motor:

$$\begin{aligned}
\frac{d\dot{q}_{s\alpha}}{dt} &= -\gamma\dot{q}_{s\alpha} + \frac{K}{T_r}\lambda_{r\alpha} + Kp\dot{q}_m\lambda_{r\beta} + \frac{1}{\sigma L_s}u_{s\alpha} \\
\frac{d\dot{q}_{s\beta}}{dt} &= -\gamma\dot{q}_{s\beta} - Kp\dot{q}_m\lambda_{r\alpha} + \frac{K}{T_r}\lambda_{r\beta} + \frac{1}{\sigma L_s}u_{s\beta} \\
\frac{d\lambda_{r\alpha}}{dt} &= \frac{L_m}{T_r}\dot{q}_{s\alpha} - \frac{1}{T_r}\lambda_{r\alpha} - p\dot{q}_m\lambda_{r\beta} \\
\frac{d\lambda_{r\beta}}{dt} &= \frac{L_m}{T_r}\dot{q}_{s\beta} + p\dot{q}_m\lambda_{r\alpha} - \frac{1}{T_r}\lambda_{r\beta} \\
y_1 &= \dot{q}_{s\alpha} \\
y_2 &= \dot{q}_{s\beta}
\end{aligned} \quad (16)$$

where

$$T_r = \frac{L_r}{R_r},\quad \sigma = 1 - \frac{L_m^2}{L_s L_r},\quad K = \frac{L_m}{\sigma L_s L_r},\quad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}$$

Our objective is to stabilize dynamic errors. Thus, by respecting following methodology:

- To define a variety (surface) $S(Y,t) \in R^p$ on which the error in estimation of the output is null (stable).
- To establish the conditions of sliding (calculation of the observer gains) for which all trajectories of system go towards surface (attractivity) and there remain (invariance).

In what follows, we briefly describe the synthesis of the observer by sliding mode [9] and [10] for the system described by:

$$\begin{cases} \dot{x} = f(x) + gu \\ y = h(x) \end{cases} \quad (17)$$

where $x \in R^n$ is the state vector, $u \in R^m$ the control, $y \in R^r$ input vector. Also let us suppose that our system is controllable and observable.

Let us define the following observer by sliding mode:

$$\dot{\hat{x}} = f(\hat{x}, u) + \Lambda I_s \quad (18)$$

where $\hat{x} \in \Re^n$ is estimation of x, $\Lambda \in \Re^{n\times r}$ is gain matrix of observer which must be synthesized to stabilize the estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$, and I_s is a vector of dimension r:

$$I_s = Sign(S) = [Sign(s_1)Sign(s_2)\ldots Sign(s_r)]^T \quad (19)$$

with

$$S = \Gamma[y - h(\hat{x})] = [s_1 s_2 \ldots s_r]^T \quad (20)$$

Γ is matrix $(n\times r)$ to specify. Let us also define the variety Ω_v $(n\times r)$ such as

$$\Omega_v = \{\tilde{x} \in R^n \,/\, S(\tilde{x}) = 0\} \quad (21)$$

It is interesting to note that the variety Ω_v can be interpreted as being the intersection of the r sliding surfaces $s_i, i \in \{1,\ldots,r\}$. As we saw later, the basic procedure of the synthesis of a sliding mode observer consists of two following stages. Initially to synthesize the variety Ω_v such as the trajectories errors in estimation brought back to the Ω_v variety have dynamic desired stability. Then, we determine the gains matrix Λ of observer, to bring back $\tilde{x}(t)$ to the variety Ω_v (property of attractivity) and to maintain it on this variety to slip towards the origin (property of invariance).

The Ω_v variety is attractive if and only if $s_i\dot{s}_i < 0, i \in \{1,\ldots,r\}$. This condition defines the area in which the sliding mode exists. During the sliding mode, the estimation error are reduced of order n (initial system) to the order $(n-r)$ (equivalent system of a reduced order). This is obtained by using the method of equivalent control, [9] and [10].

Supposing that $y = h(x) = Cx$ where C is a matrix of dimension adapted and determining the equivalent vector

I_s to satisfy the invariance condition $\dot{S}=0$ for $S=0$ one obtains

(22) $$I_s=(\Gamma C\Lambda)^{-1}\Gamma C\left[f(x,u)-f(\hat{x},u)\right]$$

The matrix $\Gamma C\Lambda\in R^{r\times r}$ must be non-singular and with a suitable choice of Γ and Λ. Then equivalent dynamics is given by

(23) $$\dot{\tilde{x}}=\left(I-\Lambda(\Gamma C\Lambda)^{-1}\Gamma C\right)\left[f(x,u)-f(\hat{x},u)\right]$$

with

$$\Gamma C\tilde{x}=0$$

To synthesize the observer in the case of the induction motor, one considers the speed $\Omega(t)$ a function bounded of class C[1] and whose derivative $\dot{\Omega}(t)$ is also bounded. Let us η_1 and η_2 are known positive parameters satisfying:

$$|\Omega(t)|\le\eta_1 \quad \text{and} \quad |\dot{\Omega}(t)|\le\eta_2$$

Let us consider only the first four equations of induction motor model. Let us, $\hat{x}_1,\hat{x}_2,\hat{x}_3,\hat{x}_4$ the estimates of the x_1,x_2,x_3,x_4 respectively which are the state variables of $\dot{q}_{s\alpha},\dot{q}_{s\beta},\lambda_{r\alpha},\lambda_{r\beta}$. The observer is only a copy of the original system to which one adds the control gains with the terms of commutation; thus

(24) $$\begin{cases}\dot{\hat{x}}_1=-\gamma x_1+\dfrac{K}{T_r}\hat{x}_3+pK\Omega\hat{x}_4+\dfrac{1}{\sigma L_S}u_{s\alpha}+\Lambda_1 I_S\\ \dot{\hat{x}}_2=-\gamma x_2+\dfrac{K}{T_r}\hat{x}_4-pK\Omega\hat{x}_3+\dfrac{1}{\sigma L_S}u_{s\beta}+\Lambda_2 I_S\\ \dot{\hat{x}}_3=\dfrac{L_m}{T_r}\hat{x}_1-\dfrac{1}{T_r}\hat{x}_3-p\Omega\hat{x}_4+\Lambda_3 I_s\\ \dot{\hat{x}}_4=\dfrac{L_m}{T_r}\hat{x}_2-\dfrac{1}{T_r}\hat{x}_4-p\Omega\hat{x}_3+\Lambda_4 I_s\end{cases}$$

where are $\Lambda_1,\Lambda_2,\Lambda_3$ and Λ_4 observer gains. $\Lambda_j=\left[\Lambda_{j1}\quad\Lambda_{j2}\right]$ for $j\in\{1,2,3,4\}$, The vector I_s is given by:

(25) $$I_s=\begin{bmatrix}sign(S_3)\\ sign(S_4)\end{bmatrix}$$

with

(26) $$S_{ob}=\begin{bmatrix}S_3\\ S_4\end{bmatrix}=\Gamma\begin{bmatrix}x_1-\hat{x}_1\\ x_2-\hat{x}_2\end{bmatrix}$$

and

(27) $$\Gamma=\frac{1}{\beta(t)}\begin{bmatrix}\dfrac{K}{T_r} & -p\Omega(t)K\\ p\Omega(t)K & \dfrac{K}{T_r}\end{bmatrix}$$

with

(28) $$\beta(t)=\left[\frac{K}{T_r}\right]^2+p^2K^2\Omega^2(t)$$

The choice Γ of is made in order to facilitate the calculation of gains observer. Let $e_i=x_j-\hat{x}_j$ for $j\in\{1,2,3,4\}$, the dynamics of estimation error are given by:

(29) $$\begin{aligned}\dot{e}_1&=\frac{K}{T_r}e_3+pK\Omega(t)e_4-\Lambda_1 I_s\\ \dot{e}_2&=\frac{K}{T_r}-pK\Omega(t)e_3-\Lambda_2 I_s\\ \dot{e}_3&=-\frac{1}{T_r}e_3-p\Omega(t)e_4-\Lambda_3 I_s\\ \dot{e}_4&=-\frac{1}{T_r}e_4+p\Omega(t)e_3-\Lambda_4 I_s\end{aligned}$$

The analysis of stability consists in determining the gains Λ_1 and Λ_2 et in order to ensure the attractivity of the sliding surface $S_{ob}=0$. Then Λ_3 and Λ_4 are given such as the reduced system obtained when $S_{ob}\equiv\dot{S}_{ob}\equiv 0$ is locally stable. The following result is obtained.

Proposition.1: *Let us suppose that the state variables $x_3(t)$ and $x_4(t)$ are bounded, let us consider the system (29) with the following gains:*

(30) $$\begin{bmatrix}\Lambda_{11}\Lambda_{12}\\ \Lambda_{21}\Lambda_{22}\end{bmatrix}=\Gamma^{-1}\Delta$$
$$\begin{pmatrix}\Lambda_{31}\Lambda_{32}\\ \Lambda_{41}\Lambda_{42}\end{pmatrix}=\begin{pmatrix}\left(q_1-\dfrac{1}{T_r}\right)\delta_1 & -p\Omega(t)\delta_2\\ p\Omega(t)\delta_1 & \left(q_2-\dfrac{1}{T_r}\right)\delta_2\end{pmatrix}$$

where

$$\begin{cases}\delta_1>\rho_3+|\hat{\varphi}_{r\alpha}|+a_{\max}|e_1|+b_{\max}|e_2|\\ \delta_2>\rho_4+|\hat{\varphi}_{r\beta}|+b_{\max}|e_1|+a_{\max}|e_2|\end{cases}$$

with

$$a_{\max}=2T_r\, p^2K\eta_1\eta_2\,,$$
$$b_{\max}=pT_r^2\eta_2\left(\frac{1}{K}+2p^2\eta_1^2\right)$$

and

$$|x_3(t)|\le\rho_3\,,\qquad |x_4(t)|\le\rho_4\,,\qquad q_1,q_2>0$$
$$\Delta=\begin{bmatrix}\delta_1 & 0\\ 0 & \delta_2\end{bmatrix}$$

Thus

(i) The variety with two dimensions $S_{ob}=0$ is attractive and $e_1(t)$, $e_2(t)$ converges towards zero

(ii) The dynamics of a reduced order obtained when $S_{ob}\equiv\dot{S}_{ob}\equiv 0$ is given by:

(31) $$\Sigma_1\ \begin{cases}\dot{e}_3=-q_1e_3\\ \dot{e}_4=-q_2e_4\end{cases}$$

where $q_1,q_2>0$ that corresponds to an exponential stability of e_3 and e_4.

Proof:

(i) Let us the Lyapunov function defined by $V=\frac{S_{ob}^{T}S_{ob}}{2}$. Its derivative is given by $\dot{V}=S_{ob}^{T}\dot{S}_{ob}$ Thus :

(ii)

(32) $$\dot{S}_{ob}=\Gamma\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\end{bmatrix}+\dot{\Gamma}\begin{bmatrix}e_1\\ e_2\end{bmatrix}$$

(33) $$\dot{S}_{ob}=\Gamma\left[\begin{pmatrix}\frac{K}{T_r} & pK\Omega(t)\\ -pK\Omega(t) & \frac{K}{T_r}\end{pmatrix}\begin{pmatrix}e_3\\ e_4\end{pmatrix}-\begin{pmatrix}\Lambda_{11}\Lambda_{12}\\ \Lambda_{21}\Lambda_{22}\end{pmatrix}\begin{pmatrix}sign(S_3)\\ sign(S_4)\end{pmatrix}\right]+\dot{\Gamma}\begin{bmatrix}e_1\\ e_2\end{bmatrix}$$

where

$$\dot{\Gamma}=\begin{pmatrix}a & -b\\ b & a\end{pmatrix}$$

with

$$a=-\frac{K}{T_r}\frac{2p^2K^2\Omega2(t)\dot{\Omega}(t)}{\beta^2}$$

$$b=pK\frac{\beta\dot{\Omega}(t)-2p^2K^2\Omega(t)^2\dot{\Omega}(t)}{\beta^2}$$

It is known that

(34) $$\begin{bmatrix}\Lambda_{11}\Lambda_{12}\\ \Lambda_{21}\Lambda_{22}\end{bmatrix}=\Gamma^{-1}\Delta\quad \Gamma^{-1}=\begin{bmatrix}\frac{K}{T_r} & p\Omega(t)K\\ -p\Omega(t)K & \frac{K}{T_r}\end{bmatrix}$$

We obtain

(35) $$\dot{V}=S_{ob}^{T}\left[\begin{pmatrix}e_3\\ e_4\end{pmatrix}-\begin{pmatrix}\delta_1 & 0\\ 0 & \delta_2\end{pmatrix}\begin{pmatrix}sign(S_3)\\ sign(S_4)\end{pmatrix}+\begin{pmatrix}a & -b\\ b & a\end{pmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}\right]$$

It is clear that $\dot{V}<0$ as long as the following conditions are respected.

(36) $$\begin{cases}\delta_1>\left|e_3+ae_1-be_2\right|\\ \delta_2>\left|e_4+be_1+ae_2\right|\end{cases}$$

However

(37) $$\begin{cases}\left|e_3+ae_1-be_2\right|\\ \left|e_4+be_1+ae_2\right|\end{cases}\leq\begin{cases}\rho_3+\left|\hat{\varphi}_{r\alpha}\right|+a_{\max}\left|e_1\right|+b_{\max}\left|e_2\right|\\ \rho_4+\left|\hat{\varphi}_{r\beta}\right|+b_{\max}\left|e_1\right|+a_{\max}\left|e_2\right|\end{cases}$$

(i) As long as surface $S_{ob}=0$ is attractive and that the matrix Γ is non-singular, convergence towards zero of $\left(e_1(t),e_2(t)\right)$ follows.

(ii) On $S_{ob}\equiv 0$ the equivalent vector $\tilde{I}_s$ is obtained, by using the invariance property of S_{ob}. Thus:

$$\begin{pmatrix}0\\ 0\end{pmatrix}=\Gamma^{-1}\left[\begin{pmatrix}e_3\\ e_4\end{pmatrix}-\begin{pmatrix}\delta_1 & 0\\ 0 & \delta_2\end{pmatrix}\tilde{I}_s\right]$$

(38) $$\tilde{I}_s=\left[\frac{e_3}{\delta_1},\frac{e_4}{\delta_2}\right]$$

The reduced system $\sum_1$ of observation error is written as:

(39) $$\sum\nolimits_1\begin{cases}\dot{e}_3=-\frac{1}{T_r}e_3-p\Omega(t)e_4-\Lambda_3\tilde{I}_s\\ \dot{e}_4=p\Omega(t)e_3-\frac{1}{T_r}e_4-\Lambda_4\tilde{I}_s\end{cases}$$

Let us replace $\tilde{I}_s$ and $\begin{pmatrix}\Lambda_{31}\Lambda_{32}\\ \Lambda_{41}\Lambda_{42}\end{pmatrix}$ in $\sum_1$, the result (ii) is obtained.

By construction, this observer is robust compared to uncertainties of modelling and the measurement errors. Let us suppose that one has uncertainties of modelling $\Delta f_1(x)$ and $\Delta f_2(x)$ in the first two equations of the motor model

(40) $$\begin{cases}\dot{x}_1=-\gamma x_1+\frac{K}{T_r}x_3+pK\Omega x_4+\frac{1}{\sigma L_s}u_{s\alpha}+\Delta f_1(x)\\ \dot{x}_2=-\gamma x_2+\frac{K}{T_r}x_4-pK\Omega x_3+\frac{1}{\sigma L_s}u_{s\beta}+\Delta f_2(x)\\ \dot{x}_3=\frac{L_m}{T_r}x_1-\frac{1}{T_r}x_3-p\Omega\hat{x}_4\\ \dot{x}_4=\frac{L_m}{T_r}x_2-\frac{1}{T_r}x_4-p\Omega x_3\end{cases}$$

where $\Delta f_1(x)$ and $\Delta f_2(x)$ are bounded by two known functions ξ_1 and ξ_2. In this case, the dynamics of observation error are given as follows:

(41) $$\begin{aligned}\dot{e}_1&=\frac{K}{T_r}e_3+pK\Omega(t)e_4-\Lambda_1 I_s+\Delta f_1(x)\\ \dot{e}_2&=\frac{K}{T_r}e_4-pK\Omega(t)e_3-\Lambda_2 I_s+\Delta f_2(x)\\ \dot{e}_3&=-\frac{1}{T_r}e_3-p\Omega(t)e_4-\Lambda_3 I_s\\ \dot{e}_4&=-\frac{1}{T_r}e_4+p\Omega(t)e_3-\Lambda_4 I_s\end{aligned}$$

Let us consider the sliding surface defined in (27) and according to the same step that previously, one can show that these surfaces are attractive provided that:

(42) $$\begin{cases}\delta_1\rangle\rho_4+\left|\hat{x}_3\right|+a_{max}\left|e_1\right|+b_{max}\left|e_2\right|+\xi_1\\ \delta_2\rangle\rho_4+\left|\hat{x}_4\right|+b_{max}\left|e_1\right|+a_{max}\left|e_2\right|+\xi_2\end{cases}$$

This result proves that our observer is robust compared to bounded uncertainties. The same result can be proven if the velocity is not exactly known $\Omega=\hat{\Omega}+\Delta\Omega$, where $\Delta\Omega$ the bounded unknown error is. What justifies the use of the sliding mode.

Simulation results

The parameters of induction motor used in the simulation are given in following table:

Table 1. The parameters of induction motor

Rotor resistance	Rr	0.07 Ω
Stator resistance	Rs	0.052 Ω
Mutual inductance	M	0.031 H
Stator inductance	Ls	0.031747 H
Rotor inductance	Lr	0.0323 H
Rotor inertia	Jm	0.41 kg m²

Pole pair	p	2
Nominal power	Pn	37 kW
Nominal speed	Ωn	1468 1/min
Nominal voltage	Usn	380 V
Nominal current	Isn	69 A
Nominal s flux	Φrn	0.93 Wb
Nominal torque	CemN	248 Nm

The benchmark shows that the load torque appears at the nominal speed. While at variable speed, the resistive torque is zero. Desired flux remains constant in the induction motor. We have done the simulation with Matlab-Simulink, as it is shown in figures (1), (2), (3) and (4), by using the benchmark and the motor parameter giving in table 1 to study the performances of passivity based control with sliding mode observer.

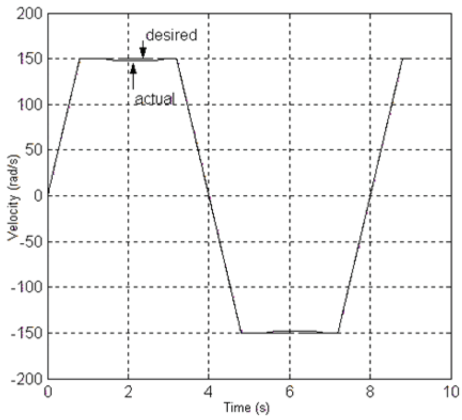

Fig.1. Curve of speed tracking

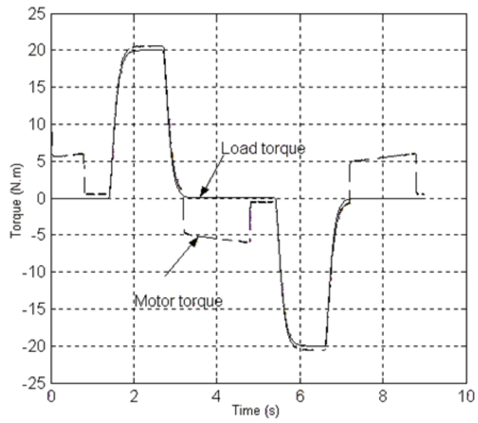

Fig. 2.Curve of torques

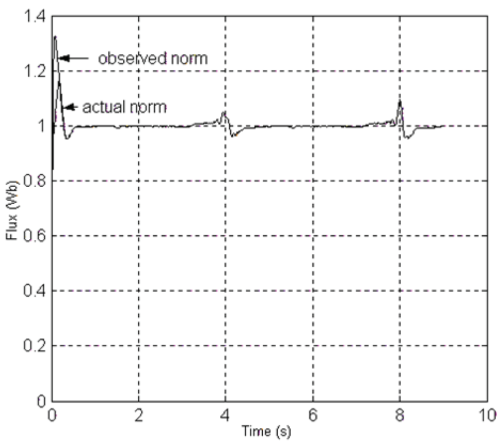

Fig. 3. Curve of flux

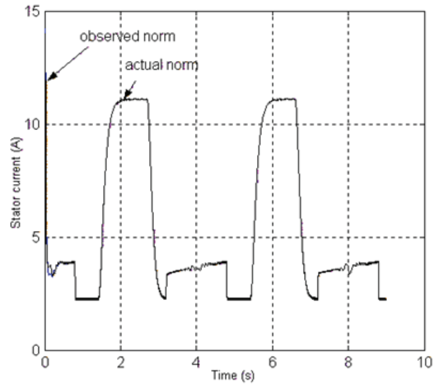

Fig.4. Curve of currents

Conclusions

In this paper, we proposed a passivity control with sliding mode observer of induction motor. The results show that this observer gives better performances in tracking of the torque, speed and estimate of flux. It presents the adjustment of the gains on the range of variation speed under the constraint of the speed at low or nominal case. A major advantage of the method is that very little tuning was required and convergence of the observation at low speed. Online simulation is in progress to validate these theoretical results.

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