Computation of electromagnetic and temperature fields of bridge transducer

Abstract. There is proposed the mathematical model of calculation of electromagnetic and thermal fields of single phase bridge converter by given voltage. The electric valve action is circumscribed by ideal key method. The spatial-time discretization algebraic equations are solved by the top relaxation method. The results of computation are given.

Keywords: mathematical model, bridge converter, electromagnetic field, temperature field

Introduction
Modern mathematical theory of high precision electronic systems indispensable run into the problem of the most modern mathematical models elaboration of devices of converter techniques. The analysis of the problem show that such mathematical models must be design with employment of electromagnetic field theory methods only because the electromagnetic circuit theory methods reach the limit of their resources completely. The theoretical results [5, 7-9] are base of subsequent our scientific research.

Mathematical model
Let iron-clad transformer is used in device scheme. The cross-section of quarter of transformer body are shown on fig. 2. The electromagnetic process in iron-clad transformer is circumscribed by quasi stationary electromagnetic field equations.

\[
\frac{\partial \mathbf{A}}{\partial t} = -\mathcal{L}^{-1} \left( \nabla \times (\nabla \times \mathbf{A}) - \mathbf{\delta} \right),
\]

where: \( \mathbf{A} \) is electromagnetic field vector-potential; \( \mathbf{\delta} \) is current density vector; \( \mathcal{L} \), \( \mathcal{R} \) are static reluctivities and conductivities matrixes; \( \nabla \) is Hamilton’s operator; \( t \) is time.

In this case the equation (1) get complicated [5,7]:

\[
\Gamma \frac{\partial \mathbf{A}}{\partial t} + \mathbf{n}_0 \frac{\gamma}{I} \int \frac{\partial \mathbf{A}}{\partial t} \, dl = -\nabla \times (\nabla \times \mathbf{A}) + \mathbf{n}_0 \frac{\gamma}{I} u(t),
\]

where: \( \mathbf{n}_0 \) is space normal or; \( u \) is electric voltage; \( \gamma \) is electric conductivity; \( I \) is length of winding wire.

Equation (1) or (2) covers whole zones of integration of electromagnetic field equation: laminated magnet conductor zone, magnetizing winding and dielectric zones. It will change in each zones.
Take into consideration conditions

\[(3) \quad A = x_0 A; \quad \delta = x_0 \delta; \quad B = B_x y_0 + B_z z_0,\]

where: \(x_0, y_0, z_0\) are Cartesian coordinate oris.

The equation (2) in magnetizing winding zone will be

\[(4) \quad \frac{\partial A}{\partial t} + c \frac{\partial}{\partial t} \int x_i \frac{\partial A}{\partial t} dA = \frac{v_k}{\gamma_{x,k}} \left( \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} \right) + \frac{c}{l_k} u_k,\]

where: \(\gamma_{x,k}\) is equivalent electric conductivity of \(k\)-th winding; \(v_0\) is constant reluctivity; \(c\) is constant coefficient.

The integral in the left part of (4) is taken over the trajectory on the surface of winding wire. In thin magnetizing winding wires the eddy currents are absent. This can make easily by adaptation \(c \to \max\).

In the case of spatial discretization (4) we substitute the equations of transient heat conduction

\[\text{Heating process we obtain from well-known differential equations of transient heat conduction}\]

\[(5) \quad \frac{\partial}{\partial t} \int x_i \frac{\partial A}{\partial t} dA \approx 4 d x_i \sum_{n=1}^{\infty} \frac{\partial A}{\partial t},\]

where: \(d\) is magnet core thickness; \(x_i, q_i\) are constant coefficients.

According to (3) in the laminated magnet conductor zone equation (1) assumes form:

\[(6) \quad \frac{\partial}{\partial z} \left( v_n \frac{\partial A}{\partial z} \right) + \frac{\partial}{\partial y} \left( v_x \frac{\partial A}{\partial y} \right) = 0,\]

where: \(v_n, v_x\) are static reluctivities of medium. The laminated magnet conductor is changed by continuous anisotropy ferromagnetic medium. The magnetisation characteristic of the medium \(H = H(B)\) we receive from recalculation of the ferromagnetic characteristic \(H_t = H_t(B_t)\) along \(x\)- and \(y\)-direction according to expression [5]

\[(7) \quad v = \chi v_i; \quad B = B_y; \quad \chi = \frac{d_y + d_b}{d_y + (d_y v_y)(B_t)/v_0},\]

where \(v(y(B))\) is static relucitivity of ferromagnetic which is calculated according to characteristic \(H_t = H_t(B_t)\); \(d_y, d_b\) are thicknesses of ferromagnetic sheets and air interval.

In air zone equation (1) will be more simplified:

\[(8) \quad v_0 \left( \frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial y^2} \right) = 0.\]

On the external border of the equations (4), (6), (8) integration zone we determine border conditions from assumption that magnetic field doesn't exist on it; on internal border – from condition of field symmetry. Magnet induction value we compute as

\[(9) \quad B_y = \frac{\partial A}{\partial z}; \quad B_z = -\frac{\partial A}{\partial y}; \quad B = \sqrt{B_y^2 + B_z^2}.\]
Logical variable $\varepsilon_1$ models pair work of first and third thyristors on the first time period ($\varepsilon_1 = 1$) and second and fourth thyristors on the second time period ($\varepsilon_1 = -1$). This presentation of valve action is possible because charged capacitor on half-period under sudden switching of control voltage impulse to second and fourth capacitors closes first and third capacitors simultaneously. This bridge scheme work is equivalent to changing of DC voltage on the next period start relatively of commutative capacitor. Therefore logical variable $\varepsilon_1$ we obtain as

$$\varepsilon_1 = \begin{cases} 1, & \text{if } jT < t < jT + 0.5T; \\ -1, & \text{if } jT + 0.5T < t < (j+1)T, \end{cases}$$

where $T$ is time period; $j = 0, 1, 2, 3,\ldots$

Secondary winding current equation we write as

$$i_2 = \frac{1}{r_2} \left( -\varepsilon_2 u_{c2} - \frac{\varepsilon_2}{x_2} \frac{dA}{dt} \right),$$

where $r_2$ is resistance of secondary magnetizing winding; $\eta$ is variable. When diodes are opened $\eta = 1$; when diodes are closed $\eta = 0$.

The secondary winding voltage is equal capacitor voltage $u_{c2}$ when $\eta = 1$ or it is equal electromotive force when $\eta = 0$. On the time interval when diodes are closed variable $\eta$ equal zero. The begin of this regime correspond to moment of secondary current nihilation ($i_2 = 0$) and continue until next condition doesn’t executes itself

$$u_{c2} + \varepsilon_2 \left( \frac{dA}{dt} \right) \geq 0,$$

where $\varepsilon_2 = \pm 1$. Positive sign of $\varepsilon$-coefficient correspond to positive semi wave of previous current

$$\varepsilon_2 = \begin{cases} 1, & \text{if } i(t-\Delta t) \geq 0; \\ -1, & \text{if } i(t-\Delta t) < 0. \end{cases}$$

According to positive signs of currents and voltages ordinary differential equation of capacitor assumes form

$$\frac{du_{c2}}{dt} = \left( \text{mod}(i) - i_{in} \right) \frac{1}{C_2},$$

where: $C_2$ is capacity of secondary condenser; $i_{in}$ is load current.

According to the Kirchhoff’s law we receive the differential equation of inductance coil

$$\frac{di_{wr}}{dt} = \frac{(u_{c2} - R_{wr} i_{wr})}{L_{wr}},$$

where $R_{wr}$ is resistance; $L_{wr} = L_{wr}(i_2)$ is differential inductance.

Simulation results

The analysis of transient processes of single-phase converter is interconnected with integration of differential equations (4), (6), (8) of transformer, first condenser (17), secondary capacitor (22) and inductance coil (23). Taking into consideration the heavy regime of transformer work in real conditions we must control it’s thermal regime. The control is very important for the normal work of device in whole.

The results of calculation of transient processes of single-phase converter in the regime of sudden switching in given DC voltage source $u(t) = 220$ V are presented. The regime is calculated under the following parameters: $C_1 = 0.002$ F; $C_2 = 0.0003$ F; $R_b = 1$ Ohm; $R_{wr} = 10$ Ohm; $L_{wr} = 0.06$ H. The magnetisation ferromagnetic curve is

$$v = \begin{cases} 274.3, & B \leq 0.7; \\ -528.854 + 2807B^2 + 1012B^4, & 0.7 < B < 1.38; \\ 19000 - 24320/B, & B \geq 1.38. \end{cases}$$
The spatial distribution in fixed time of temperature and magnetic induction on cross-section of two winding iron-clad transformer body quarter is shown on fig.5 and fig.6.

The mathematical model of single-phase convertor is realized on algorithmic language "Visual FORTRAN". The spatial discretization of differential equations is realized by finite difference method; the time discretization of differential equations is realized according to implicit Crank-Nicolson method. The problem of solution of non linear algebraic equation system was realized by top relaxation method [5].

We created mathematical models of many convert devices: bridge rectifiers, inverters, frequency dividers and etc.

Fig. 6. The spatial distribution of magnetic induction in fixed time of transient process (fig. 2.)

REFERENCES


