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Image methamorphosis based on universal morphological interpolator

Abstract. *The paper presents the method for interpolating digital images that automatically generates a sequence of images showing the methamorphosis of the content of the first image into the content of the second. Morphological interpolation performs this transformation by the modification of shapes of objects present in both images. The method is based on universal morphological interpolator, which – thanks to its parameters – allows obtaining various results of transformation. The method makes use of interpolation function that is computed based on geodesic distance functions. The method, originally applied only to interpolate binary images, may also be used to morph graytone and (in some extent) color images, what is also presented in the paper.*

Streszczenie. *W artykule przedstawiono metodę interpolacji obrazów cyfrowych polegającą na automatycznym generowaniu sekwencji obrazów przedstawiających przemianę zawartości pierwszego obrazu w zawartość drugiego. Interpolacja morfologiczna realizuje tę przemianę poprzez modyfikację kształtów obiektów znajdujących się na obrazie. Metoda jest oparta o uniwersalny interpolator morfologiczny, który – dzięki możliwości doboru pewnych jego parametrów – pozwala na uzyskiwanie zróżnicowanych wyników przekształcania. W metodzie jest wykorzystywana funkcja interpolująca wyznaczana na podstawie geodezyjnych funkcji odległościowych. Metoda oryginalnie wykorzystywana jedynie do interpolacji obrazów binarnych może być także skutecznie stosowana do przekształcania obrazów szarościowych i (z pewnymi ograniczeniami) kolorowych z paletą kolorów, co również jest przedstawione w artykule.**(Przekształcanie obrazów z wykorzystaniem uniwersalengo interpolatora morfologicznego)***

Keywords: morphological image processing, image methamorphosis, morphing
Słowa kluczowe: morfologiczne przetwarzanie obrazów, przekształcanie obrazów, morfing

Introduction

The paper presents the generic framework for automatic metamorphosis of images using the universal morphological interpolator. It allows transforming one binary image into another by using the interpolation function. Image metamorphosis consists in obtaining a sequence of intermediary objects, whose shape is transformed from an initial object into the final one (morphing sequence). The proposed approach is based on interpolation functions which are computed on the basis of geodesic distance functions. In the paper a unified formula of morphological interpolation is used – the *universal morphological interpolator*. It allows interpolating between two objects which can be either nested, intersected or disjoint. The area where the interpolation in performed is defined by the separate image called a *mask*. Several possible choices of a mask are described in the paper: union of input objects, its dilation, closing and convex hull. Due to its flexibility the method allows for the construction of new interpolators for 2D and 3D images according to the particular requirements.

An approach to interpolation of graytone and indexed color images is also presented in the paper. Such interpolation is possible thanks to the umbra transform which converts n-dimensional multivalued image into $(n+1)$-dimensional binary one. Similiarily to the binary case, in this one, the proposed interpolation method results in the metamorphosis of images by modification of shapes of objects located within images. In the case of color images, visually correct results can be obtained only for particular kinds of color table associated with input images.

Related works

The morphological interpolation methods that allow creating intermediary images between two given ones have been developed since 1994. Two approaches to this task have been investigated. The first one (which the method proposed in this paper is based on) consists in applying the interpolation function [23]. The method can be applied to both binary and mosaic images containing objects with non-empty intersection. The continuation of research [16] allowed combining the morphological interpolation with an affine transform, which made the method applicable also to disjoint objects. Another approaches to disjoint objects was based on geodesic mask defined by polynomial curves [15] and by a convex hull [10]. The problem of homotopy preservation in the morphological interpolation was treated in [2].

The second morphological approach, originally described in [20, 21], is based on the morphological median operator and can be applied to any kind of image: binary, mosaic and graytone. In [9, 18] the area of applications of this method was extended into color images by using a lexicographic ordering of color vectors in the comparative color space. A 3D binary interpolation using morphological median was proposed in [13]. Morphological interpolation combined with surface reconstruction was also applied to create a 3D representation of tomographic data [5].

Binary image interpolation

Interpolation between nested objects

The *morphological interpolator* has a form of a function which returns an interpolated binary image at a given level. It has four principal arguments: two input images (initial and final), the mask and the interpolation level α. The latter is a real number such that $0 \leq \alpha \leq 1$, which indicates how far from both input objects the interpolated object is located. The interpolator is denoted as: $Int^R_{P\to Q}(\alpha)$, where P represents an initial object and Q - the final object. R is a mask defining an area where the interpolation is performed. It is an object in which both input as well as every interpolated object are included. The shapes of interpolated objects are transformed from the shape of the input object P to the final object Q. For $\alpha = 0$, the interpolated object is equal to the initial object ($Int^R_{P\to Q}(0) = P$); for $\alpha = 1$, to the final one ($Int^R_{P\to Q}(1) = Q$). The interpolation sequence (or morphing sequence) consists of interpolated images produced for increasing values of α. To obtain a sequence consisting of given number of n frames, consecutive values of α equal $\alpha_i = \frac{i}{n-1}$ for $i = 0, ..., n-1$.

Let's assume that two input binary images X and Y contain a single connected component (object) each. Moreover, let X and Y be the nested objects: $X \subset Y$. The transformation of shape of X into Y is performed inside their difference $Y \setminus X$. The position of a pixel within this area can be described by two geodesic distances [17, 6]: to the boundary of X and to the boundary of Y. Consequently, two distance

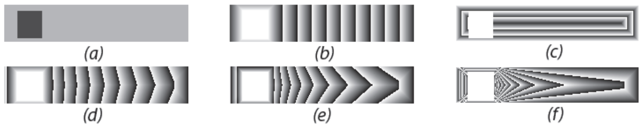

Fig. 1. Nested objects $X \subset Y$ (a), distance $D_Y(X)$ (b), distance $D_{X^c}(Y^c)$ (c), interpolation functions for various values of k: $k = 0.33$ (d), $k = 0.66$ (d), $k = 1$ (e).

functions can be computed in $Y \setminus X$. The first one, $D_Y(X)$, assigns to each pixel in $Y \setminus X$ its distance to the boundary of X. The second, $D_{X^c}(Y^c)$ describes a geodesic distance to the boundary of Y (see Fig. 1(b) and (c)). Considered together, they describe the relative position of each point in $Y \setminus X$. In order to combine both distances in a flexible way, the *universal interpolation function* is defined as:

(1)
$$I_Y(X) = \frac{D_Y(X)}{k(D_Y(X) + D_{X^c}(Y^c)) + (1-k)\max(D_Y(X))}.$$

Additions and multiplications of distance functions are performed pixel-wise. The parameter $k \in [0, 1]$ controls in (1) the form of the interpolation function. For $k = 1$ the interpolation function can be called the relative distance function since it describes the relative position of pixels in $Y \setminus X$ - such a form of the interpolation function has been proposed in [23].

Examples of geodesic distance functions and interpolation functions for various values of k are shown in Fig. 1. The geodesic distance functions were computed by 8-connected propagation (max-norm distance).

The interpolator based on the interpolation function defined by the Eq. 1 is the following:

(2) $$Int^{Y}_{X\to Y}(\alpha) = T_{[\alpha]}(I_Y(X)),$$

where $T_{[\alpha]}(f) = \{p : f(p) > \alpha\}$ represents thresholding of the interpolation function.

The parameter k can be set-up manually. However, an automated choice is also possible. For example one can assume $k_{X,Y} = \frac{\max(D_{X^c}(Y^c))}{\max(D_Y(X))}$. In this case k depends on the shapes of both objects. If Y is elongated comparing to X, the maximal value of $D_Y(X)$ is higher than the maximal value of $D_{X^c}(Y^c)$. In such a case the parameter k is closer to 0 and the universal distance depends less on $D_{X^c}(Y^c)$. Otherwise, if the shape is more compact, then the parameter k becomes closer to 1. Such a choice of k results in a constant growth of the interpolated object. The parameter k can be also set-up locally, depending on the position of a pixel within the image.

Case of any two objects

Let P and Q be the initial and final objects respectively. They can now be either nested, intersected or disjoint. The interpolation, in this case, is performed within the area defined by an auxiliary object - *a mask* R. The mask should meet two requirements: it must be a single connected component and both P and Q must be subsets of this mask: $P \cup Q \subseteq R$.

The result of the interpolation at a given level α is obtained as the intersection of two results of interpolations between each of two input objects and a mask, first at level α and second at level $(1-\alpha)$:

(3) $$Int^{R}_{P\to Q}(\alpha) = Int^{R}_{P\to R}(\alpha) \cap Int^{R}_{Q\to R}(1-\alpha),$$

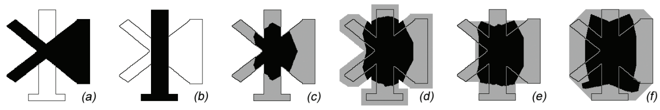

Fig. 2. Two test images (a),(b); interpolation with various masks: union (c), dilation (d), closing(e) and convex-hull(f). Masks in gray, interpolation result for $k = 0.7$ and $\alpha = 0.5$ in black, outline represents the union of input object.

where R stands for the mask, $Int^{R}_{P\to R}(\alpha)$ and $Int^{R}_{Q\to R}(1-\alpha)$ are the interpolated objects between P and R and between Q and R respectively. Eq. 3 describes the *universal morphological interpolator* for any two input objects P and Q inside a mask R.

The simplest mask is defined as the union of the input objects [23]. This solution, $R = P \cup Q$, can be applied only in the case of two input sets with a non-empty intersection ($P \cap Q \neq \emptyset$). This mask cannot be applied to the interpolation of disjoint objects.

A more flexible solution which considers also disjoint objects, must define a larger area where the interpolation is performed. In fact, any object which includes both input ones can be considered as a mask. Due to this fact, an application of any extensive morphological operator to the union of both object can create correct mask. It can be thus created e.g. by using morphological dilation or closing of the union:

(4) $$R_1 = \delta_B(P \cup Q)\ ;\ R_2 = \phi_B(P \cup Q),$$

where δ_B and ϕ_B stand for dilation and closing with the structuring element B, respectively.

In case of disjoint input objects ($P \cap Q = \emptyset$) there exists one important condition which must be fulfilled by the morphological operator applied to create a mask, or – more precisely – by the structuring element B used by this operator. Separated objects which are being dilated or closed, for certain structuring elements will become connected. Owing to this fact, the dilated or closed union can be used also in the case on empty intersection of input objects but under the condition that size and shape of the structuring element must guarantee that the resulting mask will be a single object.

The mask can also be computed as the *convex hull* (CH) of input objects, i.e. the smallest single convex object containing both input objects [1]: $R = CH(P \cup Q)$.

Examples of various masks in 2D case are presented in Fig. 2. Convex-hull (Fig. 2(f)) was created using morphological thickening in 8-connected grid.

Another possible solution, which could be applied in case of empty intersection of input objects, is usage of object *matching* before starting the interpolations. This approach applies the affine transform to match both objects before the interpolation functions are computed. The parameters of the affine transform can be computed on the basis of the shape and position of objects in various ways. In the simple approach only translation and scaling is considered [13]. More sophisticated methods computes also rotation angles using the covariance matrix [12].

Extension to graytone and indexed color images

Graytone case

The method can also be applied to interpolate between graytone images. In this case graytone images must be transformed into binary. The transformation that al-

[1] A 2D convex hull algorithm is described e.g. in [6] and for 3D objects a convex hull computation is presented in [24].

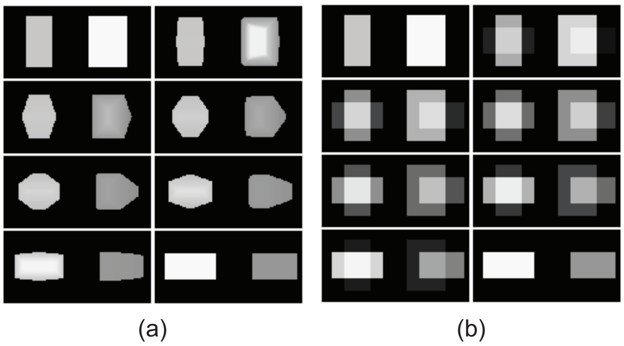

Fig. 3. Comparison of results of morphological interpolation (a) and cross-dissolving (b) of test images.

lows converting n-dimensional graytone image into $(n+1)$-dimensional binary one is called the *umbra* transform [25, 6].

Let $f : \mathcal{D} \to \mathcal{V}$ be the graytone image where: $\mathcal{D}$ is the set of pixels coordinates - image definition domain and $\mathcal{V}$ is set of possible graytones. The umbra transformation is defined as follows:

(5) $$U[f] = \{(p, q) \in \mathcal{D} \times \mathcal{V} : f(p) \geq q\}.$$

Thanks to this transform scalar grayvalues are transformed into new dimension of the image. An important advantage of this approach is the fact that graytone images can be processed using the binary morphological transformations. In order to interpolate between two graytone images, the input ones must be converted into their umbras. The latter are interpolated as described previously, and finally the results of interpolation which are also umbras are converted back to graytone images. Considering the fact that the intersection of umbras refer to pixel-wise minimum respectively, the interpolated image g at level α can be described using the following equation:

(6)
$$g_\alpha = U^{-1}\left[Int^{U[m]}_{U[f_a]\to U[m]}(\alpha)\right] \wedge U^{-1}\left[Int^{U[m]}_{U[f_b]\to U[m]}(1-\alpha)\right],$$

where g_α stands for the output graytone image at level α; U^{-1} for the inverse umbra transform i.e. transformation of $(n+1)$-dimensional umbra into n-dimensional graytone image; f_a, f_b, m are two input images and mask image respectively. Similiarily to the binary case, the mask image m can be computed as e.g. point-wise maximum of input images $m = f_a \vee f_b$ (which refers to the union of respective umbras), as its dilation $m = \delta(f_a \vee f_b)$ or closing $m = \phi(f_a \vee f_b)$. Also graytone convex-hull [6] of the union can be applied.

An example of graytone morphing is shown in Fig. 3. The morphological approach is comapred there with cross-dissolving. Contrary to the latter, which produces a point-wise graytone mixing, the morphological method generates both the transformation of the shape of objects and the tranformation of the graytone within the objects.

Fig. 4 shows umbras of input images, their union, intersection and interpolation result at level $\alpha = 0.5$.

Indexed color image interpolation

Similar approach can be used to generate morphing between two color indexed images. Input images which must be scalar ones, are in this a case images containing indeces of colors in color map. Such an approach has however some restrictions as far as the input images are concerned. First, the method requires both input images to have the same color tables. The second (and more important) refers to the

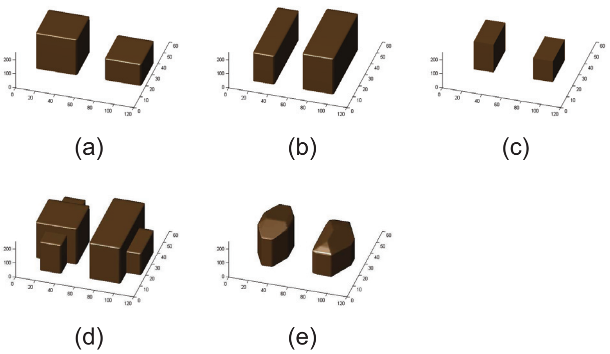

Fig. 4. Umbras of test images (shown in Fig. 3): (a),(b) input images, (c) - their intersection, (d) - union, (e) interpolation result for $\alpha = 0.5$.

Fig. 5. Graytone morphing sequence.

color map itself. The colors stored there must be ordered on the basis of human perception of colors. This ordering should guarantee that colors of consecutive indeces are visually close one to another. Owing to that, when increasing the pixel value during the interpolation process, the resulting color change would look naturally. The above considerations restrict, as far as color images are concerned, the applicability of the method only to images with images either with few different colors or with color table consisting of ordered colors which differs by just one component (no matter the color space used) with other two components similar one to another. Interpolation between color images whith color tables which does not follow above requirements is also possible, but results might be far from satisfactory.

Results

The first example shows the application of the proposed method to 2D graytone morphing. In order to produce the morphing sequence, at first, two input images (shown in Fig. 5(a) and Fig. 5(l), size 256x256 pixels) was transformed into their 3D binary umbras according to the Eq. 5. The union of both umbras (equal to the umbra of point-wise maximum of input graytone images) was used as a mask, while parameter $k = 1$. The resulting morphing sequence of $n = 12$ frames is shown in Fig. 5.

The second example presented in Fig. 6 shows the result of interpolation of color images of size 256x256 pixels. Both images (”candle” and ”sunset”) have the same color map, which is shown on the right-hand side of the figure. The map is visually ordered, i.e. such that when observing colors of increasing indeces within the map one can see smooth

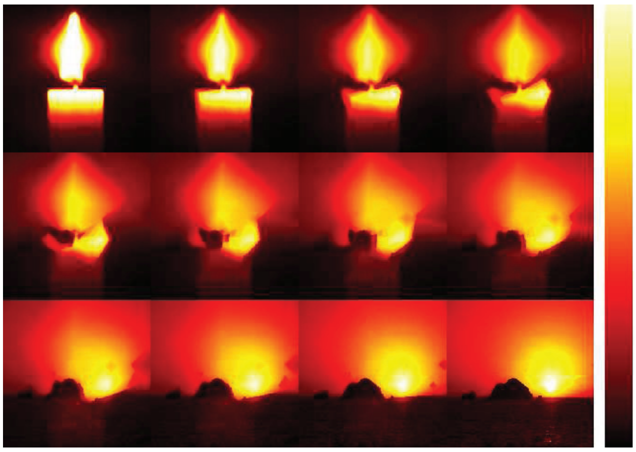

Fig. 6. Morphing of color indexed images with visually ordered color map (show on the right).

transformation of colors (this color map is sometimes referred to as "black body" color map).

Conclusions

In this paper an universal formula of interpolation-function-driven binary image metamorphosis was presented. The proposed formula defines the interpolation which is performed inside a given mask restricting a space where the interpolated objects are located. An additional parameter k controls the shape of the interpolation function on which the interpolation is based. The method can be applied to produce a metamorphosis of both 2D and 3D binary images. It can also create 3D binary object from 2D slices. Moreover, using the umbra transformation, it can be applied to 2D gray-tone images and to indexed color images with visually ordered color tables. The method can be used in various image processing application areas. For example, it can be used to reconstruct 3D volumes from 2D slices in medical imaging (e.g. tomographic data), geology etc. In computer graphics it may be used as an alternative to simple cross-dissolving to produce the transformation of one image into another one in fully automatic manner.

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