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Accurate Performance Analysis of DS/SS Systems in Single-tone Interference over Flat Fading Channels

Abstract*. The performance analysis of DS/SS system in single-tone interference is usually presented for AWGN channel. While, there usually exists fading over the actual wireless channels. Using the well-known MGF-based analysis approach, the exact symbol error rate of DS/SS system in single-tone interference over flat fading channel is derived in this paper, and the application conditions of two familiar performance analysis methods —standard Gaussian approximation and improved Gaussian approximation for DS/SS system in single-tone interference are discussed.*

Streszczenie. *W artykule analizuje się wskaźnik błędu systemu telekomunikacyjnego DS/SS w kanale z zanikami przy jednotonowej interferencji. Do analizy wykorzystuje się metodę MGF.* (***Analiza dokładności systemu telekomunikacyjnego DS./SS z interferencją jednotonową w kanale z zanikami***)

Keywords: single-tone interference; direct sequence spread spectrum; fading channels; Gaussian distribution.
Słowa kluczowe: system telekomunikacyjny DS/SS (direct sequence spectrum spread), metoda MGF (moment generating function)

Introduction

It is well known that the direct sequence spectrum spread (DS/SS) system has been widely applied in the military and civil communications as its excellent anti-jamming ability, in which the signals transmitted are not only interfered by the channel noise, but also by some intentional interferences. For example, the single-tone interference is a familiar interference. Up to now, there are lots of works about the DS/SS system in single-tone interference in recent years [1-7]. Note that most of the results are on the additive white Gaussian noise (AWGN) channels, however there usually exists fading over the actual wireless channels, which make the existing results not be used directly in practice. Moreover, for the DS/SS system in single-tone interference, the traditional performance analysis method is the standard Gaussian approximation (named SGA) based on the central limit theorem [8]. While, there are some works which regard SGA not to produce accurate error probabilities, and propose some improved Gaussian approximation methods (named IGA) [5-7]. In fact, whether SGA or IGA is derived based on certain system model, and has its own application condition. But to the best of our knowledge, no paper has presented the clear conclusion about the application conditions of SGA and IGA up to now. Aiming at the fading channel, the performance of DS/SS system in single-tone interference is investigated in this paper, and the application conditions of SGA and IGA are discussed.

The remainder of this paper is organized as follows. In Section II, we describe the system model that is under consideration. Using the moment generating function (MGF) based analysis approach [9], the expression of the exact symbol error rate (SER) of DS/SS system in single-tone interference over flat fading channel is derived in Section III. In Section IV, we present some simulation and numerical results. Finally, the conclusions are drawn in Section V.

System Model

The DS/SS system model with binary phase-shift-keying (BPSK) modulation [5] over flat fading channel is shown in Fig. 1. The data signal transmitted by the user is

(1) $$d(t)=\sum_{n=-\infty}^{\infty} d_n P_{T_b}(t-nT_b)$$

where: d_n – data bit, T_b – bit width, only when $t \in [0, T_b]$, $P_{T_b}(t)=1$, and other is always zero.

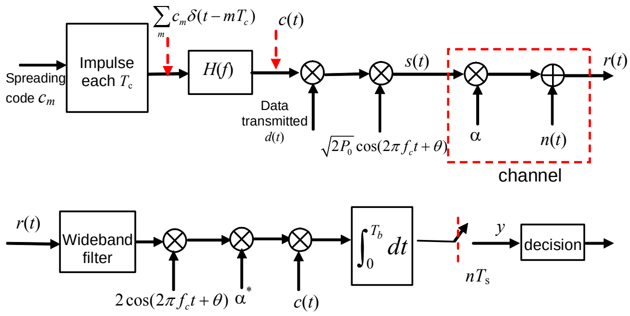

Fig.1. A DS/SS system model with BPSK modulation

For convenience, let the period of the spreading sequence c_m be infinite, and both d_n and c_m be regarded as sequences of independent and identically distributed (i.i.d.) random variables taking values on {–1, +1} with equal probability. In fact, the period of c_m is usually finite in practical system. While it maybe very large in some system, such as GPS system, where the c_m can be regarded as ideal random sequence as above. Though in some system, it is not very large and the nature of the c_m is not i.i.d., which makes the performance analysis be troublesome, but it can be obtained by simply modifying the analytical result for the ideal random sequence. Suppose the spreading factor be given by $N = T_b/T_c$. The impulse modulator generates an impulse each T_c seconds. The impulse are shaped by a base band filter with the transfer function $H(f)$, and then multiplied by $\sqrt{2P_0}\cos(2\pi f_c t+\theta)$ to form the RF signal $s(t)$

(2) $$s(t)=\sqrt{2P_0}\sum_{m=-\infty}^{\infty} c_m d_{\lfloor m/N\rfloor} h(t-mT_c)\cos(2\pi f_c t+\theta)$$
$$=\sqrt{2P_0}\,c(t)d(t)\cos(2\pi f_c t+\theta)$$

where: P_0 – average transmit power of the transmitter, $\lfloor x\rfloor$ – integer portion of x, f_c – carrier frequency, θ – carrier initial phase, $c(t)=\sum_{m=-\infty}^{\infty} c_m h(t-mT_c)$. The inverse Fourier transform of $H(f)$ is a real chip waveform $h(t)$. To normalize the chip waveform, we assume the energy constraint $\int_{-\infty}^{\infty}|h(t)|^2\,\mathrm{d}t=\int_{-\infty}^{\infty}|H(f)|^2\,\mathrm{d}f=T_c$. For convenience, we assume $h(t)$ is a rectangular chip waveform over the interval [0, T_c] in this paper, so $|H(f)|=T_c \sin c(fT_c)$.

Let the jammer be close to the receiver, and the total received signal can be given by

(3) $$r(t) = \alpha s(t) + j(t) + n(t)$$

where: α – channel fading coefficient, $n(t)$ – channel noise, regarded as white Gaussian one with double-sided power spectrum density $N_0/2$, $j(t)$ – single-tone interference

(4) $$j(t) = \sqrt{2P_j}\cos(2\pi f_j t + \varphi)$$

where: P_j – average signal power, f_j –carrier frequency, φ – phase of the interference signal at the receiver. Let φ be a random variable that is uniformly distributed on [0, 2π]. The frequency f_j is assumed to be close enough to the carrier frequency f_c that the tone can pass through the wideband filter without distortion, so here $f_d = |f_c - f_j| << |f_c + f_j|$.

In this paper, we consider the flat slow fading channel, so that the channel coefficients are constant over a frame and vary independently from frame to frame. The channel coefficient α can be modeled as a complex Gaussian random variable with mean μ_α, and variance $0.5\sigma_\alpha^2$ per dimension. The probability density function (PDF) of α [9] is

(5) $$f(\alpha) = \frac{1}{\pi\sigma_\alpha^2}\exp\left\{-\frac{|\alpha - \mu_\alpha|^2}{\sigma_\alpha^2}\right\}$$

The channel fading amplitude R follows the distribution

(6) $$f(R) = \frac{2R}{\sigma_\alpha^2}\exp\left\{-\frac{R^2}{\sigma_\alpha^2} - \kappa\right\} I_0\left(2R\sqrt{\frac{\kappa}{\sigma_\alpha^2}}\right), \qquad R \ge 0$$

where: $\kappa = |\mu_\alpha|2 / \sigma_\alpha^2$ – ratio of the power of direct line-of-sight (LOS) component to the average power of multipath fading components. If κ is small, the average power of multipath fading components is large relative to the power of LOS component, which means the channel fading is heavy. When $\kappa = 0$, the channel fading follows the Rayleigh distribution. When $\kappa \neq 0$, the channel fading follows the Rician distribution, and κ is called Rician factor.

Performance Analysis

Let the synchronization and channel estimate of the system be perfect, and the perfect channel state information can be obtained at the receiver. According to [5, 7], the signal $r(t)$ received is dealt with according to the system in Fig. 1, and the final decision statistic $y = \sum_{m=0}^{N-1} y_m$ can be yielded, where

(7) $$\begin{aligned} y_m &= |\alpha|^2\sqrt{2P_0}T_c d_{\lfloor n/N \rfloor} + \alpha^* J(mT_c)c_m + \alpha^*\eta(mT_c)c_m \\ &= |\alpha|^2 S + \alpha^* J_m + \alpha^*\eta_m \end{aligned}$$

where: "*" – conjugate operation, $J(mT_c)$, $\eta(mT_c)$ – sampled interference and noise at the output of the matched filter, and they are mutually independent. The noise is independent from one sample to another, with the same variance

(8) $$\sigma_\eta^2 = N_0\int_{-\infty}^{\infty}|H(f)|^2 df = N_0 T_c$$

The interference signal J_m can be expressed as

(9) $$J_m = \sqrt{2P_j}T_c \operatorname{sinc}(f_d T_c)\cos(2\pi f_d m T_c + \varphi - \theta + \pi f_d T_c)c_m$$

where: c_m – mth dispreading code during the current symbol, and is supposed as the ideal random sequence taking values on {–1, +1} with equal probability.

When the system synchronization has been established perfectly, the carrier phase θ can be obtained. For convenience, here we assume $\theta = 0$. From (9), the interference signal J_m is regarded as a random variable with joint distribution of the dispreading code c_m and carrier phase φ of the interference signal. Since c_m is ideal random sequence, when φ is fixed, the samples in interference $J = \sum_{m=0}^{N-1} J_m$ are mutually independent, and the variance of J is given by

(10) $$\begin{aligned} \Psi &= Var(J) = \sum_{m=0}^{N-1} Var(J_m) \\ &= P_j T_c^2 \sin c^2(f_d T_c)\left[N + \sum_{m=0}^{N-1}\cos(4\pi m f_d T_c + 2\varphi + 2\pi f_d T_c)\right] \end{aligned}$$

In what follows, we will discuss according to the random phase φ which is unknown at the receiver.

CASE I: The random phase φ of the interference signal varies randomly during one jamming process. Without loss of generality, we suppose φ be constant over a chip, and vary independently from chip to chip with uniform distribution over [0, 2π]. In fact, here φ can be regarded as a random process with uniform distribution. Since φ varies randomly from chip to chip, so Ψ for one symbol at the receiver also varies randomly. Based on the central limit theorem, when N >> 1, the distribution of interference J tends to be Gaussian, and the mean of J is given by

(11) $$Mean(\Psi) = P_j T_c T_b \sin c^2(f_d T_c)$$

When the channel coefficient α is given, the instantaneous receive signal-to-noise ratio (SNR) for one symbol can be obtained as

(12) $$\gamma_1 = \frac{E_b|\alpha|^2}{N_0 + P_j T_c \sin c^2(f_d T_c)} = \frac{E_b|\alpha|^2}{N_0 + N_{j1}} = \bar{\gamma}_1|\alpha|^2$$

where: $E_b = P_0 T_b$ – bit energy at the transmitter, $N_{j1} = P_j T_c sinc^2(f_d T_c) = P_j T_c b_{tone1}$, regarded as the noise density introduced by the single-tone interference at the receiver, $b_{tone1} = sinc^2(f_d T_c)$ is defined as the interference factor. $\bar{\gamma}_1 = \frac{E_b}{N_0 + P_j T_c b_{tone1}} = \left(\frac{1}{E_b/N_0} + \frac{b_{tone1}}{SIR \cdot N}\right)^{-1}$, and $SIR = P_0/P_J$ is defined as the signal-to-interference ratio.

Since the channel coefficient α follows the distribution as (5), so the MGF of γ_1 can be obtained as [9]

(13) $$M_{\gamma_1}(s) = \frac{1+\kappa}{1+\kappa+\bar{\gamma}_1 s}\exp\left(\frac{\kappa s\bar{\gamma}_1}{1+\kappa+\bar{\gamma}_1 s}\right)$$

Using the unified MGF-based approach for the performance evaluation of digital modulations over fading channels, we can obtain the average SER of the DS/SS system, as

(14) $$P_{e1} = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_1}\left(\frac{1}{\sin^2\vartheta}\right)d\vartheta$$

If the channel coefficient α is not considered, (12) is the same as the results based on SGA in reference. This means SGA is fit for the case that the random phase of the interference is a random process with uniform distribution. So, we call the method using (12) ~ (14) to evaluate the SER of DS/SS system in single-tone interference as SGA over fading channel.

CASE II: The random phase φ of the interference signal is constant during one jamming process, and varies independently from process to process with uniform distribution over [0, 2π]. In fact, here φ can be regarded as a random variable with uniform distribution and this instance is usually encountered in some practical systems. Since φ for one symbol is constant, so Ψ for one symbol at the receiver is also fixed value.

Using the equation [7]

(15) $$\sum_{m=0}^{n-1}\cos(a + mb) = \cos\left(a + \frac{n-1}{2}b\right)\frac{\sin(nb/2)}{\sin(b/2)}$$

Equation (10) can be rewritten as

(16) $$\Psi = P_j T_c T_b \sin c^2(f_d T_c)\left[1+\frac{\sin c(2 f_d T_b)}{\sin c(2 f_d T_c)}\cos 2\phi\right]$$

where: $\phi=\varphi+\pi f_d T_c$. Since φ is a random variable with uniform distribution over [0, 2π], it is reasonable to model ϕ as a random variable that is uniformly distributed over [0, 2π]. Based on the central limit theorem, when $N >> 1$, the condition distribution of interference J tends to be Gaussian for fixed ϕ (or φ). When the channel coefficient α and random phase φ of the interference signal are given, the instantaneous receive SNR for one symbol can be obtained as

(17) $$\gamma_2 = \frac{E_b|\alpha|^2}{N_0 + N_{j2}} = \bar{\gamma}_2|\alpha|^2$$

where: $N_{j2}=P_j T_c b_{tone2}$ is regarded as the noise density introduced by the single-tone interference at the receiver, $\bar{\gamma}_2 = \left(\frac{1}{E_b/N_0}+\frac{b_{tone2}}{SIR\cdot N}\right)^{-1}$, the interference factor $b_{tone2} = \sin c^2(f_d T_c)\left[1+\frac{\sin c(2 f_d T_b)}{\sin c(2 f_d T_c)}\cos 2\phi\right]$ is a function of the random phase ϕ.

As **CASE I**, the MGF of γ_2 can be obtained as

(18) $$M_{\gamma_2}(s) = \frac{1+\kappa}{1+\kappa+\bar{\gamma}_2 s}\exp\left(\frac{\kappa s\bar{\gamma}_2}{1+\kappa+\bar{\gamma}_2 s}\right)$$

For the given ϕ, the condition SER of DS/SS system can be written as

(19) $$P_{e2}(\phi) = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_2}\left(\frac{1}{\sin^2\vartheta}\right)d\vartheta$$

To obtain the average SER, it is necessary to evaluate the error probability with respect to ϕ, as follows [5, 7]

(20) $$P_{e2} = \frac{2}{\pi}\int_0^{\pi/2} P_{e2}(\phi)d\phi$$

If the channel coefficient α is not considered, (17) is the same as the result based on IGA in reference. This means IGA is fit for the case that the random phase of the interference is a random variable with uniform distribution. So, we call the method using (17) ~ (20) to evaluate the SER of DS/SS system in single-tone interference as IGA over fading channel.

Through the SER in (14) and (20) are both given in the form of integration, they can be evaluated with simple numerical integration techniques in common mathematical software such as Matlab. Moreover, comparing the above SGA with IGA, it can be found that the main different is the interference factor in γ_1 and γ_2. When there is no interference, the SER obtained by (14) and (20) (i.e. SGA and IGA) are the same.

Numerical and Simulation Results

In this section, we provide some numerical results of our analysis and compare them with simulation results in order to verify the analysis. Suppose the spreading code c_m used in this paper is m sequence with 25 order, and $N = 128$, the length of the frame equals to 50 bits.

For the DS/SS system over fading channel, the average SNR per receive symbol is

(21) $$SNR = \frac{E_b}{N_0}E\left[|\alpha|^2\right]$$

which is relative to the channel coefficient α. For convenience, the channel coefficients used in simulation are normalized so that $E[|\alpha|^2]=1$. Hence, the average SNR per receive symbol is given by

(22) $SNR=E_b/N_0$

Fig.2 shows the SER of the DS/SS system without interference over different fading channels. From these plots, we can see that our analysis agree exactly with the simulation results always over the same fading channel.

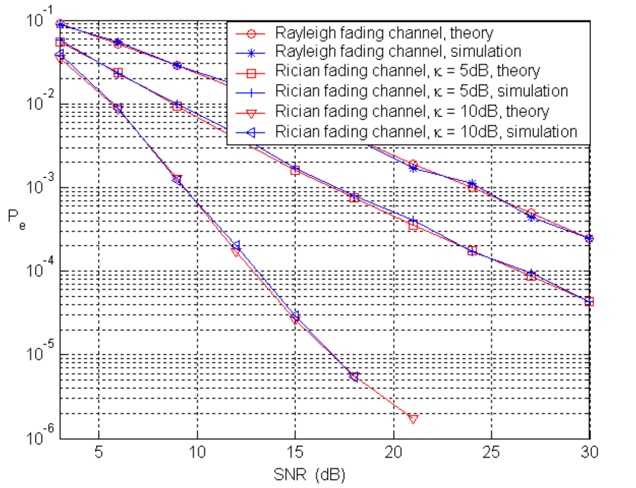

Fig.2. SER of the DS/SS system without interference over different fading channels

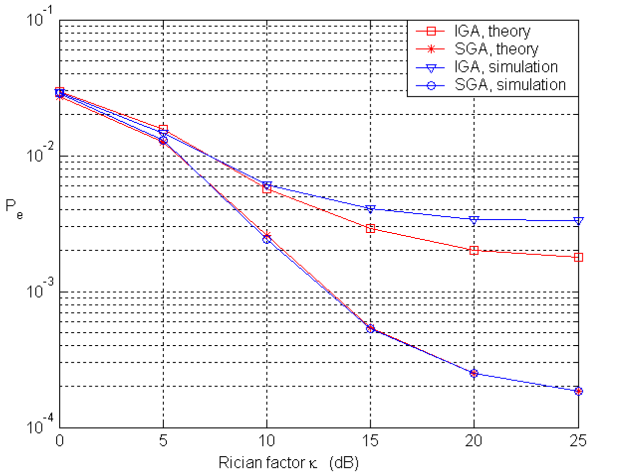

Fig.3. SER of the DS/SS system without channel noise and with single-tone interference, $f_d = 0$, SIR = -13dB

Fig. 3 shows the SER of the DS/SS system without channel noise and only with single-tone interference, $f_d = 0$, SIR = -13dB, over different fading channels. From these plots, we can see that our analysis agree exactly with the simulation results always for SGA; while for IGA, when the Rician factor κ is small (i.e. the channel fading is heavy), our analysis agree exactly with the simulation results, and when the Rician factor κ is large (i.e. the channel fading is light), our analysis underestimates the SER slightly. The main reason is that when the channel fading is very light, there exists relativity among the effect of interference J on different chips which results in the distribution of interference J not to be Gaussian fully when N>>1. As the above analysis, the random phase of the interference has different manner to take values for SGA and IGA. When the Rician factor κ is small, the performances for two cases are close, and there are very small differences for the SER obtained by SGA and IGA. However, when the Rician factor κ is large, the performance for two cases has large difference, so there are big differences for the SER obtained by SGA and IGA. Usually, the random phase of the interference takes values according to **CASE II** in practical system. If SGA is used as the conventional, the performance will be undervalued greatly when the channel fading is not too heavy. Especially, when the Rician factor κ is very large, the channel has almost no fading, and tends to the AWGN. From this point of views, we can say that our results agree with that in reference which is not accurate using SGA. The performance is also undervalued using IGA when the Rician factor κ is very large, but there is a great improvement comparing with that using SGA. So, it is seen that SGA is not inaccurate in fact, but it has its own application condition. Moreover, the above results can be used in the design of the jamming system, too. To obtain the effective jamming, the random phase should be taken values according to **CASE II**, i.e. it is a slow changing process. If it changes rapidly, the effective jamming may be not obtained, and the implement complexity may be increased, too. Fig. 4 shows the SER of the DS/SS system with channel noise and single-tone interference, f_d = 0, SIR =-10dB, over different fading channels. From these plots, the same conclusion as above can also be obtained.

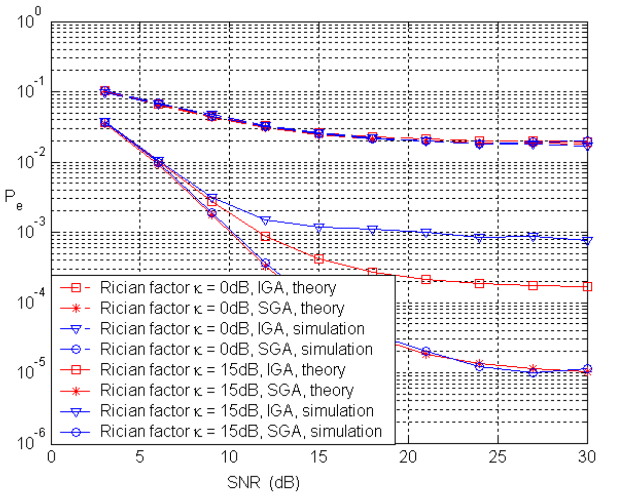

Fig.4. Performance of the DS/SS system with channel noise and single-tone interference, f_d = 0, SIR = -10dB

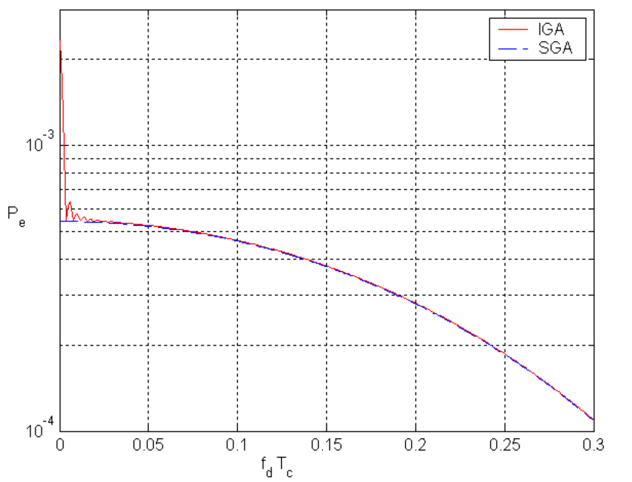

Fig. 5. The relation between the SER and f_d for the DS/SS system without channel noise and with single-tone interference, SIR = -13dB over fading channel with the Rician factor κ = 15dB

For the DS/SS system without channel noise and with single-tone interference, SIR = -13dB over fading channel with the Rician factor κ = 15dB, the relation between the SER and f_d is given in Fig. 5, where f_d is normalized by the chip interval T_c. From these plots, it is seen that for DS/SS system in single-tone interference over the same fading channel, the SERs obtained using SGA and IGA are the same almost except for the region with very small f_d . This means the performance of DS/SS system in single-tone interference obtained using SGA as the conventional is accurate mostly except the very small f_d. So taking into account the above results together, we can find that the SER obtained by SGA is accurate mostly for the DS/SS system in single-tone interference whose phase is a random variable over fading channel (except that the channel fading is light, for example the Rician channel, and f_d is very small simultaneously), which is different to that over AWGN channel.

Conclusion

Usually, the performance analysis of DS/SS system in single-tone interference is on the AWGN channel using the SGA method. While, the actual wireless channels are usually the fading ones, and there are some works which regard SGA not to produce accurate error probabilities and present some methods based on IGA. Using the well-known MGF-based analysis approach, the SER of DS/SS system in single-tone interference over flat fading channel is derived in this paper, and the application conditions of SGA and IGA are discussed. The analysis results in this paper can not only present the direction in theory for performance evaluation of anti-jamming communication system, but also direct the design of the jamming system.

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