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Robust H_∞ state feedback control for singular systems: A parameter-dependent approach

***Abstract**. The robust H_∞ state feedback control problem for both continuous- and discrete-time singular systems with polytopic-type uncertainties is revisited via a parameter-dependent approach. Attention is focused on the design of a parameter-dependent state feedback controller, such that the closed-loop system is admissible with prescribed H_∞ noise attenuation level for all parameter uncertainties. Without using decomposition technique to the singular model, sufficient condition for the existence of an H_∞ state feedback controller is expressed in terms of strict linear matrix inequalities (LMIs). In case that the LMI conditions are feasible, a suitable state feedback control law is explicitly given. The proposed approach is expected to be less conservative compared with previous results. Numerical examples are also provided to show the effectiveness of the approach.*

***Streszczenie.** Analizowany jest odporny system sterowania ze sprzężeniem zwrotnym H_∞ dla przypadku systemu dyskretnego i ciągłego i z niepewnościami typu polytopic. Do analizy wykorzystuje się metodę zależności parametrycznych. Sterownik opisany jest liniową macierzą nierówności LMI. (**Odporne sterowanie ze sprzężeniem zwrotnym H_∞ - metoda zależności parametrycznych**)*

Keywords: state feedback, singular system, parameter dependent, robust control, linear matrix inequality.
Słowa kluczowe: sterowanie, macierz nierówności, sprzężenie zwrotne.

Introduction

Analysis and design of singular systems (also referred to as descriptor systems, generalised state-space systems or differential-algebraic systems) have received great attention in the last decades. This is because, for many practical systems such as robotics, power systems, networks, economical systems, and highly interconnected large-scale systems, singular model is a natural mathematical representation and provides a description of algebraic constraints between physical variables (see, e.g. [1]-[4] and the references therein).

In the last years, many fundamental system theories developed for standard state space system have been successfully extended to their counterparts for singular system, including the analysis and design of robust control [5]-[12] and filtering systems [13]-[16]. More precisely, necessary and sufficient conditions for H_∞ control of singular systems with or without uncertainties have been derived in [7] and [6], respectively. In the discrete-time setting, the sufficient and necessary condition for the H_∞ control problem has been presented by Xu and Yang [8]. For nonlinear singular systems, [10] and [12] have considered both the state feedback and output feedback control for continuous and discrete-time case, respectively; a necessary conditions for the output feedback control problem to be solvable are obtained in terms of two Hamilton-Jacobi inequalities plus a weak coupling condition. Recently, [13] presented the robust H_∞ sate feedback control for uncertain discrete singular systems in terms of strict LMIs. Very recently, the reduced-order H_∞ and L_2 - L_∞ filtering are addressed for singular systems in [16]-[18]; the necessary and sufficient conditions for the solvability of this problem in terms of LMIs and a coupling non-convex rank constraint are obtained, and an explicit parameterisation of all desired reduced-order filters is presented. The monograph [1] gives an excellent overview of the robust control and filtering for singular systems. In those results, only norm-bounded parameter uncertainties are considered. Although norm-bounded parameter uncertainties are important to consider, most uncertain systems models are much better described by polytopic structures (see e.g., [19]). Indeed, polytopic structures arise naturally when there are multiple real-valued uncertain parameters. Using norm-bounded structures typically over-estimates the uncertainties in the system [20].

On the other hand, for standard state space systems, many efforts have been made in the direction of reducing the conservativeness of the analysis and design methods for improving the systems performance (see e.g. [21]-[26] and the references therein). In order to reduce the conservatism of traditional Lyapunov function methods, [21]-[23] proposed a new approach known as the parameter-dependent Lyapunov method, to the study of robust stability of systems with parametric uncertainties. Similar ideas have been subsequently developed to investigate the stability analysis, control and filtering synthesis problems in a few contexts (see, [24]-[29], and the references therein). To further reduce the conservativeness, in [30], a structured polynominal parameter-dependent approach is proposed for robust H_2 filtering of linear uncertain systems. For uncertain singular systems, the problem of robust filtering is discussed in [20] in the minimum-variance (or H_2) setting, using LMI method and singular value decomposition. In recent years, uncertain singular systems with time-delay obtained much attention and a great number of fundamental results on controller or filter design have been reported. For example, Ma and Liu *et al.* have discussed robust stochastic stability and stabilization of time-delay discrete Markovian jump singular systems with norm-bounded parameter uncertainties [31]. Zhu and Zhang *et al.* have investigated the delay-dependent robust stability criteria for two classes of singular time-delay systems with norm-bounded uncertainties without using model transformation and bounding technique for cross terms [32]. Very recently, the problem of delay-dependent robust stabilization for norm-bounded uncertain singular systems has been investigated based on the free-weighting-matrix approach and LMIs in [33]. The works mentioned are concerned with the norm-bounded uncertainties, and only delay-dependent results are derived. To the best of our knowledge, however, for the parameter-dependent robust H_∞ state feedback controller design for singular systems with polytopic uncertainties, there is no result in the literature so far, which still remains open and challenging.

This paper is devoted to studying the robust H_∞ state feedback control problem for singular systems with polytopic-type uncertainties, namely the matrices in the system model are uncertain and assumed to belong to a given polytopic set. Attention is focused on the design of a parameter-dependent state feedback control law, such that the resulting closed-loop system is admissible while the norm of close-loop transfer function is optimised or lower than a prescribed H∞ norm. Both continuous- and discrete-time cases are considered in this paper. The remainder of this paper is organised as follows. In Section 2 the problem under consideration is stated. The discrete- and continuous-time cases are considered in Section 3 and Section 4, respectively. Two practical examples are given in Section 5, by which followed Section 6 concluding the paper.

Problem formulation

Consider the following singular system:

(1a) $$E\delta\left[x(t)\right] = Ax(t) + B_1u(t) + B\omega(t)$$

(1b) $$z(t) = Cx(t) + Du(t)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $z(t) \in \Re^q$ are the state vector, control input, and the controlled output, respectively; $\omega(t) \in \Re^p$ is the disturbance input which belongs to $L_2[0,\infty)$. $\delta\left[\cdot\right]$ stands for the shift operator for discrete-time systems and the derivate operator for continuous-time systems. The matrix $E \in \Re^{n\times n}$ may be singular with $\mathrm{rank}E = r \le n$. *A*, B_1, *B*, *C,* and *D* are appropriately dimensioned matrices. We first introduce the following definitions for the continuous-time (Definition 1) and discrete-time (Definition 2) system, respectively.

Definition 1 [1-3]:

(i) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero.

(ii) The pair (E, A) is said to be impulse-free if $\deg(\det(sE - A)) = \mathrm{rank}(E)$.

(iii) The pair (E, A) is said to be stable if all the roots of $\det(sE - A) = 0$ have negative real parts.

(iv) The pair (E, A) is said to be admissible if it is regular, impulse-free and stable.

Definition 2 [1-3]:

(i) The pair (E, A) is said to be regular if $\det(zE - A)$ is not identically zero.

(ii) The pair (E, A) is said to be casual if $\deg(\det(zE - A)) = \mathrm{rank}(E)$.

(iii) The pair (E, A) is said to be stable if all the roots of $\rho(E, A) < 1$.

(iv) The pair (E, A) is said to be admissible if it is regular, casual and stable.

The operators $\det(\cdot)$, $\deg(\cdot)$, and $\rho(\cdot,\cdot)$ stands for the determinant of a matrix, degree of a polynomial, and the generalized spectral radius of the matrices pair, respectively.

In this paper, we assume that the matrices *A*, B_1, *B*, *C,* and *D*, which have partially unknown parameters, belong to the following uncertain polytope

(2) $$\Omega = \{\left(A, B_1, B, C, D\right) | = \sum_{i=1}^{N} \tau_i (A^{(i)}, B_1^{(i)}, B^{(i)}, C^{(i)}, D^{(i)}), \tau_i \ge 0, \sum_{i=1}^{N} \tau_i = 1\}$$

In this paper, we consider the following linear state feedback controller

(3) $$u(t) = Kx(t)$$

where the sate feedback gain $K \in \Re^{m\times n}$ is a constant matrix to be determined. Applying the controller (3) to (1) results in the following closed-loop system

(4a) $$E\delta\left[x(t)\right] = A_c x(t) + B\omega(t)$$

(4b) $$z(t) = C_c x(t)$$

where $A_c = A + B_1K$, $C_c = C + DK$

The robust H∞ state feedback control problem we address in this paper is to obtain the state feedback gain *K* in (3) such that the closed-loop system (4) is regular, causal (for discrete-time systems) or impulse-free (for continuous-time systems), and stable for zero initial condition of *x*(*t*) and $\omega(t) = 0$, while the transfer function of the closed-loop system satisfies a prescribed H∞ performance level for all admissible parameter uncertainties in (2). It is worth mentioning that the robust H∞ state feedback control problem for uncertain systems has been addressed before, (see e.g. [7], [13]), but the results therein are parameter-independent and the uncertainties therein are norm-bounded, which will result in performance conservatism. This paper is contributed to design the parameter-dependent robust H∞ state feedback controller for polytopic uncertain singular systems using a strict LMI technique.

Discrete-time case

In this section, we shall concentrate our attention on the robust H∞ state feedback controller design for discrete-time singular systems. First, a strict LMI condition concerning the H∞ performance for singular systems is given in the following lemma [1].

Lemma 1: Given a scalar $\gamma > 0$, the closed-loop system (4) is admissible with guaranteed H∞ noise attenuation level γ if and only if there exist matrices $P > 0$ and *Q* such that the following LMI holds:

(5) $$\begin{bmatrix} A_c^T P A_c + C_c^T C_c - E^T P E & A_c^T P B_c \\ * & B_c^T P B_c - \gamma^2 I \end{bmatrix} + \begin{bmatrix} A_c^T \\ B_c^T \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} A_c^T \\ B_c^T \end{bmatrix}^T < 0$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$, asterisks (*) stand for entries that are easily inferred from symmetry.

Note that

$$\left\|\Omega\right\|_\infty = \left\|\Omega^T\right\|_\infty$$

where Ω denotes the transfer function of the system $\{E, A_c, B_c, C_c\}$, while Ω^T the transfer function of the dual system $\{E^T, A_c^T, C_c^T, B_c^T\}$. Then, by Lemma 1, it is not difficult to obtain the following corollary.

Corollary 1: Given a scalar $\gamma > 0$, the closed-loop system (4) is admissible with guaranteed H∞ noise attenuation level γ if and only if there exist matrices $P > 0$ and *Q* such that

(6) $$\begin{bmatrix} A_c P A_c^T + B B^T - E P E^T & A_c P C_c^T \\ * & C_c P C_c^T - \gamma^2 I \end{bmatrix} + \begin{bmatrix} A_c \\ C_c \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} A_c \\ C_c \end{bmatrix}^T < 0$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $ES = 0$.

Based on the results in Corollary 1, we give the following parameter-dependent strict LMI condition for the closed-loop system (4).

Theorem 1: Given a scalar $\gamma > 0$, the system (4) is admissible with guaranteed H∞ noise attenuation level γ if and only if there exist matrices $\left(P, Q_1, Q_2, F, G, J\right)$ with $P > 0$ satisfying

(7)
$$\begin{bmatrix} -G-G^T+P & 0 & 0 & GA_c^T-F^T+SQ_1^T & GC_c^T+SQ_2^T \\ * & -J-J^T & 0 & 0 & JC_c^T \\ * & * & -I & B^T & 0 \\ * & * & * & -EPE^T+A_cF^T+FA_c^T & FC_c^T \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

where Q_1, and Q_2 are partition of Q in (6), i.e. $Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$ with $Q_1 \in \Re^{n\times(n-r)}$, and $Q_2 \in \Re^{p\times(n-r)}$.

Proof: *(Necessity)* Suppose that there exist matrices $\left(P, Q_1, Q_2, F, G, J\right)$ with $P > 0$ such that (7) holds. Then, pre- and post- multiplying (7) by
$$\begin{bmatrix} A_c & 0 & 0 & I & 0 \\ C_c & C_c & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 \end{bmatrix}$$
and its transpose, we obtain (6) immediately.

(Sufficiency) Suppose that there exist matrices $P > 0$ and Q satisfying (6). Using Schur complement lemma [19], we have

(8)
$$\begin{bmatrix} -P & 0 & PA_c^T & PC_c^T+SQ_2^T \\ * & -I & B^T & 0 \\ * & * & \Theta_{33} & Q_1S^TC_c^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

where $\Theta_{33} = -EPE^T + A_cSQ_1^T + Q_1S^TA_c^T$. Then, (7) holds by choosing
$$G = P,\ F = A_cP - A_cG + Q_1S^T,\ J = I/2. \qquad \square$$

Remark 1: In the case when $E = I$, it is to see that $S = 0$; then, Corollary 1 reduces to the bounded real lemma (BRL) for standard state space systems. Accordingly, Theorem 1 reduces to the parameter-dependent bounded real lemma same as in [29], which has been shown, both theoretically and through numerical examples, to be less conservative than the results using a common Lyapunov matrix for the entire uncertainty. Therefore, Theorem 1 can be viewed as an extension of parameter-dependent BRL for standard state space systems to singular systems.

Remark 2: Comparing to the result in (6), the slack variables $\left(F, G, J\right)$ in (7) provides free dimensions in the solution space for the robust H∞ control problems and the matrices $\left(P, Q_1, Q_2\right)$ in (7) is allowed to be dependent on the uncertain domain. Moreover, note that the (1,1) block of (7) implies that $-G - G^T + P < 0$, from which we can easily to conclude that $G > 0$ and, in turn, G is nonsingular.

Now, we are in the position to design the robust H∞ state feedback controller using a parameter-dependent approach. Note that directly applying Theorem 1 would lead to bilinear matrix inequalities (BMIs), which are difficult to be solved numerically. However, it turns out that, applying appropriate congruence transformations, these BMIs can be transformed into LMIs. The next theorem gives a solution for the robust H∞ state feedback control problem using convexity arguments.

Theorem 2: Given a scalar $\gamma > 0$, there exists a state feedback controller (3) such that the closed-loop system (4) is regular, casual and stable with disturbance attenuation level γ for all uncertainties in (2), if there exists a solution $\left(P^{(i)}, Q_1^{(i)}, Q_2^{(i)}, F^{(i)}, J^{(i)}, M^{(i)}, N^{(i)}, G, \bar{K}\right)$ such that the following LMI holds:

(9)
$$\begin{bmatrix} -G-G^T+P^{(i)} & 0 & 0 & \Theta_{11}^{(i)} & \bar{K}D^{(i)T}+GC^{(i)T}+SQ_2^{(i)T} \\ * & -J^{(i)}-J^{(i)T} & 0 & 0 & N^{(i)}D^{(i)T}+J^{(i)}C^{(i)T} \\ * & * & -I & B^{(i)T} & 0 \\ * & * & * & \Theta_{12}^{(i)} & M^{(i)}D^{(i)T}+F^{(i)}C^{(i)T} \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

for $i = 1, \cdots, N$, where $\Theta_{11}^{(i)} = \bar{K}B_1^{(i)T} + GA^{(i)T} - F^{(i)T} + SQ_1^{(i)T}$,
$\Theta_{12}^{(i)} = -EP^{(i)}E^T + A^{(i)}F^{(i)T} + B_1^{(i)}M^{(i)T} + F^{(i)}A^{(i)T} + M^{(i)}B_1^{(i)T}$. Then a suitable state feedback control law can be determined as

(10) $$u(t) = \left(G^{-1}\bar{K}\right)^T x(t)$$

Proof: Define the (new) linearisation transformations

(11) $$GK^T = \bar{K},\ F^{(i)}K^T = M^{(i)},\ J^{(i)}K^T = N^{(i)}$$

Then, the LMI in (9) follows immediately by substituting (4c) into (7) and taking the uncertainty set (2) into consideration. This completes the proof. □

Remark 3: In Theorem 2, not only $\left(P^{(i)}, Q_1^{(i)}, Q_2^{(i)}\right)$ but the general slack matrices $\left(F^{(i)}, J^{(i)}, M^{(i)}, N^{(i)}\right)$ are allowed to be dependent on the uncertain parameter. This is different from the results entails fixed matrices for the entire uncertainty domain, or the ones that need the slack variables to be fixed. It is worth mentioning that the uncertain-domain-dependent slack matrices provide further reduction of conservativeness, as depicted in Section 5.

In the case $E = I$, the singular system (1) reduces to a standard state space system, we have the following parameter-dependent result on robust H∞ state feedback control.

Corollary 2: Considering the discrete-time state space system described by (1) when $E = I$. There exists a control law in (3) to solve the robust H∞ state feedback control problem, if a solution $\left(P^{(i)}, F^{(i)}, M^{(i)}, N^{(i)}, G, \bar{K}\right)$ such that the following LMI holds:

(12)
$$\begin{bmatrix} -G-G^T+P^{(i)} & 0 & \Theta_{11}^{(i)} & \bar{K}D^{(i)T}+GC^{(i)T} \\ * & -I & B^{(i)T} & 0 \\ * & * & \Theta_{12}^{(i)} & M^{(i)}D^{(i)T}+F^{(i)}C^{(i)T} \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

for $i = 1, \cdots, N$, where $\Theta_{11}^{(i)} = \bar{K}B_1^{(i)T} + GA^{(i)T} - F^{(i)T}$,
$$\Theta_{12}^{(i)} = -P^{(i)} + A^{(i)}F^{(i)T} + B_1^{(i)}M^{(i)T} + F^{(i)}A^{(i)T} + M^{(i)}B_1^{(i)T}$$
. Then a suitable state feedback control law can be determined as in (10).

Remark 4: Design of optimal robust H∞ state feedback controller requires the solution of the following minimisation

(13) $$\min_{P^{(i)}, Q_1^{(i)}, Q_2^{(i)}, F^{(i)}, J^{(i)}, M^{(i)}, N^{(i)}, G, \bar{K}} \gamma$$

subject to the LMI constraint in (9).

Continuous-time case

In this section, we consider the robust H∞ state feedback controller design for continuous-time singular systems. The results of Theorem 1 and Theorem 2 can be easily extended to the continuous-time case.

Theorem 3: The closed-loop system (4) is admissible with guaranteed H∞ noise attenuation level γ if and only if there exists a solution (P,Q,F,G) with $P>0$ satisfying

$$(14)\quad \begin{bmatrix} -G-G^T & PE+SQ+GA_c^T-F & GC_c^T & 0 \\ * & A_cF+F^TA_c^T & F^TC_c^T & B \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

Then, by following similar procedures as the discrete-time case, we can obtain the following solution for the robust H∞ state feedback controller problem for continuous-time singular systems.

Theorem 4: Given a scalar $\gamma>0$, there exists a state feedback controller (3) such that the closed-loop system (4) is regular, impulse-free and stable with disturbance attenuation level γ for all uncertainties in (2), if a solution $\left(P^{(i)},Q^{(i)},F^{(i)},M^{(i)},G,\bar{K}\right)$ satisfies the following LMI:

$$(15)\quad \begin{bmatrix} -G-G^T & \Phi_{11}^{(i)} & \bar{K}D^{(i)T}+GC^{(i)T} & 0 \\ * & \Phi_{12}^{(i)} & M^{(i)}D^{(i)T}+F^{(i)}C^{(i)T} & B^{(i)} \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

for $i=1,\cdots,N$, where $\Phi_{11}^{(i)}=P^{(i)}E+SQ^{(i)}+\bar{K}B_1^{(i)T}+GA^{(i)T}-F^{(i)}$, $\Phi_{12}^{(i)}=A^{(i)}F^{(i)}+B_1^{(i)}M^{(i)}+F^{(i)T}A^{(i)T}+M^{(i)T}B_1^{(i)T}$. If the LMI (15) are solvable, a suitable state feedback control law can be determined as (10).

Remark 5: Similar to Theorem 2, the uncertain-domain-dependent slack matrices $\left(F^{(i)},M^{(i)}\right)$ provide further reduction of the conservativeness of the solutions. Design of optimal robust H∞ state feedback controller requires the solution of the minimisation problem in (13) with appropriate decision variables subject to the LMI constraint in (15).

Similarly, in the case $E=I$, we have the following parameter-dependent result on design of robust H∞ state feedback controller for continuous-time state space systems.

Corollary 3: Considering the continuous-time state space system described by (1) when $E=I$. There exists a state feedback controller (3) to solve the robust H∞ state feedback control problem, if there exists a solution $\left(P^{(i)},F^{(i)},M^{(i)},G,\bar{K}\right)$ satisfies the following LMI:

$$(16)\quad \begin{bmatrix} -G-G^T & \Phi_{11}^{(i)} & \bar{K}D^{(i)T}+GC^{(i)T} & 0 \\ * & \Phi_{12}^{(i)} & M^{(i)}D^{(i)T}+F^{(i)}C^{(i)T} & B^{(i)} \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

for $i=1,\cdots,N$, where $\Phi_{11}^{(i)}=P^{(i)}+\bar{K}B_1^{(i)T}+GA^{(i)T}-F^{(i)}$, $\Phi_{12}^{(i)}=A^{(i)}F^{(i)}+B_1^{(i)}M^{(i)}+F^{(i)T}A^{(i)T}+M^{(i)T}B_1^{(i)T}$. If the LMI in (16) is feasible, then a desirable state feedback control law according to the solution can be constructed as in (10).

Illustrative examples

In this section, we give two examples to demonstrate the effectiveness of the proposed approach.

Example 1: Consider the electrical uncertain circuit system, which is adapted from [34], described by (1) with the following parameter:

$$E=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A=\begin{bmatrix} -2+0.1\alpha & -1 & 1 \\ 0 & -1+0.1\alpha & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$B_1=B=\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, C=\begin{bmatrix} 0 & 1+0.1\beta & 0 \end{bmatrix},\ D=0$$

where $|\alpha|\le 1$ and $|\beta|\le 1$. Our objective is to construct a state feedback controller in (3) such that the resulting close-loop system is admissible with H∞ noise attenuation level γ. Note that the system can be represented as a four-vertex polytopic system, and this system without parameter uncertainties is not impulse free. To this end, we choose $S=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. When setting $\gamma=0.3$ and using MATLAB LMI Control Toolbox to solve the LMI (15), we obtain the following solution

$$G=\begin{bmatrix} 2.1368 & -0.0332 & -0.3339 \\ -0.1387 & 0.0860 & 0.0555 \\ 0.8085 & 0.0263 & 2.7155 \end{bmatrix},\ \bar{K}=\begin{bmatrix} 1.1668 \\ -2.0628 \\ 0.2676 \end{bmatrix}$$

Then, by Theorem 4, a parameter-dependent robust H∞ state feedback control law can be obtained as

$$u(t)=\begin{bmatrix} 0.2173 & -23.8039 & 0.2640 \end{bmatrix} x(t).$$

Example 2: Consider the discrete-time model for a DC motor in a hydraulic system borrowed from [35]:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix}=\begin{bmatrix} 0.4121+\alpha & 0.8113+\alpha \\ -0.345+0.345\beta & -1 \end{bmatrix}.$$
$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u(t)+\begin{bmatrix} 1 \\ 1 \end{bmatrix}\omega(t)$$

where $x_1(t)$ is the axis speed, $x_2(t)$ is the armature current, and $\omega(t)$ is the disturbance with unknown statistics. The uncertain parameters, $|\alpha|\le 1$ and $|\beta|\le 1$, are resulted from the viscous-friction coefficient, torque constant, and the motor back-EMF constant may not be measured exactly. Here we suppose that the computational delay are relative small and can be neglected, and that the controlled output $y(t)=x_1(t)$.

Table 1 presents a comparison of the number of variables (complexity) and the obtained minimum feasible γ^* by using Theorem 2 according the following two cases:

(a) $\left(F^{(i)},J^{(i)},M^{(i)},N^{(i)}\right)$ parameter-independent with $\left(P^{(i)},Q_1^{(i)},Q_2^{(i)}\right)$ parameter-dependent;

(b) Both $\left(F^{(i)},J^{(i)},M^{(i)},N^{(i)}\right)$ and $\left(P^{(i)},Q_1^{(i)},Q_2^{(i)}\right)$ are parameter-dependent.

In order to show the less conservatism of our results, the parameter-independent robust H∞ state feedback control in [13] is provided in Table 1. From the table, it can be seen that the proposed approach achieves better H∞ performance bounds with the fixed $\left(F^{(i)},J^{(i)},M^{(i)},N^{(i)}\right)$ than the parameter-independent ones. The performance is further improved when both $\left(F^{(i)},J^{(i)},M^{(i)},N^{(i)}\right)$ and

$\left(P^{(i)}, Q_1^{(i)}, Q_2^{(i)}\right)$ are parameter-dependent. The optimal performance index achieved is $\gamma^* = 0.1007$ and the corresponding parameter and control law are

$$G = \begin{bmatrix} 0.0082 & -0.2968 \\ 0.2924 & 2.5250 \end{bmatrix}, \ \bar{K} = \begin{bmatrix} -0.0321 \\ 0.2486 \end{bmatrix}$$

$$u(t) = \begin{bmatrix} -0.0668 & 0.1062 \end{bmatrix} x(t) .$$

Besides, the number of decision variables, the computational burden (we run the algorithms on the same PC with 2.0G CPU, 1.0G RAM, and WINDOWS XP operation system) for the three scenarios are compared (see Table 1). Through comparison, we can see that although the number of decision variables increases as a tradeoff to the reduced H_∞ bound. However, the computational complexity does not increase seriously like the number of decision variables. This is because, when optimizing the parameters, the polytope-dependent searching algorithms (Theorem 1 in this paper) need less iterations than the parameter-independent searching algorithm (Theorem 3 in [13]).

Table 1. Comparison of close-loop performances

Method	Theorem 1 case (a)	Theorem 1 case (b)	Theorem 3 in [13]
Minimum γ	0.1592	0.1007	0.1875
No. of decision variables	18	30	7
Computational complexity (ms)	59.8	67.2	42.3

Conclusions

The parameter-dependent robust H_∞ state feedback control problem for singular systems with polytopic-type uncertainties has been addressed in this paper. Both continuous- and discrete-time cases have been considered. Sufficient condition for the existence of an H_∞ state feedback controller was expressed in terms of strict LMIs. When these inequalities were feasible, explicit parameterisation of a desired H_∞ state feedback control law has been presented. It was worth mentioning that the proposed approach was obtained without decomposition technique to the singular model and was less conservative.

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