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PD and LQR controllers applied to vibration damping of an active composite beam

***Abstract**. Presented results describe an application of MFC actuator for vibration reduction of a composite beam. Characteristics of the studied system are formed in a closed loop control. The paper describes two possible approaches. The first bases on a feedback control with a PD regulator. The second one operates with a state control with a gain tuned by LQR index. On the basis of the performed tests the LQR controller demonstrated better results. The PD controller was very sensitive to parameter changes, which led to an increase of oscillations, even up to saturation limits.*

***Streszczenie.** Przedstawione wyniki opisują zastosowanie aktuatora MFC do redukcji drgań belki. Charakterystyki tego układu kształtowane są w układzie regulacji zamkniętej. W artykule opisano dwa układy regulacji. Pierwszy oparty jest na strukturze z regulatorem PD. Drugi układ działa ze sprzężeniem od wektora stanu, a jego wzmocnienie strojone jest według funkcjonału LQR. Zdecydowanie lepsze wyniki uzyskano dla regulatora LQR. Regulator PD był bardzo czuły na zmiany parametrów układu, na które reagował wzrostem oscylacji aż do granicznych wartości nasycenia. (**Zastosowanie aktuatora MFC do redukcji drgań belki**)*

Keywords: MFC piezoelectric actuator, active vibration damping, PD and LQR controllers
Słowa kluczowe: piezoelektryczny aktuator MFC, aktywne tłumienie drgań, regulatory PD i LQR.

Introduction

Mechanical vibrations are a type of movement, which often coexist with a useful motion of technical devices. In most cases vibrations have negative influence on mechanical properties and are treated as a parasitic effect of the dynamic properties.

Vibrations reduce control accuracy and in particularly unfavourable conditions can cause divergent and resonance effects. Therefore methods for reducing vibrations are being sought strongly. In bibliography we can find many different examples which solve this problem. The most common one is an application of mechanical damper which we can find for example in vehicles. The other utilizes eddy currents which generate braking force proportional to the speed. Example of this phenomenon we can observe in analogue electricity meter.

To obtain better effectiveness and possibility to form a specific motion requirements, active damping systems are applied. Such an example represents hoisting drive system in deep mine shaft. A long steel ropes with lifted car act as a spring with very dangerous vibrations. To reduce these vibrations, electric motors with special control algorithms are applied. Similar methods we can use while working elements are connected by rotating long shaft. This system can be described as a multi-mass drive and we can also use active methods for the vibration reduction. In most of the mentioned cases the main electric motor plays a fundamental role in damping oscillations. Corresponding methods exist in many systems, even in wind driven turbine with electric generator and its drive train. Special generator procedures enable to reduce vibrations and receive better operational characteristics.

Unfortunately, turbine blades are under influence of nonlinear aerodynamic effects and vibrate in different directions. These movements can be reduced mainly by damping properties of the blades. Almost until the present days only passive methods have been applied, but current development of active composite structures has changed this situation. Among modern active sheathing components piezoelectric composites have characteristics which fulfil most requirements of aerodynamical blades. These composites are known as Macro Fiber Composites (MFC) which operate with large deflections and generate sufficiently big forces.

They are formed in a multilayer composite structure with a smooth surface. Under the influence of an electric charge, they change the shape. For these reasons they are applied in control and in vibration damping systems.

In this paper a short description of MFC piezoelectric actuator embedded to a cantilever beam is presented. The active MFC element is used to reduce beam vibrations. The effectiveness of this action depends on the type of the algorithm. In described tests PD and LQR control algorithms are applied. A comparison of received results shows benefits and risks of these algorithms.

Structures of Macro Fiber Composites and test bed scheme

Piezoelectric MFC are transducers, converting electric energy into mechanical energy or vice versa. This corresponds to the actuator and strain sensor's work.

The structure of MFC actuators usually refer to the matrix construction, which is closed by the outer epoxy plates (Fig.1). Polarizing comb-shaped electrodes located inside the MFC, produce an electric field which directly affects the generation of mechanical stress in the piezoelectric fibres. The use of the MFC as a sensor assures that the electric charge stored on the electrodes is proportional to the measured voltage and capacity between electrodes.

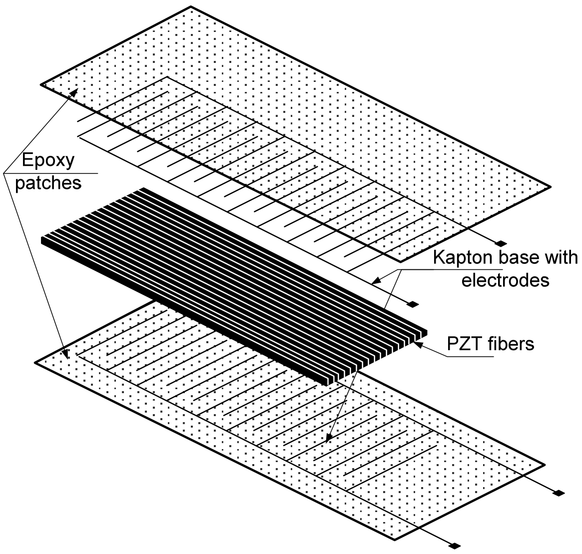

Fig.1 Matrix structure of the Macro Fiber Composite

In contrast to classical PZT elements, the MFC actuators have much bigger distortions, which are measured in millimetres rather than micrometers [2].
In terms of electrical properties MFC have capacitive nature. This nature influences current rate of rise [3].

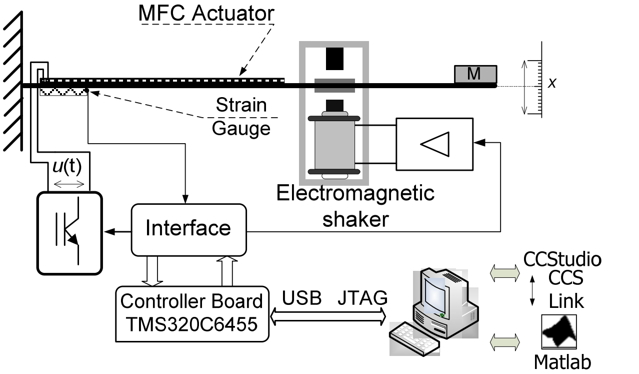

Fig.2 Test bed of active MFC cantilever beam controlled by computer aided DSP controller.

The heart of the control system is a DSP controller. Both the program of direct control and the program to perform test cycle are written in their ROM memory. Moreover, in LQR parameter derivations, the DSP is supported by Matlab software in PC.

Model of the tested beam

In a cantilever beam, during its vibration, the bending plays the most important role. If vibration amplitude is large, then nonlinear terms have to be taken into account in the beam model. The equation of motion have been derived by the Hamilton's principle of least action [7]. The partial differential equations of the beam model are reduced to ordinary differential equations by the Bubnov-Galerkin method. Taking into account only the first vibration mode, the model of the vibrating beam takes the form

$$\ddot{x} + 2\mu\omega\dot{x} + \omega^2 x + \beta x^3 - \delta\left(x\dot{x}^2 + x^2\ddot{x}\right) = a(t) \tag{1}$$
$$a(t) = a_m \sin(\Omega t)$$

where: x – beam deflection, ω – natural frequency of cantilever beam, μ – viscous damping coefficient, β, δ – geometrical and inertia nonlinearity coefficients, a_m – amplitude of external excitation force, Ω - frequency of the external force,

Application of PD controller

In a control process the critical parameters for control algorithm selection are the natural frequency of cantilever beam (ω) and the viscous damping coefficient (μ). Nonlinear factors β and δ cause a shift of the natural frequency and have influence on the force generated in the beam structure. At small deflections, components standing at these factors have negligibly small influence on the motion of the cantilever beam. Therefore, at the beginning stage of tests, the idea of a linear model, omitting nonlinear components, seemed to be natural choice of study. The linear model corresponds to the Laplace transmittance determined for *x(t)* output deflection and $a(t)$ input external force.

$$\mathrm{G}_{beam}(s) = \frac{X(s)}{A(s)} = \frac{1}{s^2 + 2\mu\omega s + \omega^2} \tag{2}$$

where: *X(s)* and *A(s)* are transforms of *x(t) and* $a(t)$.

For a control system with a PD controller the control scheme has a feedback loop with *x(t)* signal. An input variable equals to zero, which corresponds to an ideal damping demand. The control scheme corresponding to these requirements is shown in Fig.3.

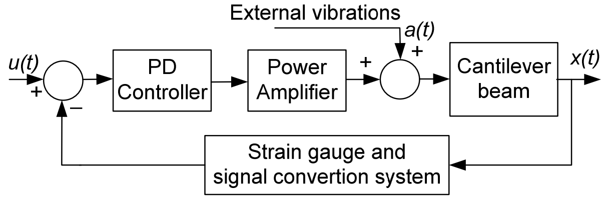

Fig.3 Feedback control with PD controller

In simulation tests, the PD algorithm changes dynamical properties of a vibrating beam. It can move the eigenvalues even to real values. Received simulation results were very promising. An exemplary result presents Fig.4.

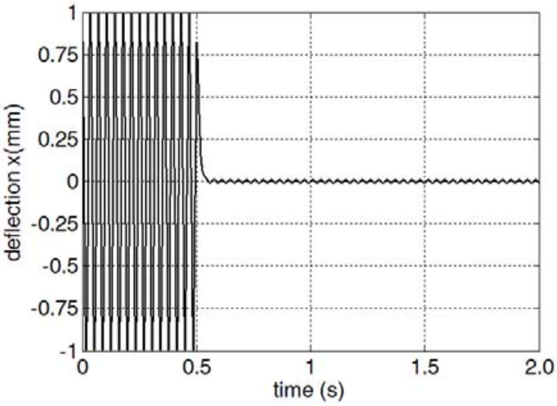

Fig.4. Simulation results for PD controller made for simplified second degree model without any parameter deviations and any time delays in measuring and conversion systems

Unfortunately, similar results were very difficult to obtain in the real system. Although sometimes results were satisfactory, in general this kind of regulator caused some dangerous excitations. Instead of reduction, beam vibrations increased up to the established mechanical limits. At the same time, the power amplifier also boosted voltage supply up to its saturation limits. This situation was very dangerous for the MFC actuator. Increased thermal losses were generated which could damage structure and led to short circuits between the MFC electrodes. Moreover, unexpectedly every time, after such excitation, the tested cantilever beam with MFC actuator changed its parameters, so the control system was completely out of tune and not fitted for use for a certain time.

Continued tests have shown that the time delay of the feedback signal was mainly responsible for these dangerous results. This hypothesis was checked for closed loop transmittance with a delay in the feedback signal (3).

$$\mathrm{G}(s) = \frac{1}{\mathrm{G}_{\mathrm{beam}}^{-1}(s) + \mathrm{G}_{\mathrm{c}}(s) \cdot K_{\mathrm{A}} \cdot K_{\mathrm{s}} e^{-s\tau}} \tag{3}$$

where: $\mathrm{G_C}(s)$ – controller transmittance, K_A - gain coefficient of a power amplifier, $K_\mathrm{S}e^{-s\tau}$ – sensor and signal conversion transmittance.

Obtained simulation tests confirmed this hypothesis. Similarly as in a real system, after reaching saturation the beam stopped in one of the extreme positions (Fig.5).

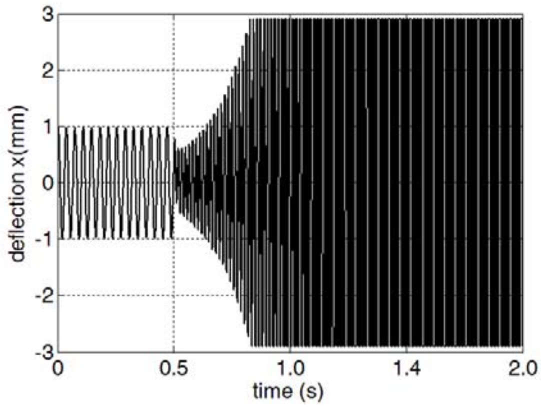

Fig.5 Loss of a control due to the feedback a signal delay

Due to the described problems, the tests with PD controller were abandoned. Next studies were performed with the LQR controller, which formed dynamical characteristics using multi dimension state feedback.

The LQR Algorithm

Choosing the type of regulator, we required the characteristic function with eigenvalues which guarantee stable operation in a closed loop state control system. Moreover the controlled system should be insensitive to small parameter changes. The choice fell on the LQR controller, because it uses state variables in a control loop and optimizes transient states with the help of the energy and cost performance index. Furthermore, for more demanding systems it enables the adaptation to the controller settings.

The used control scheme is based on differential state equation (Fig.6). Matrices A, B, C, and D represent respectively matrices: state, input, output and feed forward matrix.

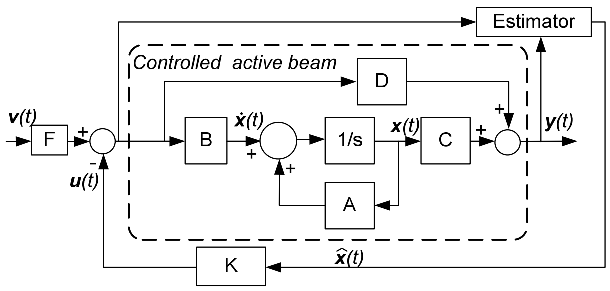

Fig.6 The state feedback control with the state estimator and the **K** gain matrix calculated from the Riccati equation.

The aim of the control is to determine control vector **u**(t) which is subjected to the LQR performance criteria and satisfies dynamical differential state equation. This control vector should minimize displacement of vibrating element.

The general form for the LQR index performance is:

$$\mathbf{J} = \frac{1}{2}\int_0^\infty \left(\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\right)\mathrm{d}t \tag{4}$$

where $\mathbf{Q}$ is a symmetric positive matrix which is the weighting matrix of the state variable $\mathbf{x}$, while $\mathbf{R}$ is control weighting matrix of input variable $\mathbf{u}$. The **J** quadratic index (4) is a compromise between quality of control and control costs. The control quality determines the first part of integrated expression. The second part of integrated expression minimizes energy consumption and costs [5,6,8].

In a feedback loop of the LQR controller, the state signal vector is necessary. If some signals are not measured and system is observable, the estimate of the remaining signals is required [5]. Such situation presents Fig.6, where a full order estimate gains $\mathbf{K}$ matrix. The optimal linear state control law is defined as

$$\mathbf{u}(t) = -\mathbf{K}\hat{\mathbf{x}}(t) \tag{5}$$

where feedback gain $\mathbf{K}$ is concluded from performance index (4) and defined by expression (6)

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{6}$$

The constant matrix $\mathbf{P}$ is delivered from Riccati equation (7).

$$0 = \mathbf{PA} + \mathbf{A}^{\mathrm{T}}\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} \tag{7}$$

The system tuned by the LQR method satisfies requirements for a steady state error, transient response, stability margins or closed loop pole location. These have positive influence on a disturbance rejection and vibration reduction. The gain and phase margins satisfy stability conditions [8].

As the performance index (4) fulfils conditions of the Riccati equation the weight of control matrix $\mathbf{Q}$ can be determined by output matrices $\mathbf{Q} = \mathbf{C}^*\mathbf{C}^{\mathbf{T}}$ and matrices $\mathbf{K}$ and $\mathbf{R}$ are found out in the Riccati equation procedure [6].

In consequence the results shown in Fig.7 characterize properties of the system.

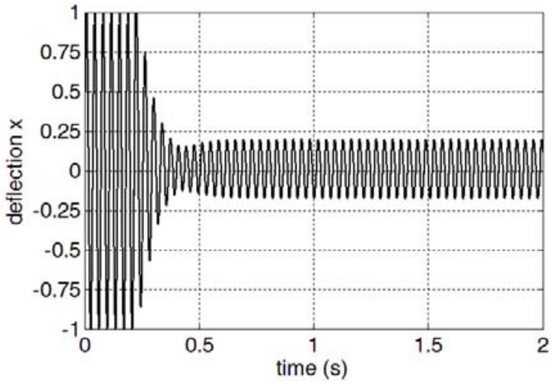

Fig.7. Vibration reduction with LQR state feedback control

The above test was repeated for the modified signal parameters, with a longer processing time. Differences in both obtained results were almost imperceptible (Fig.8).

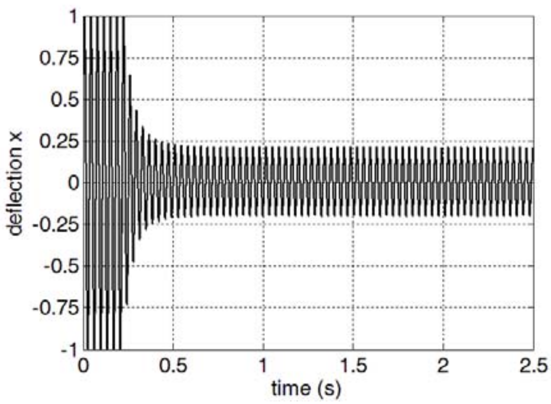

Fig.8. Vibration reduction with implemented LQR state feedback control and extended conversion time up to 5ms in feedback signals

Obtained results confirm that LQR control algorithm has good properties even in a case of parameter changes. The multi dimension feedback loop takes into account, the all state variables. Application of the LQR index performance enables the selection of the optimal solution within desired conditions.

In the tested system, we measured only deflection of the end of cantilever beam. We checked simple simulator and reduced order observer. The proposed approach guaranteed stable and satisfactory results.

Conclusions

Active composite structures create new application possibilities. Due to the flat shape and a very small thickness, they may be embedded inside multilayer composite structure. They may also be applied to control and to reduce vibration of the helicopters or wind turbine's blades.

In the paper, the piezoelectric Macro Fiber Composite (MFC) was studied. While this active element is in an inner layer of a multilayer composite, all the construction can be controlled and called as smart composite structure. High possible deflections during electrical charging, the multi-layered active composite can be applied to control and to reduce mechanical vibrations.

The main subject of the presented paper is to propose an efficient method for a cantilever beam vibrations control. Among many regulators in the current investigation the Proportional Differential (PD) feedback control and the Linear Quadratic Regulator (LQR) state control were studied.

According to the second order differential equation, the PD operation should move eigenvalues location into real values. Unfortunately the predicted results did not fulfil the expectations. A study of the real system showed that a use of this controller is potentially dangerous, both for the active MFC system and for the power supply electronic system. Existing additional time delays in a signal converter caused the conditions to generate vibrations that could damage main components. Therefore, this approach was excluded from the future study.

Following the need for an application of a regulator which can operate with a multi-loop feedback and allow to receive both high efficient with minimal value of input signal, the LQR regulator was selected.

The carried out studies and obtained results confirmed expectations. We received good damping properties, which in a certain range, were only slightly sensitive to the signal time delay.

Moreover, applied estimator of beam's velocity didn't make any problems in the control strategy. However, we found some limits in application of the LQR regulator. The most important restriction concerns the detuning problem. While the controlled system changes parameters in a wide range then the LQR regulator may not work properly. This problem can be overcome by taking into account adaptive control techniques.

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