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Numerical analysis of YBCO coated conductors

Abstract. *Second generation high-T_C superconducting wire has a layered structure. The ratio of thickness of superconducting layer to overall width of HTS tape can be as high as 1:10000. FEM computer simulation of such thin subdomains is difficult and very time-consuming. Creation of a finite element mesh of acceptable quality and execution of calculations in reasonable time are the main opposing problems. In this paper, authors present possibilities of overcoming the difficulty of unfavourable geometric ratio of thin HTS layers.*

Streszczenie. *Taśma nadprzewodnikowa drugiej generacji ma strukturę warstwową. Stosunek grubości do szerokości warstwy nadprzewodnika w takiej taśmie może osiągać wartość 1:10000. Symulacja numeryczna metodą elementów skończonych tak cienkich obszarów jest bardzo utrudniona. W artykule autorzy przedstawiają wybrane możliwości obejścia problemu niekorzystnej geometrii cienkiej warstwy nadprzewodnikowej. (**Analiza numeryczna cienkowarstwowych taśm nadprzewodnikowych**).*

Keywords: YBCO coated conductors, FEM computer simulation.
Słowa kluczowe: nadprzewodniki cienkowarstwowe, symulacja numeryczna metodą elementów skończonych.

Introduction

Second generation (2G) high-T_c superconducting (HTS) wire has a layered structure. In most cases, a thin coating of superconductor compound, usually $YBa_2Cu_3O_7$ (YBCO), is applied or grown on a flat metallic substrate by means of chemical vapour deposition (CVD) or pulsed laser deposition (PLD). The superconductor coating in this coated conductor (CC) wire typically has the thickness of the order of one micrometer (fig.1).

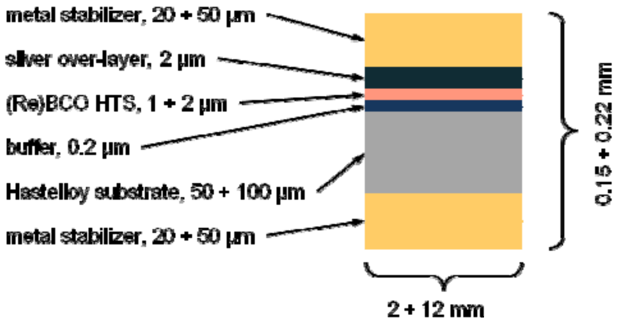

Fig.1. Cross-section of 2G HTS tape (anisotropic scale)

Numerical simulation appears as an effective and relatively inexpensive tool of designing of novel electromagnetic devices. Unfortunately, a finite element method (FEM) simulation of superconducting coated conductors provokes a few major difficulties i.e. creation of a finite element mesh of acceptable quality and execution of calculations in reasonable time.

Computer simulation of high-T_c superconductors

For the sake of simplicity, highly non-linear electrical behaviour of HTS can be described by equation (1) binding the resistivity and the current density:

$$\rho = \frac{E_c}{J_c}\left(\frac{|J|}{J_c}\right)^{n-1} \tag{1}$$

where: ρ – resistivity, E_c – constant (electric field intensity, 10^{-4} V/m), J_c, – critical current density, J – current density, n – constant exponent.

Maxwell equations can be written in a few forms depending on the chosen state variables. As proposed in [1], magnetic field strength can be calculated by solving equation (2):

$$\mu\frac{\partial \mathbf{H}}{\partial t} + \nabla\times(\rho\nabla\times\mathbf{H}) = 0 \tag{2}$$

where: μ – magnetic permeability, $\mathbf{H}$ – magnetic field vector.

This problem formulation permits for a convenient incorporation of the fundamental superconductor feature (1) in the numerical problem. Because of formula (3), which holds for two-dimensional problems, relation (1) is transformed into FEM solvable task.

$$J_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \tag{3}$$

where: J_z – perpendicular component of current density, H_x, H_y – in-plane magnetic field components.

The AC power losses (in W/m) in superconducting subdomain can be calculated using equation (4):

$$P = \frac{1}{T}\int_0^T \mathrm{d}t \int_S \mathbf{E}\cdot\mathbf{J}\,\mathrm{d}S \tag{4}$$

where: P – power losses, T – current period, $\mathbf{E}$ – electric field vector, S – superconductor subdomain cross-section.

Geometrical aspect ratio

Models with large geometric scale differences are always problematic to mesh. The ratio of thickness of superconducting layer to HTS tape width can be high as 1:10000. These extremely thin subdomains are very difficult to analyse using FEM computer simulation because of gigantic number of degrees of freedom. Creation of a finite element mesh of acceptable quality and execution of calculations in reasonable time are the main opposing problems. There are three resolutions to these obstacles: thickness manipulation, mesh scaling and shell region usage.

Subdomain thickness manipulation

Thin superconducting or resistive layers can be geometrically rescaled in order to obtain lower aspect ratio for meshing process [2, 3, 4]. The thickness of HTS and metal over-layers is multiplied by a factor of 10÷1000. In such situation, for approximation of the real solution, five physical parameters need also to be scaled: critical current density of superconducting subdomain, resistivity or electric field intensity, magnetic permeability, specific heat and heat transfer coefficient.

Table 1 contains basic simulation parameters utilized in each attempt described in the following parts of this paper.

Table 1. Basic FEM model parameters

HTS layer width [m]	$4\cdot10^{-3}$
HTS layer thickness [m]	$1\cdot10^{-6}$
J_C [A/m^2], at self field, at T = 77 K	$20\cdot10^{9}$
n	20
f [Hz]	50

Selected computer simulation results of the rescaled problem are shown in figures 2 and 3. In this attempt, HTS thickness was increased r-fold, where r = 1000.

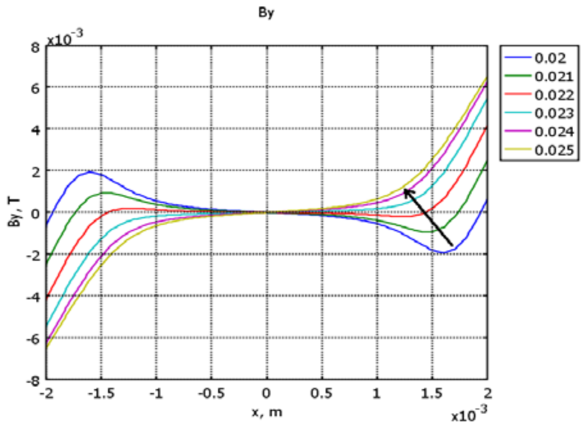

Fig.2. Perpendicular component of magnetic flux density on the upper surface of HTS (I/I_C = 0.65, t = 20÷25 ms, r = 1000)

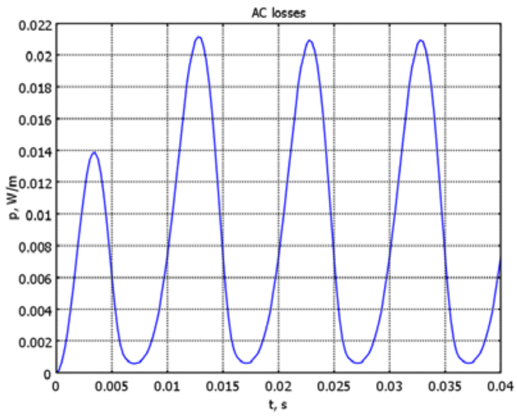

Fig.3. Instantaneous AC losses (I/I_C = 0.65, r = 1000))

Aspect ratio manipulation shortens computation time meaningfully, maintaining quasi-realistic results of magneto-thermal modelling. Simulation results demand cautious interpretation because of the parameters modification.

Mesh mapping and multiscale mesh

One way to reduce meshing problem is to split the geometry into several regions and mesh them independently. Some FEM packages allow to use structured meshes, which density and basic element type is chosen independently for a particular subdomain. The connection of the different meshes into one single model is achieved by identity boundary conditions [5].

It is also possible to lower the number of degrees of freedom significantly and to accelerate calculation process using so-called "mesh mapping" (fig.4). This simple manipulation may give reliable results of AC losses estimation [6].

Figures 5 and 6 contain chosen results of the multiscale mesh simulation. This approach produces highly consistent values, which are comparable to the outcome described in the next section. Moreover, calculation time is many-fold shorter than in the case of standard triangular meshing.

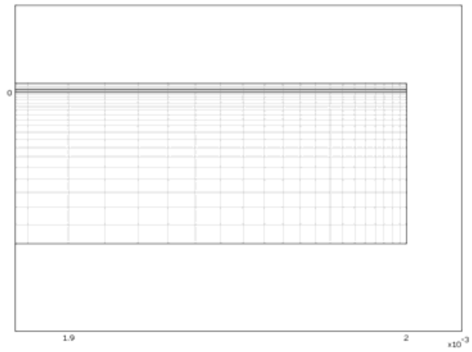

Fig.4. Anisotropic mapped mesh of HTS tape (wire)

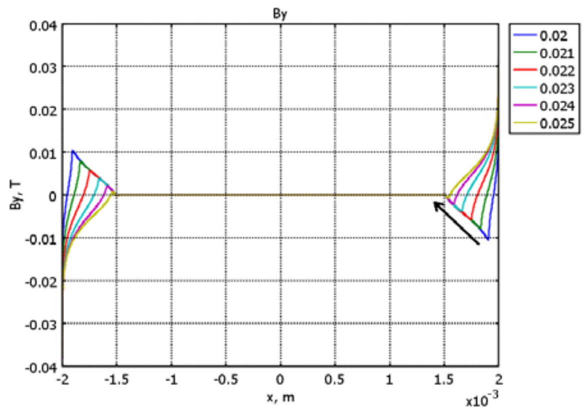

Fig.5. Perpendicular component of magnetic flux density on the surface of HTS (I/I_C = 0.65, t = 20÷25 ms)

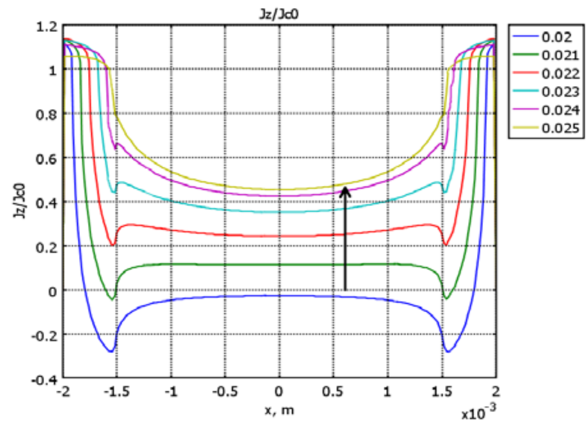

Fig.6. Relative current density along HTS layer x-axis (I/I_C = 0.65, t = 20÷25 ms)

Integral equation or shell regions

In order to decrease the number of degrees of freedom and computation time, it is possible to substitute thin two-dimensional strips by a corresponding one-dimensional segments [7, 8]. It is also possible to replace three-dimensional thin strips with two-dimensional equivalents [9].

Contemporary FEM packages offer the ability to model thin layers with equation-based modelling using tangential derivative variables. Some of them poses specialized application modes, other ones require user to define problem in the weak form.

In the case of 2G HTS tapes, current density distribution in the cross-section of 2D strip can be approximated by one-dimensional Brandt's integral equation. It can be formulated as a partial differential equation (PDE) and solved by FEM software. To describe electromagnetic interactions of one-dimensional strips with other subdomains, it is necessary to couple integral equation with two-dimensional problem.

Idea of sheet current density in depicted in figure 7 and may be described as equation (5).

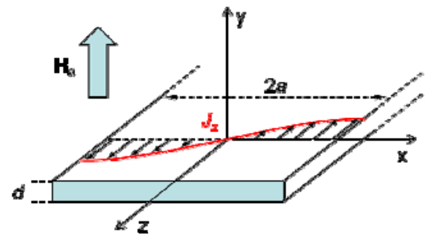

Fig.7. Idea of sheet current density

(5) $$J_z(x,t) = \int_{-d/2}^{d/2} j_z(x,y,t)\,\mathrm{d}y$$

where: $J_z(x,t)$ - sheet current density, $j_z(x,y,t)$ - current density, d - conductor thickness.

To find the sheet current density distribution under different transport conditions, it is necessary to solve formula (6) derived from Brandt's integral equation, taking also under consideration additional constraint (7).

(6) $$J_z(x,t) = \frac{\mu d}{\rho}\left(\int_{-a}^{x} \dot{H}_{ay}(u,t)\mathrm{d}u + \right. \left. + \frac{1}{2\pi}\int_{-a}^{a} \dot{J}_z(u,t)\ln|x-u|\mathrm{d}u \right) + C(t)$$

where: $H_{ay}(x,t)$ - y-component of external magnetic field, a - half-width of the conductor.

(7) $$\int_{-a}^{a} J_z(x,t)\,\mathrm{d}x = I(t)$$

where: $I(t)$ - transport current.

Usage of integral equations demands a formulation of coupled design: 1D model for every thin strip and auxiliary 2D model for computation of magnetic field (fig.8). Instantaneous AC loses obtained from this method are shown in figure 9.

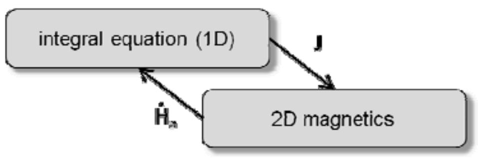

Fig.8. Coupling between models

Conclusion

This work was intended to select a reliable and efficient method of FEM simulation of ReBCO coated conductors, use of which is planned in our superconductor laboratory. Direct approach provokes major computational difficulties – large number of mesh nodes and consequently very long calculation time. Three main methods helping to overcome high aspect ratio problem are known: geometrical scaling, mesh mapping and constrained integral equation usage.

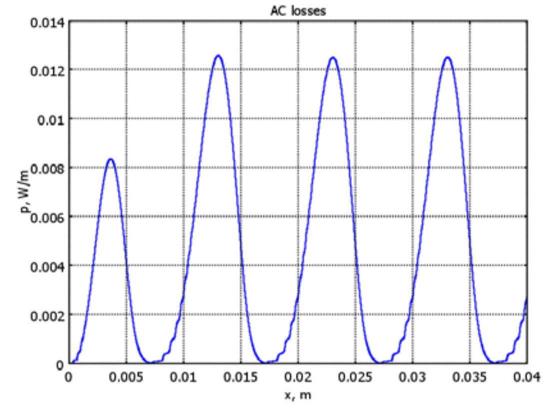

Fig.9. Instantaneous AC losses (I/I_C = 0.65)

The simplest solution – sub-domain scaling – distorts magnetic phenomena. Therefore, correct evaluation of power losses appears very difficult. This technique may be useful for simulation of over-J_C regimes (current limiters), what is also in the scope of our interest. Mesh mapping allows for efficient decrease of problem complexity. It maintains physical aspect ratio of ReBCO layers, electromagnetic phenomena are simulated reliably also in complex configurations. Dimensional reduction of thin layers is the most computationally effective. It gives consistent results and can be used for AC losses estimation. However, each HTS layer needs additional coupled model, what may be perceived as an impediment.

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