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The use of fuzzy logic to evaluate the nonlinearity of human multi-criteria used in decision making

***Abstract**. In this paper, author proposes how to evaluate the nonlinearity of human multi-criteria by using fuzzy logic. In the article three potential indicators have been selected to describe level of nonlinearity. In order to obtain data, author conducted a survey on a group of 300 people. As a result, author received 300 nonlinear functional surfaces of the human, individual multi-criterion, for which the proposed indictors were calculated. Finally, it turned out that values of proposed indicators correctly identified the nonlinearity of human multi-criteria.*

***Streszczenie.** W niniejszym artykule, autor proponuje w jaki sposób można dokonać oceny nieliniowości ludzkich wielokryteriów decyzyjnych przy zastosowaniu logiki rozmytej. W tym celu zaproponowano trzy potencjalne wskaźniki nieliniowości. W celu zebrania danych, autor przeprowadził badanie na grupie 300 osób, w wyniku którego otrzymał 300 nieliniowych powierzchni ludzkiego wielokryterium. Następnie dla otrzymanych powierzchni zostały wyliczone proponowane wskaźniki, których wartości poprawnie zidentyfikowały nieliniowość ludzkich wielokryteriów. (**Ocena nieliniowości ludzkich wielokryteriów decyzyjnych przy zastosowaniu logiki rozmytej**)*

Keywords: multi-criteria analysis, nonlinear multi-criteria, fuzzy logic, indicator of nonlinearity.
Słowa kluczowe: analiza wielokryterialna, nieliniowe wielokryteria, logika rozmyta, wskaźnik nieliniowości.

Introduction

People use more than one criterion to make a decision. Then we talk about human multi-criteria. Research conducted by D. Kahneman and A. Tversky in 60-ties and 70-ties of the XX century has shown that human M-Cr are nonlinear [3]. In this paper, author try to answer the main question: "How to evaluate the nonlinearity of human multi-criteria". How to do it? Author propose the following procedure as the problem solution. In the first step author identifies linear and nonlinear criterion-functions describing the preferences of decision maker. The linear function is identified by the method of least squares [5]. The method of characteristic objects in the space of the problem, created by A. Piegat [4] is used to identify a nonlinear, multi-criterion-function. It is a new method based on fuzzy logic which can be used for identification of human multi-criteria.
In the second step author compares values of the identified criterion-functions. Following indicators are used for comparison and for evaluation of the nonlinearity of human multi-criteria:

- $N-Ind_K$ - quantitative indicator of nonlinearity proposed in [4],
- $1-r_{K,K_L}$ - difference between one and Pearson product-moment correlation coefficient (in short PPMCC),
- φ^2 - coefficient of linear indetermination [2].

All these indicators are normalized to interval [0, 1]. When the value of an indicator is close to zero it means a very weak nonlinearity. When it is close to one, it means a very strong nonlinearity. The indicator equal zero describes a linear criterion-function. Evaluation of the indicators will be further discussed in the section entitled "Discussion of results". In the next chapter is presented experiment.

Experimental Procedures

An experiment will be presented in which a simple, nonlinear, human multi-criterion function representing inner human preferences will be identified. The multi-criterion K represents the dependence between the attractiveness degree of a mixed color and the degree of brightness of green color (in short G) and of blue color (in short B). K, G and B are normalized to the interval [0, 1].

Author gathered the data for research using a survey. The interviewed person has to make decisions described:
"In the survey, please indicate, which color of the pair of colors is more attractive (please mark this color by X). If both colors have similar or identical level of attractiveness, please mark a draw. Attractiveness of color is telling you which color you prefer more from the pair of colors."

In this way, author collected 300 questionnaires, which have been used in the research. To identify the nonlinear criterion-function all membership functions were determined. The membership function for component B is presented as formula (1):

$$\mu_L = \frac{0.5-B}{0.5} \quad \mu_{ML} = \frac{B-0.0}{0.5} \quad \mu_{MR} = \frac{1.0-B}{0.5} \quad \mu_H = \frac{B-0.5}{0.5} \tag{1}$$

On the other hand, membership function for component G is presented as formula (2):

$$\mu_L = \frac{0.5-G}{0.5} \quad \mu_{ML} = \frac{G-0.0}{0.5} \quad \mu_{MR} = \frac{1.0-G}{0.5} \quad \mu_H = \frac{G-0.5}{0.5} \tag{2}$$

L – low, *ML* – medium left, *MR* – medium right, *H* – high.
On the basis of the formula (1) and (2) nine characteristic objects were generated. Evaluation of the characteristic objects is determined with the tournament-rank method. Values of the attractiveness degree of characteristic objects (in short K_i for $i=\overline{1:9}$) are the basis for calculation the linear criterion-function, which is presented as formula (3):

$$K_L = \omega_0 + \omega_1 \cdot G + \omega_2 \cdot B \tag{3}$$

where: ω_i – weight coefficients of particular component criteria for i = 0, 1, 2. Then using formula (3) and characteristic objects values K_{Li} were calculated.
The same values K_i are used to present fuzzy rules (4-12):

$$IF(G \sim 0.0) AND(B \sim 0.0) THEN(Atr \sim K_1) \tag{4}$$
$$IF(G \sim 0.0) AND(B \sim 0.5) THEN(Atr \sim K_2) \tag{5}$$
$$IF(G \sim 0.0) AND(B \sim 1.0) THEN(Atr \sim K_3) \tag{6}$$
$$IF(G \sim 0.5) AND(B \sim 0.0) THEN(Atr \sim K_4) \tag{7}$$
$$IF(G \sim 0.5) AND(B \sim 0.5) THEN(Atr \sim K_5) \tag{8}$$
$$IF(G \sim 0.5) AND(B \sim 1.0) THEN(Atr \sim K_6) \tag{9}$$
$$IF(G \sim 1.0) AND(B \sim 0.0) THEN(Atr \sim K_7) \tag{10}$$
$$IF(G \sim 1.0) AND(B \sim 0.5) THEN(Atr \sim K_8) \tag{11}$$
$$IF(G \sim 1.0) AND(B \sim 1.0) THEN(Atr \sim K_9) \tag{12}$$

On the base fuzzy rules (4-12) we can identify functional surface of the human, individual multi-criterion, which is synonymous nonlinear criterion-function. This surface will be presented for an empirical visualization of its nonlinearity in chapter titled Results.
When the linear and nonlinear criterion-functions were identified indicators of nonlinearity can be calculated. The formula of the first indicator is presented as (13):

$$N - Ind_K = \frac{\sum_{i=1}^{m} \left| K_i - K_{Li} \right|}{0.5 \cdot m \cdot (K_{\max} - K_{\min})} \qquad (13)$$

It is based on the absolute difference of points K_i and K_{Li}. The denominator in formula (13) realizes normalization of the indicator to interval [0,1]. The second indicator is the difference between one and the PPMCC, which defines a strength of nonlinear correlation between K_i and K_{Li}. It is presented as formula (14):

$$1 - r_{K,K_L} = 1 - \frac{\operatorname{cov}(K, K_L)}{S(K) \cdot S(K_L)} \qquad (14)$$

The last one is the coefficient of linear indetermination (15). This indicator determines how much variability of nonlinear criterion-function was not explained by the linear criterion-function.

$$\varphi^2 = 1 - \frac{\operatorname{cov}^2(K, K_L)}{S^2(K) \cdot S^2(K_L)} \qquad (15)$$

In this paper, author is presented only selected values of indicators and functional surface of the human, individual multi-criterion. The next chapter presents characteristic values of indicators.

Results

In the experiment there were identified together 600 linear and nonlinear (using fuzzy logic) functional surfaces of the human, individual multi-criterion. It was a basis for computing overall 900 values of nonlinear indicators. For example, sample #146 (Fig.1.) shows the visualization of high nonlinearity of surface of the human, individual multi-criterion.

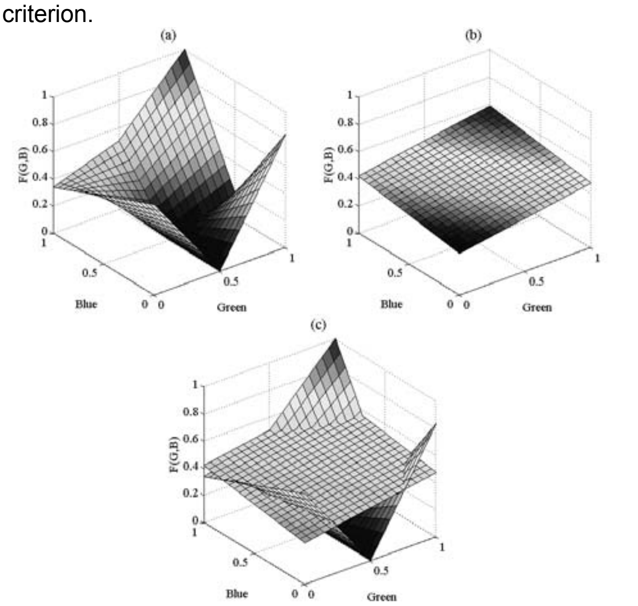

Fig.1. Example of functional nonlinear (Fig.1a) and linear (Fig.1b) surfaces of the human, individual multi-criterion. The empirical visualization of its strong nonlinearity (high nonlinearity), Fig.1c.

The values of the nonlinear indicators are presented in Table 1, they indicate a high level of nonlinearity.

Table 1. The values of indicators for surface from Fig.1a.

Indicator	Value of indicator
$N - Ind_K$	0.5679
$1 - r_{K,K_L}$	0.7547
φ^2	0.9398

For another example, sample #152 (Fig.2.) shows the visualization of low nonlinearity of surface of the human, individual multi-criterion.

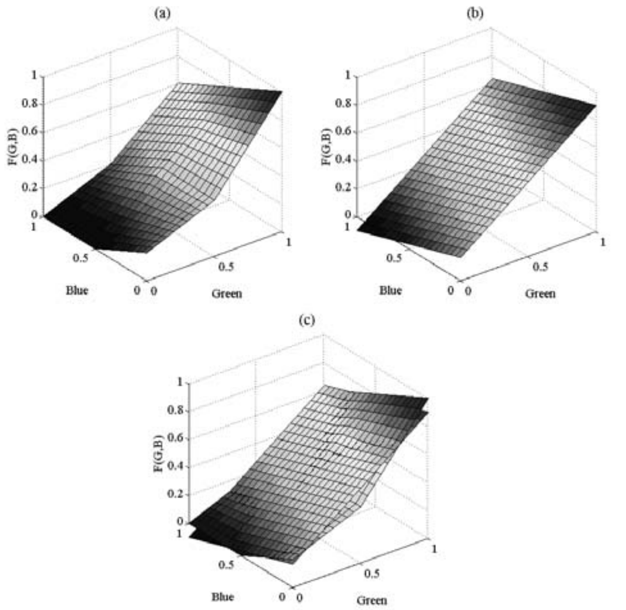

Fig.2. Example of functional nonlinear (Fig.2a) and linear (Fig.2b) surfaces of the human, individual multi-criterion. The empirical visualization of its strong nonlinearity (low nonlinearity), Fig.2c.

The values of the nonlinear indicators are presented in Table 2, they indicate a low level of nonlinearity.

Table 2. The values of indicators for surface from Fig.2a.

Indicator	Value of indicator
$N - Ind_K$	0.1185
$1 - r_{K,K_L}$	0.0246
φ^2	0.0486

The most important set of statistical coefficient calculated for the distribution of nonlinearity indicators is presented in Table 3. For example, set of statistical coefficients calculated for $N - Ind_K$ is interpreted as follows. Values of the indicator $N - Ind_K$ concentrate to the effective interval [0, 0.6667]. This means a moderate volatility of interval. The arithmetic average of this indicator is 0.4094. The standard deviation is 28.37% of the arithmetic mean, which indicates a moderate variation. Twenty-five percent of the data have a value of the indicator not greater than 0.3333, and further seventy-five percent of the data has a value not less than 0.3333. Seventy-five percent of the data have a value of the indicator not greater than 0.4938, and further seventy-five percent of the data have a value not less than 0.4938. The median value of the indicator $N - Ind_K$ is equal 0.4233. The distribution of indicator has a weak negative coefficient of skewness measured by Bowley's formula[1] and a moderate negative coefficient of skewness measured by Yule's formula[1].

Table 3. Set of statistical coefficient of the nonlinearity indicators.

	$N-Ind_K$	$1-r_{K,K_L}$	φ^2
min value	0.0000	0.0000	0.0000
max value	0.6667	0.9408	0.9965
lower quartile	0.3333	0.2254	0.4000
upper quartile	0.4938	0.6171	0.8534
median	0.4233	0.3778	0.6128
arith. mean	0.4094	0.4157	0.6031
Std deviation	0.1161	0.2356	0.2651
Coeff. of variation	0.2837	0.5667	0.4395
Bowley's coeff. of skewness	-0.6080	0.2866	-0.3020
Yule's coeff. of skewness	-0.1215	0.2220	0.0615
number of indicators	300	300	300

In addition, the experiment results will be presented in two selected sets of samples. The first set of samples presents the six lowest values of nonlinearity indicators. Summary of them is shown in Table 4.

Table 4. The lowest values of nonlinearity indicators with corresponding numbers of samples.

No.	$N-Ind_K$	$1-r_{K,K_L}$	φ^2
75	0.1623	0.0440	0.0861
122	0.1564	0.0442	0.0865
131	0.1340	0.0363	0.0713
146	0.1185	0.0246	0.0486
148	0.1481	0.0385	0.0756
189	0.1193	0.0483	0.0942

The data in Table 4. are characterized by very low values. This means that the corresponding functional surfaces of them are quasi-linear. The visualization of functional surfaces is presented in Fig. 3.

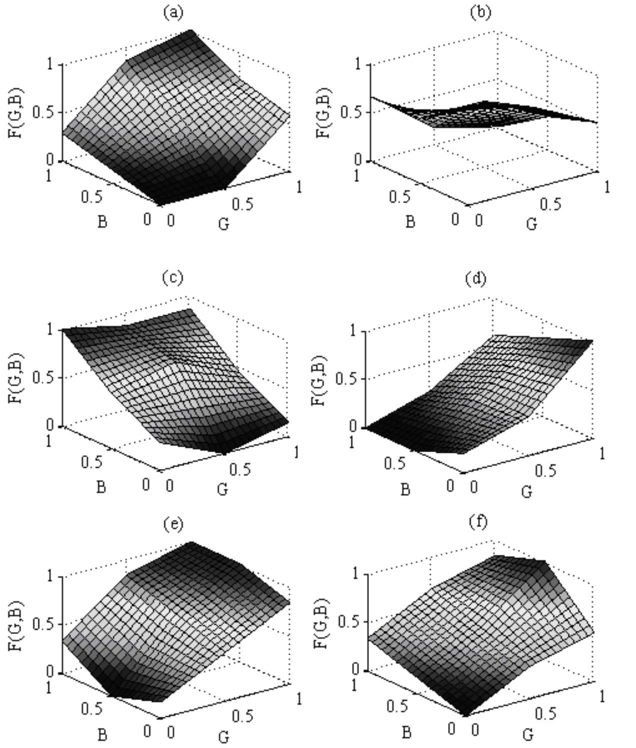

Fig 3. The visualization of samples, which have the lowest values of nonlinearity indicators. (a) Sample #75 (b) Sample #122 (c) Sample #131 (d) Sample #146 (e) Sample #148 (f) Sample #189.

Although the data in Fig.3. show the functional surfaces of quasi-linear character, there is a clear deviation from linearity. The second set of samples presents the six highest values of nonlinearity indicators. Summary of them is shown in Table 5.

Table 5. The highest values of nonlinearity indicators with corresponding numbers of samples.

No.	$N-Ind_K$	$1-r_{K,K_L}$	φ^2
36	0,4988	0,9092	0,9918
38	0,5679	0,9408	0,9965
55	0,5885	0,9013	0,9903
97	0,6420	0,8913	0,9882
155	0,5597	0,9288	0,9949
215	0,5679	0,8676	0,9825

The data in Table 5. are characterized by very high values. This means that the corresponding functional surfaces of them are an certainly nonlinear.

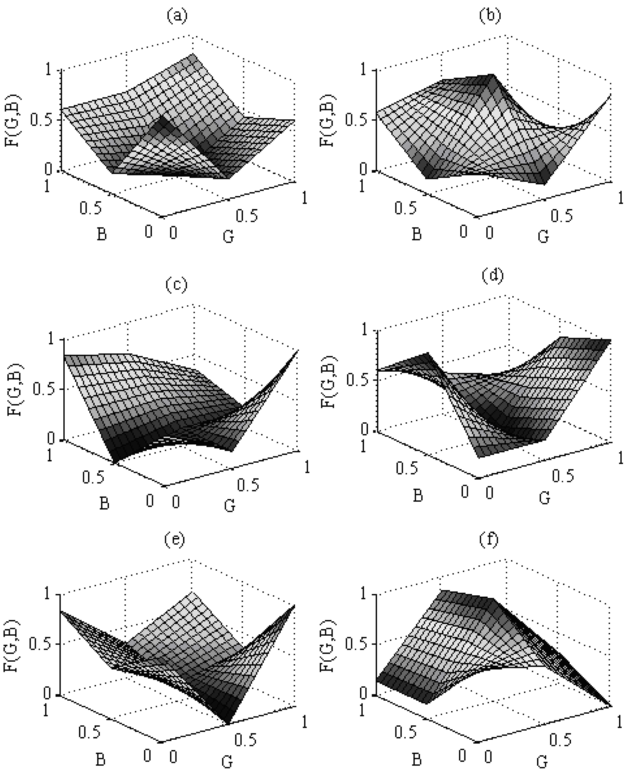

Fig 4. The visualization of samples, which have the highest values of nonlinearity indicators. (a) Sample #36 (b) Sample #38 (c) Sample #55 (d) Sample #97 (e) Sample #155 (f) Sample #215.

Fig. 4. shows a much higher nonlinearity of functional surfaces than is showed in Fig.3. This is a result that confirms the suitability of fuzzy logic to evaluate the nonlinearity of human multi-criteria used in decision making.

Discussion of results

Values of indicators correct represent the real nonlinearity of human multi-criteria. Indicators φ^2 and $1-r_{K,K_L}$ have the largest rate of variability. This means that the interval [0,1] is rather only a theoretical interval for the indicator $N-Ind_K$. It is a very important for the interpretation of values of this indicator, because in this case values higher than 0.5 imply high or very high level of nonlinearity. On the basis of arithmetic mean and standard deviation, intervals of levels of human multi-criteria nonlinearity were determined. Limits of intervals were found, by addition or subtraction, arithmetic mean and multiple of standard deviation. Set of the intervals of levels of human multi-criteria nonlinearity was presented in table 6. (This set was established only for this experiment)

Table 6. Set of the intervals of levels of human multi-criteria nonlinearity

Level of nonlinearity	$N - Ind_K$	$1 - r_{K,K_L}$	φ^2
Low	0,00 - 0,18	0,00 - 0,06	0,00 - 0,07
Moderate	0,18 - 0,29	0,06 - 0,18	0,07 - 0,34
High	0,29 - 0,53	0,18 - 0,65	0,34 - 0,87
Very high	0,53 - 0,64	0,65 - 0,89	0,87 - 0,96
Extreme	0,64 - 0,67	0,89 - 0,94	0,96 - 1,00

This summary also shows the relationship between the analyzed values of indicators. Their relationship can be described also by coefficient of linear determination. Thus, the indicator φ^2 and $1 - r_{K,K_L}$ are linearly dependent in 95.73%, and the indicator $N - Ind_K$ and $1 - r_{K,K_L}$ in 76.90%. This implies a strong relationship of these three indicators

Conclusions

This paper shows, that procedure proposed by author in the chapter Introduction was effective. The main evidence for this was the result of the experiment. The fuzzy logic is useful for evaluation of the nonlinearity of human multi-criteria used in decision making. The proposed nonlinear indicators are described adequately to the level of nonlinearity criterion-function. The each indicator has a different distribution, but practically all of them have a positive correlation. This means that they are consistent indicators.

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