The impact of incorrectly selected soil permittivity value on focused GPR image

Abstract. This article presents simulation and measurement results of ground penetrating radar with FMCW (Frequency Modulated Continuous Wave) signal. The author shows the effect of incorrectly chosen permittivity value of tested soil on final SAR (Synthetic Aperture Radar) image. Novel iterative AutoESP (Auto Estimation of Soil Permittivity) algorithm is tested on actual measurement GPR data.

Introduction

Ground penetrating radars are widely used in military and civilian applications. GPR are used among other thing, to detect explosive materials and roadway damages or for archaeological explorations. However, digital signal processing applied to GPR is much more sophisticated then in traditional radars [1][2]. The main problem is the heterogeneity of tested medium. Appearance of buried objects change electromagnetic and mechanical properties of the earth. The shape of received echo signals after range compression does not correspond to geometrical dimensions of located objects [4][5]. It has the shape of a curve. The greater depth of an object or lower permittivity value, the greater radius of received curve. Soil in general is a medium composed of many layers with different permittivity coefficient values. Propagated wave is reflected, refracted and scattered on the border of two mediums. What is more, transmitted electromagnetic waves are strongly attenuated by surrounding soil. This directly translates into a maximum GPR range. Typically, the range of ground penetrating radar is several meters.

In order to improve radar resolution in movement direction, SAR processing has been applied. Fully focused image requires information about soil permittivity value [4]. Incorrectly matched permittivity causes the final image is blurred. Presented AutoESP after few iterations allows to estimate correct value of permittivity.

FMCW GPR properties

Presented simulation and actual measurement ground penetrating radar transmit continuous signal with linear frequency modulation FMCW. Range resolution improvement for such radars is carried out in the following steps. Received echo signal is multiplied by reference signal. For that purpose mixer system is used. Intermodulation components are generated at the output of the mixer. Information about distance between radar and object is contained in intermediate component, other components are filtered out. Intermediate frequency signal is then considered in the frequency domain. In order to do that, FFT algorithm is used. FMCW modulation function is given as a continuous line in figure 1. Dashed line corresponds to received echo.

Signal sweep time $T$ for simulated and actual GPR device is equal to 1 ms. Frequency shift connected with received echo signal delay $dt$ is indicated by $df$ and can be described by the relationship:

$$df = \frac{2RB}{cT}\sqrt{\varepsilon_r}$$

where: $R$ – the distance between object and radar, $B$ – bandwidth, $c$ – the speed of light in vacuum, $\varepsilon_r$ - permittivity coefficient.

![Fig.1. FMCW modulation function.](image)

The spectrum bins of received echo contain information about depth of an object. In order to properly scale the frequency axis, following equation is used:

$$R(n) = \frac{n}{M} N \Delta f$$

where: $n$ – the spectrum bins <1;M>, $M$ – the number of spectrum bins, $N$ – the number of signal probes connected with sweep time $T$.

$\Delta f$ parameter is range resolution and is given by the relationship [2]:

$$\Delta f = \frac{c}{2B\sqrt{\varepsilon_r}}$$

GPR with signal bandwidth equal to 2 GHz, which propagate electromagnetic wave in the soil of permittivity coefficient 6, has range resolution equal to 3 cm.

The main purpose of the ground penetrating radar developed by Telecommunication Research Institute (Przemysłowy Instytut Telekomunikacji S.A.) is detection of explosive materials. Workplace of such radar is generally sandy soil. What is more the depth at which explosive materials are located do not exceed 1 m [5]. In such a situation it can be assumed that tested soil is homogeneous. Simplified GPR scene is showed in Fig. 2.
GPR antenna is located at a constant height $h$ above the ground. Even such a simple radar scene requires calculation of fourth-degree polynomial in order to obtain place $x_0$ where the signal cross the border of two layers. The solution can be obtained using Ferrari’s algorithm to reduce that polynomial to the cubic form. The distance between radar and object is given in equation (4):

$$R(x_0) = \sqrt{h^2 + x_0^2 + H^2 + (x-x_0)^2}$$

That equation defines a hyperbolic shape of received signal. The distance travelled by the signal is composed of two factors. The first factor corresponds to the distance travelled in the air, while the second one corresponds to the distance travelled in tested soil. Relationship (4) reaches minimum value when ground penetrating radar is located directly above the object.

Simulated range compressed data of echo signal received by GPR form six objects located at different depths is presented in Fig. 3.

SAR processing

Azimuth resolution improvement is obtained using two-dimensional SAR filtering. The most important parameters of matched filter is its phase and geometrical shape of impulse response. These parameters include information about the depth of an object and permittivity value of tested soil. Two-dimensional convolution of range compressed data $S(n,m)$ and SAR matched filter $h(n,m)$ can be calculated [1][3][5]:

$$S_{SAR}(n,m) = \sum_{k=-N_{LB}}^{N_{LB}} \sum_{l=-N_{LB}}^{N_{LB}} S(k,l) \cdot h(n-k,m-l)$$

SAR matched filters are initially multiplied by hamming window function. This procedure results in sidelobes suppression in movement direction. Two-dimensional filtering is carried out according to following scheme:

Range compressed data is initially divided into blocks. Each block $b_i$ is composed of several range cells. 2D SAR matched filter is prepared for the object located at the depth corresponding to the middle range cell of each block $b_i$. Selected block and the nearest neighbourhood are then filtered. SAR filters amplitude is selected in an appropriate manner to compensate signal power losses related to travelled distance. In order to smooth final image, filtered blocks need to be multiplied by window function before they can be merged with filtered data from other blocks. Hamming window have been chosen for this purpose [1][3].

$$w(n) = 0.5 - 0.5 \cdot \cos \left( \frac{2\pi n}{3 \cdot N_{LB}} \right)$$

where: $n = 0,1,2, \ldots , 3\cdot N_{LB}$, $N_{LB}$ - the number of range cells in block.
Finally achieved azimuth resolution after coherent SAR processing is determined by the aperture of used antenna. Combining range compression of frequency modulated continuous wave with SAR processing algorithms provide fully focused visualisation of underground objects. The result of two-dimensional SAR filtration applied for data presented in Fig. 3 is given in the following figure.

Increase of noise level for higher depths is connected with two-dimensional matched filter. SAR filters compensate signal power losses related to travelled distance.

### Permittivity mismatch

Presented SAR filtering results, in the previous section, are obtained on the assumption that actual permittivity value of tested soil is known. In fact, GPR does not have any information about the electrical properties of the medium. Permittivity value of tested soil affects both the shape and phase of received echo \([4][5]\). Received signal phase after range compression is given by the relationship:

\[
\phi(x_0) = \exp \left(-j \frac{2}{\lambda} \left(\sqrt{R^2 + x_0^2} + \sqrt{H^2 + (x-x_0)^2}\right)\right)
\]

where: \(\lambda\) - wavelength in vacuum.

Incorrectly selected permittivity blurs the final SAR image. The difference in shapes of two-dimensional SAR matched filters for a different permittivity values is shown in Fig. 6.

![Matched filter difference](image1)

**Fig. 6.** Matched filter difference.

Matched filter for permittivity \(\varepsilon_r = 6\) has greater radius of curvature than filter for \(\varepsilon_r = 16\). Permittivity mismatch influence on received signal phase form, for two objects located at a depth of 1 m in different soils, is presented in Fig. 7. The black line is denoted to permittivity equal to 5, and the red one refers to permittivity equal to 10.

Analysis of the above results shows that incorrectly selected permittivity during SAR filtration causes significant deterioration of azimuth resolution. Having an object buried in the ground at a depth of 1 m, matched filtration have been carried out. Permittivity of tested soil is equal to 5, while used filters have been prepared for permittivity values 5 and 10. The results of obtained azimuth resolution shown in Fig. 8 leave no doubt. In order to obtain fully focused SAR image actual permittivity value is necessary.

![Matched filter phase](image2)

**Fig. 8.** Matched filter phase.

Matched filter output for correct permittivity value has three times better azimuth resolution.

### AutoESP algorithm

AutoESP iterative algorithm in each iteration estimates permittivity adjustment. Actual permittivity value is obtained after few iteration of the algorithm. AutoESP is based on multilook technique \([4][5]\). Multilook applied in AutoESP assumes division of the main antenna into two sub-beams. Matched filtration is carried out for each of the received sub-beams. Focused images from two sub-beams need to be shifted at a specific value before they can be merged together. The concept of multilook algorithm is presented in Fig. 9:

![Multilook concept](image3)

**Fig. 9.** Multilook concept.

![Additional displacement](image4)

**Fig. 10.** An additional displacement.
In the case of well-defined permittivity coefficient, images are merged in the correct way. Otherwise the additional displacement $dx$ for these images is observed and the final SAR image is blurred. The following chart presents an additional displacement for different permittivity values. Actual permittivity of tested soil is 10.

Equations considered in autoESP algorithm are derived from the geometrical model shown in Fig. 2. By measuring the displacement $dx$ in pixels between two multilook images, the value of the shift can be determined [4],[5]:

$$\Delta x = dx \cdot \hat{\delta}_r$$  \hspace{1cm} (6)

where: $\hat{\delta}_r$ - the size of azimuth cell (in movement direction).

On the other hand, the shift between two multilook images can be calculated with AutoESP formula [4]:

$$\Delta x = \frac{H \sin(\theta_1/2)}{[\varepsilon_r - \sin^2(\theta_1/2)]^{1/2}} \hat{\delta}_r$$  \hspace{1cm} (7)

where: $\hat{\delta}_r$ - permittivity adjustment connected with an additional displacement $dx$.

Dependence on the permittivity correction for each iteration is given by equation [5]:

$$\hat{\delta} e_r = -dx \cdot \frac{\left(\varepsilon_r - \sin^2(\theta_1/2)\right)^{1/2}}{H \sin(\theta_1/2)}$$  \hspace{1cm} (8)

In order to obtain the correct value of permittivity, the adjustment (8) should be taken into account during calculating in each $i$ iteration:

$$\varepsilon_r(i+1) = \varepsilon_r(i) + \hat{\delta} e_r$$  \hspace{1cm} (9)

**Measurement results**

The GPR frequency modulated with continuous wave, which is presented in the paper, has been developed by PIT S.A. Transmitted signal bandwidth is about 2 GHz and the sweep time $T$ is equal to 1 ms. By measuring the received echo signal frequency, the time delay between transmission and reception can be measure and therefore the range determined. Detected objects gives an extra amplitude modulation of received echo (Fig. 11).

![Received signal in time domain](image1.png)

**Fig.11. Raw GPR data.**

This modulation is dependent on radar cross-section of the object. The signal of received echo is given to first input of the mixer. On the second input reference signal is given. After low-pass filtering signal component of intermediate frequency is obtained. Presented Ground penetrating radar is characterized by 3 antennas – one antenna is transmitting signal, the other two antennas receive echo signals. What is more the radar has ability to change polarization. Unfortunately polarimetric components dose not give any finally image improvement. All the data presented in the article has been taken for horizontal polarization. GPR can move at the maximum speed equal to 3 meters per second and is steered by remote control. The following image of range compressed measurement data has been received for a dry, sandy soil.

![Raw data after range compression](image2.png)

**Fig.12. Measurement data.**

The initial permittivity value for SAR processing is equal to 8. Without having full information about electrical properties of the ground, it is necessary to carry out an additional permittivity measurement using appropriate equipment. Unfortunately it is expensive and impractical solution. Much better way is to find an algorithm which estimate this value. AutoESP makes it possible to estimate that coefficient of the soil without any additional measurements. Estimated permittivity values for each iteration of AutoESP algorithm is presented in Fig. 13. Finally estimated permittivity is equal to about 4.77.

![Estimated permittivity](image3.png)

**Fig.13. Estimated permittivity.**

![Final autoESP image](image4.png)

**Fig.14. Final SAR image.**
Final SAR image obtained for estimated permittivity coefficient is fully focused.

Regardless of the initial value of soil permittivity, AutoESP algorithm converges to the same, actual value. However, the greater difference between actual and initial permittivity values the higher number of iterations required to obtain correct permittivity. The results of this experiment are shown in following figure.

![Fig.15. Estimated permittivity for different initial values.](image)

**Conclusions**

Simulation and measurement results of FMCW ground penetrating radar confirm the possibility of using 2D SAR processing for underground targets imaging. Presented AutoESP algorithm is good solution for permittivity estimation of homogeneous soil. What is more, AutoESP results do not depend on an initial value of soil permittivity.

**REFERENCES**


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