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Fault Diagnosis Method Integrated Fuzzy Logic and Particle Filter for Nonlinear Systems

***Abstract**. A new fault diagnosis method based on integrated fuzzy logic and particle filter for nonlinear systems is proposed to improve the accuracy of fault diagnosis. The Water Level and Temperature Control System is taken as test-bed process, with different switching states simulating possible system faults. The simulation result show that the proposed method could diagnose fault more accurately than that based on two-valued logic.*

***Streszczenie.** W artykule zaprezentowano nową metodę wykrywania awarii w opartą na logice rozmytej i filtrze cząsteczkowym. Metoda dedykowana układom nieliniowym, zwiększa dokładność detekcji stanów niepożądanych. Przeprowadzone badania symulacyjne i eksperymentalne w układzie regulacji temperatury oraz poziomu cieczy, potwierdziły zwiększoną skuteczność algorytmu dla różnych przypadków awarii. (**Algorytm wykrywania awarii w systemach nieliniowych – wykorzystanie logiki rozmytej i filtru cząsteczkowego**)*

Keywords: fault diagnosis, particle filter, fuzzy logic, nonlinear system
Słowa kluczowe: Wykrywanie awarii, filtr cząsteczkowy, logika rozmyta, system nieliniowy.

Introduction

For different systems, researchers have proposed different diagnosis methods[1,2,3].The fault diagnosis methods could be divided into three categories: the diagnosis method based on knowledge, the diagnosis method based on data-driven and the diagnosis method based on model[4]. The diagnosis method based on model has the solid theoretical foundation and has been realized by many researchers. In engineering, the most commonly used model-based fault diagnosis method is to use the Kalman filter, Kalman filter is optimal filtering for linear system with Gaussian noises[5], But for the nonlinear system, various approximate methods have been proposed, such as extend Kalman filter (EKF), unscented Kalman filter, approximate grid-based methods and particle filters. The EKF approximates the models used for the dynamic and measurement process in order to be able to approximate the probability density by a Gaussian. If the true density is non-Gaussian (e.g., if it is bimodal or heavily skewed), then a Gaussian can never describe it well. Approximate grid-based filter approximate the continuous state space as a set of discrete regions[6]. As the dimensionality of the state space increases, the computational cost of the approach therefore increases dramatically. The advantage of particle filtering over other methods is in that the exploited approximation does not involve linearization around current estimation, but rather than approximation in the presentation of the desired distributions by discrete random measures[7]. In this paper, the researched system is nonlinear and system model is given, so particle filter is adopted to estimate the state of nonlinear system.

In fault diagnosis of nonlinear systems, particle filter utilizes sample particles to describe the posterior distribution of estimated state, with better diagnosis performance than that using the extended Kalman filter[8,9]. In the fault diagnosis approach based on likelihood, the complete probability density function information of the estimated state is utilized to diagnose fault, thus the diagnosis effect is improved[10]. When the variance of the state noise is less than that of measurement noise, the rate of the missing alarm generated by the fault diagnosis approach based on sampling importance resampling(SIR) state estimation, and smoothed residual is lower than that generated by fault diagnosis approach based on SIR likelihood[11]. So the fault diagnosis approach of integrating the SIR state estimation and smoothed residual will be adopted in nonlinear systems in this paper.

The fault diagnosis method based on two-valued logic lacks the description and disposition for system transitional states which are between "normal" and "non-normal" state. If the transitional state is diagnosed as the "non-normal" state, the operation of the equipment will be suspended to eliminate the fault and leading to efficiency decrease; If the transitional state is diagnosed as the "normal" state, potential dangers threatening the safe and reliable production will be laid. However, in some real processes, the fault occurrence is more of the gradual accumulation of mild modifications, rather than an abrupt change which jumps directly from "normal" to "non-normal" state with no transition at all. So the system should not be characterized simply as "normal" or "non-normal". It is healthy or damaged to some extent, and damage is a matter of degree. The introduced fuzzy decision system detects the degree of the damage and warns for the need of preventive maintenance to avoid unsafe situations.

Bootstrap Particle Filter Algorithm

A stochastic nonlinear system is defined by a state transition equation and a measurement equation:

(1) $$x_k = f(x_{k\text{-}1}, v_{k\text{-}1})$$

(2) $$z_k = h(x_k, n_k)$$

v_{k-1} is the state noise at time point of $k-1$ and n_k is the measurement noise at time point of k. Eq.1 describes the conditional transition probability $p(x_k|x_{k-1})$, predicting the current state x_k. Eq.2 describes the likelihood probability $p(z_k|x_k)$, predicting current measurement z_k. The measurement set $D_k=\{z_i : i=1,...,N\}$ is the available information of the system at the time step k.

In the particle filter algorithm, the sample particles propagate chronologically via the system model. Where the state samples are obtained, the posterior probability density function of system state can be calculated[12]. As the basis of particle filter, the Sequential Importance Sampling (SIS) algorithm has a problem – with the increasing iteration, the weight of most particles becomes negligible, which leads to serious degeneracy. However, based on the SIR algorithm, the Bootstrap Particle Filter can reduce the effects of degeneracy by resampling[13,14]. Following paper[13], The steps of particle filter are as follows: Suppose there are a set of random samples $\{x_{k-1}(i) : i=1,2,...,N\}$ from probability density function $p(x_{k-1}|D_{k-1})$.

(1) Prediction: the particle state at the next time point is obtained from prior particle state through the system state equation at time step k : $x_k^*(i)=f_{k-1}(x_{k-1}(i),v_{k-1}(i))$, where v_{k-}

$_{1}(i)$ is a sample drawn from the PDF of the system noise $p(v_{k-1})$.

(2) Update: when measurement z_k is obtained, assign each $x_k^*(i)$ a weight $\omega_k(i)$: $i=1,2,...,N$. The weights are given by

$$\omega_k(i) = \frac{p\left(z_k \left| x_k^*(i)\right.\right)}{\sum_{j=1}^{N} p\left(z_k \left| x_k^*(j)\right.\right)}$$

This defines a discrete distribution over $\{ x_k^*(i): i=1,2,...,N\}$, with probability mass $\omega_k(i)$ associated with element $x_k^*(i)$.

(3) Estimation: $\tilde{x}_k = \sum_{i=1}^{N} x_k^*(i) * \omega_k(i)$, $\tilde{x}_k$ is the estimation of system state at time point of k.

(4) Resample: Resample N times from the discrete distribution to generate samples $\{x_k(i) : i=1,2,...,N\}$, which satisfy $p_r\{ x_k(i)= x_k^*(j)\}= \omega_k(j)$ for any i.

The prediction, update and resample steps form a single iteration of the recursive algorithm and is recursively applied at each time point k.

Fuzzy Logic

In 1965, L.A.Zadeh proposed the concept of fuzzy set, which utilizes membership function to express the ambiguity of things[15]. Different from two-valued logic, the fuzzy logic is a continuous logic. In the fuzzy logic, the objective world is regarded as full of continuous changes of gray levels which are described by memberships.

Fuzzy reasoning offers an alternative to Boolean logic such that the truth value of a proposition can take any value between 0 and 1. Fuzzy logic is thus more representative of the continuous nature of many real world problems than the inflexibility of all-or-nothing Boolean logic, whilst avoiding the cryptic and complex nature of traditional mathematical model. It is not as precise, yet for many applications an approximation is adequate and this may be a small price to pay for the ease of implementation, comprehensibility and processing speed afforded by fuzzy logic[16].

A series of gray levels in gradual change is utilized to depict the fault in different levels. Fault membership function in fuzzy logic accurately describes different fault levels, so as to reflect the gradually changing process of system fault.

Fault diagnosis method based on integrated particle filter and fuzzy logic

Most fault diagnosis methods based on model utilize residual as the diagnosis criterion. Residual is the difference between real state and estimated state, There are two steps in this method, namely, residual generation and residual evaluation. Residual evaluation detects the fault based on the following decision rules: a large residual indicates the occurrence of a fault, while a small residual indicate the normal state[17]. The novel aspects of this paper are that the fault diagnosis method based on integrated fuzzy logic and Bootstrap Particle Filter describes the gradually changing process of system fault, from mild to serious fault. Steps are presented as follows:

(1) Estimating the system state: samples $\{ x_k^*(i): i=1,2,...,N\}$ are generated as in the Bootstrap particle filter prediction step. Weights of all particles $\{\omega_k(i)$: $i=1,2,...,N\}$ are generated in the algorithm update step. Estimation of system state $\tilde{x}_k$ is generated in algorithm estimation step.

(2) Comparing the real system state with the estimated system state to obtain the residual. Residual $\Delta_k = |x_k - \tilde{x}_k|$, where x_k is the real system state, $\tilde{x}_k$ is the estimated system state. To improve accuracy, Smoothed residual is used, $d_k = \frac{1}{M} \sum_{j=k-M+1}^{k} \Delta_k$,

where M is sliding window.

(3) Calculating the mean value of smoothed residual. The mean value of smoothed residual R is the arithmetic mean value of smoothed residual in fault existence period.

$$R = \frac{1}{n-m+1} \sum_{k=m}^{n} d_k ,$$

where m is the starting time for the fault, while n is the ending time of fault.

(4) Diagnosing fault. The mean value of smoothed residual is applied to make diagnosis decisions. Fault membership function in fuzzy logic describes different fault levels over [0, 1], corresponding to different mean value of smoothed residual ranges. The fault at corresponding level is detected when the mean value of smoothed residual is falling into the specific range of threshold h. The final diagnosis results include fault level information.

The determination of the sample size N and threshold h is an important issue. In practice, for a given system, the sample size N can be determined through simulations. The threshold is often chosen as a trade-off between minimizing false alarm rates and minimizing miss alarm rates[18].

Simulation

The Water Level and Temperature Control System is as follows. The nonlinear system is shown in Fig. 1. This example has been frequently used in many earlier publications[19,20].

The state transition equation of system is described by equation.

$$\begin{aligned} x_1(t) &= x_1(t-1) + dt[\alpha_1(t-1)Q_1 + \alpha_3(t-1)Q_3 \\ &\quad - \alpha_2(t-1)Q_2] + v_1(t-1) \\ x_2(t) &= x_2(t-1) + \frac{dt}{x_1(t-1)}[(\alpha_1(t-1)Q_1 + \alpha_3(t-1)Q_3) \\ &\quad (\theta_m - x_2(t-1)) + 23.88915] + v_2(t-1) \end{aligned} \quad (3)$$

where $x_1(t)$ and $x_2(t)$ stand for water level and temperature at time point of t respectively.Q_1,Q_2,Q_3 are setting flow rates [m/h], θ_m is the reference temperature [□]. $V_1(t-1),v_2(t-1)$ are uncorrelated zero mean Gaussian white noise with variance 0.1. $\alpha_i(t), i=1,2,3$ describe the switch states of unit 1,2,3 at time point of t .

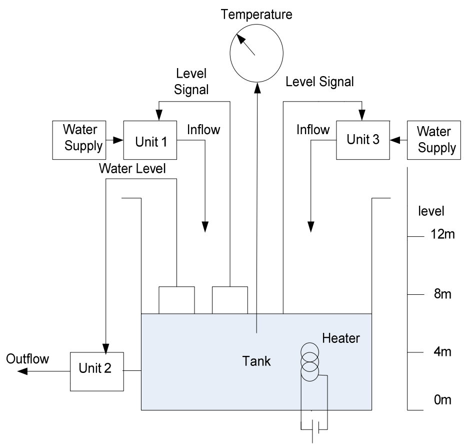


Fig. 1 Water Level and Temperature Control System

When the unit i should turn on, $\alpha_i(t)=1$ denotes that unit i is in normal turning-on state. When the unit i should turn off, $\alpha_i(t)=0$ denotes that unit i is in normal turning-off state.

The evolution equation, namely, the measurement equation is described by Eq. 4.

$$\alpha_1(t)=\begin{cases}1 & ,x_1(t)<l_1\\ 0 & ,x_1(t)>l_2\\ 0 \text{ or } 1 & ,\textit{depending on previous switching}\end{cases}$$
$$(4)\qquad \alpha_2(t)=\begin{cases}1 & ,x_1(t)>l_1\\ 0 & ,x_1(t)\le l_1\end{cases}$$
$$\alpha_3(t)=\begin{cases}1 & ,x_1(t)<l_1\\ 0 & ,x_1(t)>l_2\\ 0 \text{ or } 1 & ,\textit{depending on previous switching}\end{cases}$$

The measurement equation is severely nonlinear, l_1 is the setting lowest level and l_2 is the setting highest level. The system parameters are provided in the Table 1.

Table 1 System parameters

Q_1	Q_3	l_1	l_2	θ_m
1[m/h]	3.5[m/h]	4[m]	10[m]	15[□]

When the water level $x_1(t)$ is lower than the setting lowest level l_1, unit 1 and unit 3 turn on automatically to supply water for the tank. When the water level $x_1(t)$ is higher than the setting highest level l_2, unit 1 and unit 3 turn off automatically, unit 2 turn on to discharge water. The objective of the control system is to maintain the water level at $4\text{m}<x_1<10\text{m}$, the temperature at $15□<x_2<100□$.

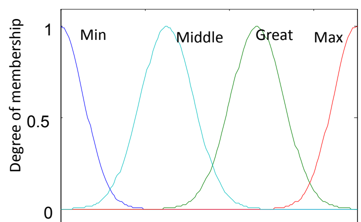
Bootstrap Particle Filter is applied in simulation. The faults of unit 1,2,3 can be divided into three categories: ① The functions of turning-on and turning-off are both in fault, which means that the switch state $\alpha_i(t),i=1,2,3$ is independent of water level, hence $\alpha_i(t),i=1,2,3$ is a constant. ② The function of turning-on is in normal state while the function of turning-off fails, which means that when the switch is turned off, it still leaks. ③ The function of turning-off is in normal state while the function of turning-on fails which means that the actual flowing rate is below the set flowing rate when the switch is turned on. Due to the limitation of paper length, Unit 3 is selected as the fault unit to simulate the gradually changing process of system fault, only the simulation of fault ③ is selected to elaborate fault diagnosis process.

Assumed that $q_3(t)$ is the actual flowing rate of unit 3 at time point t, $q_3(t)=\alpha_3(t)$, while $\alpha_3(t)$ and mean value of smoothed residual are defined as linguistic variables in fuzzy logic. The relationship among the three linguistic variables and fault levels are shown in the Table 2. Fig. 2,(a)-(d) provide the membership functions of $q_3(t)$, $\alpha_3(t)$, mean value of smoothed residual for water level and temperature respectively. The switch state $\alpha_3(t)$ controls actual flowing rate. When $\alpha_3(t)=1$, $q_3(t)=3.5$[m/h]; $\alpha_3(t)=0.5$, $q_3(t)=1.75$[m/h]; $\alpha_3(t)=0$, $q_3(t)=0$[m/h].

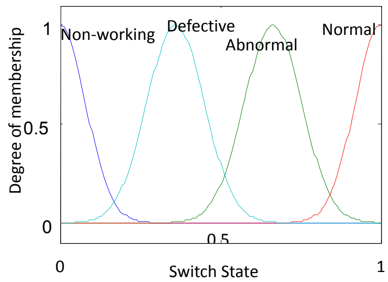
Table 2 The relationship among switch state, residual and fault level

$q_3(t)$	Maximum	Great	Middle	minimum
$\alpha_3(t)$	normal	abnormal	defective	Non-work
residual	minor	low	middle	Large
Faultlevel	Fault-free	mild	moderate	serious

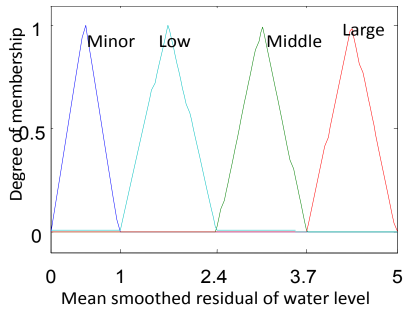
Membership function reflects the gradually changing process. But there is no mature method to determine it. Most methods are established on the basis of experiences. Fig. 2,(a)-(b) use normal distribution curve, Fig. 2,(c)-(d) use the triangle as membership function curve.



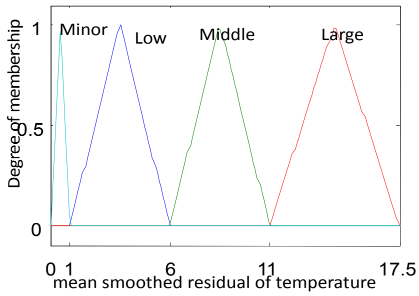
(a) Membership function of $q_3(t)$



(b) Membership function of $\alpha_3(t)$



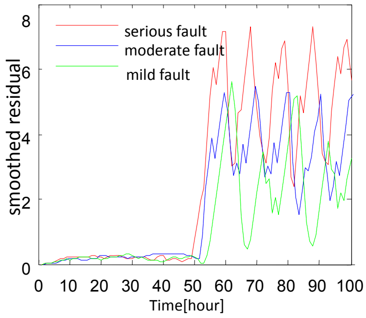
(c) Membership function of water level



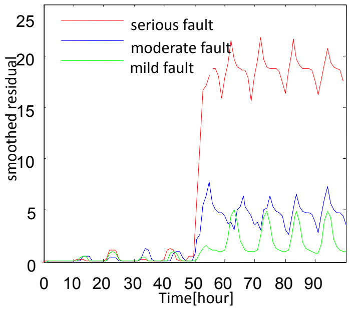
(d) Membership function of temperature

Fig. 2 Membership function

In the simulation, the particle number $N=1000$, the initial level $x_1(0)=6$m, the initial temperature $x_2(0)=10$℃, the sampling time point $k=0,1,...,100$h, and the sliding window $M=3$. The system works normally when $0\le k<50$. At the time point of $K=50$h, fault occurs on unit 3. After $K=50$h, when the water level $x_1(t)$ is lower than the setting lowest level l_1, the actual flowing rate $q_3(t)$ is less than the set flowing rate Q_3, $q_3(t)=\alpha_3(t)*Q_3$, $0\le\alpha_3(t)<1$. As shown in Eq.4, $\alpha_3(t)=1$ when $x_1(t)<l_1$ with free fault. When $x_1(t)<l_1$, different $\alpha_3(t)$ values in $0\le\alpha_3(t)<1$ reflect different fault levels. Mean values of residuals for water level and temperature with different $\alpha_3(t)$ values are shown in Fig. 3,(a) and (b). Red solid lines, blue dash lines and green dash dot lines represent the mean values of smoothed residuals for water level and temperature on the condition of $\alpha_3(t)=0$, $\alpha_3(t)=0.4$, and $\alpha_3(t)=0.8$ respectively. According to the previous definition of fault ③, the three lines represent serious, moderate and mild fault respectively. Fig. 3,(a)-(b) both indicate that smoothed residuals have an abrupt increase at the fault occurrence time point of $k=50$h, with drastic change in the fault existence period. Changes of smoothed residual are used to detect whether a fault occurs, and the size of smoothed residual is used to determine fault levels.

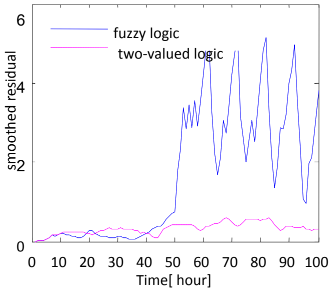


(a) Smoothed residual of water level

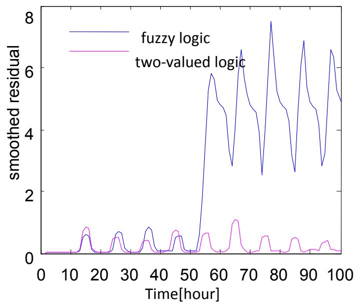


(b) Smoothed residual of temperature

Fig. 3 Smoothed residual



(a) The diagnosis results for water level



(b) The diagnosis results for temperature

Fig. 4 The diagnosis results of two-valued logic and fuzzy logic

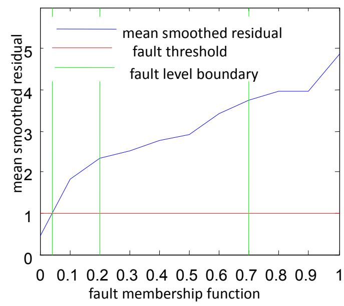
When the moderate fault occurs on the condition of $\alpha_3=0.4$, corresponding to blue dashed lines in Fig. 3(a)-(b), two diagnosis methods based on fuzzy logic and two-valued logic are adopted for the system. Assuming the diagnosis rules for the method based on two-valued logic is that diagnosis decision showing the occurrence of fault is made if $\alpha_3 \neq 1$; The diagnosis decision showing free fault is made if $\alpha_3=1$. The diagnosis thresholds of smoothed residual for water level and temperature are both 1. As shown in Fig. 4(a)-(b), the blue dashed lines are the diagnosis results of the method based on fuzzy logic. It is larger than the threshold, which indicates the occurrence of fault. The magenta dash dotted lines are the diagnosis results of the method based on two-valued logic. It is less than the threshold, which indicates free fault.

Mean value of smoothed residual $R=\frac{1}{n-m+1}\sum_{k=m}^{n} d_k$. In the simulation, $m=50$, $n=100$, so $R=\frac{1}{51}\sum_{k=50}^{100} d_k$. When the water level $x_1(t)$ is lower than the setting lowest level l_1, turning-on function of unit 3 is in fault at time point of $k=50$h just as described in fault ③. When switch state $\alpha_3=0$, the value of the fault membership function that describes the fault level of turning-on function is 1, and when $\alpha_3=1$, the fault membership function becomes 0. When unit 3 is turned on, with the fault levels from serious to mild, fault membership function decreases from 1 to 0, with the step-length of 0.1 (that is, 1,0.9,0.8,0.7…,0), corresponding to that switch state α_3 equals 0, 0.1, 0.2, 0.3,…,1.

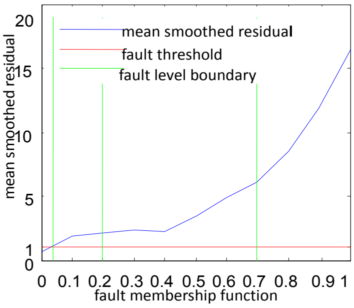
Table 3 The relationship between fault membership function(MF) and α_3

α_3	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
MF	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

The mean value of smoothed residuals for water level and temperature are shown in Fig. 5(a)-(b). The mean value of smoothed residual is selected as the diagnosis criterion. The two red lateral lines represent threshold of the mean value of smoothed residuals, with the setting value of 1. According to Fig. 2(c), the mean value of smoothed residual for water level falling into the range of [0, 1), [1, 2.4), [2.4, 3.7) and [3.7, 5] describes free, mild, moderate and serious fault, corresponding to fault membership function falling into the range of [0, 0.04), [0.04, 0.2), [0.2, 0.7) and [0.7, 1] respectively in Fig. 5(a),(b). As shown in Fig. 5(a),(b), three green dashed lines partition horizontal axis namely the fault member function into four parts, representing free, mild, moderate and serious fault from left to right respectively. In Fig. 5(a) for water level, when fault membership function equals 1, mean smoothed residual $R=4.873$ which falls into the range [3.7,5), hence diagnosis decision showing the occurrence of serious faults is made; likewise, when fault membership function equals 0.5, $R=2.924$ (within [2.4, 3.7)), then middle fault diagnosis decision is made; when fault membership function equals 0.1, $R=1.81$ (within[1, 2.4)), then mild fault diagnosis decision is made; when fault membership function equals 0, $R=0.4518$ (within (0, 1)), then the diagnosis decision indicate that unit 3 works normally. According to the Table 3, when fault membership function equals $1,0.5,0.1,0$, $\alpha_3=0,0.5,0.9,1$ respectively. Corresponding to mean value of smoothed residual range (0, 4.873] for water level, the range for temperature is (0, 16.46]. The analysis of mean value of smoothed residual for temperature is similar to that for water level. The mean value of smoothed residual reflects fault level quantitatively. The mean smoothed residual increases with rise of fault membership function.



(a) Mean smoothed residual of water level



(b) Mean smoothed residual of temperature

Fig. 5 Mean smoothed residual

Conclusion

This diagnosis method based on particle filter and fuzzy logic proposed in this paper can describe gradually changing process of fault and demonstrate transitional boundaries. The simulation results indicate that the detection performance of this new method is superior to that based on two-valued logic. The diagnosis result includes fault level information. The diagnosis method based on integrated fuzzy logic and particle filter is also applicable to general non-linear systems with non-Gaussian noise.

Fault isolation would require further investigations, which is as important as fault detection. In addition, another problem is what response suggestion will be proposed according to the results of fault detection and isolation, such as stop production or maintain on-line. Further work needs to be done to solve the problems.

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REFERENCES

[1] D.R.Espinoza-Trejo,D.U.Campos-Delgado. Detection and isolation of actuator faults for a class of non-linear systems with application to electric motors drives, IET Control Theory and Applications. , 3(2009), No.10, pp.1317-1329.
[2] Mingxing Jia, Xiaoping Guo, Chunhui Zhao, Dong Xiao. Nonlinear Fault Diagnosis based on RBF with Sliding Window Error Feedback, Proc. Int. Conf. on Computational Engineering in Systems applications, Beijing, China, October (2006), pp. 1980-1983.
[3] H. Xue, J.G. Jiang. Fault Detection and Accommodation for Nonlinear Systems Using Fuzzy Neural Networks, Proc. 5th Int. Conf. Power Electronics and Motion Control, (2006), pp.1-5.
[4] Frank P.M. Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge-based Redundancy—A Survey and Some New Results, Automatica, 26(1990),no.3: 459-474.
[5] B.D.Anderson, J.B.Moore. Optimal Filtering, Englewood Cliffs, NJ: Prentice-Hall, (1979).
[6] M.Sanjeev Arulampalam, Simon Maskell, Neil Gordon, Tim Clapp. A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking, IEEE TRANSACTION ON SIGNAL PROCESSING, 50(2002), no.2, pp. 174-188.
[7] Petar M.Djuric, Jayesh H.Kotecha, Jianqui Zhang, Rufei Huang, Tadesse Ghirmai, Monica F.Bugallo, Joaquin Miguez. Particle Filtering, IEEE SIGNAL PROCESSING MAGZINE, September, pp.19-38. (2003)
[8] Fredrik Gustafsson, Fredrik Gunnarsson, Niclas Bergman, Urban Forssell, Jonas Jansson, Rickard Karlsson, Per-Johan Nordlund. Particles Filters for Positioning, Navigation, and Tracking, IEEE TRANSACTIONS ON SIGNAL PROCESSING, 50(2002),no.2, pp. 425-437.
[9] V.Kadirkamanathan, P.Li, M.H.Jaward, S.G.Fabri. A sequential Monte Carlo filtering approach to fault detection and isolation in nonlinear systems, Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia, December (2000), pp.245-250.
[10] V.Kadirkamanathan, P.Li, M.H.Jaward, S.G.Fabri. Particle filtering-based fault detection in non-linear stochastic systems, International Journal of Systems Science, 33(2002),no.4, pp.259-265.
[11] LIANG J, QIAO L Y, PENG X Y. Fault Detection Based on SIR State Estimation and Smoothed Residual(in Chinese), ACTA ELECTRONICA SNICA, 35(2007),no.12, pp.32-36.
[12] LIANG J, PENG X Y. A particle filter with the resampling based on the similarity of observation path for single station passive target tracking with bearing-only measurement' (in Chinese), Journal of Electronic Measurement and Instrument. 23(2009),no.2, pp.10-14.
[13] Gordon N, Salmond D. Novel approach to nonlinear and non-Gaussian Bayesian state estimation, Proc. Institute Electric Engineering, 140(1993),no.2, pp.107:113.
[14] James V.Candy. Bootstrap Particle Filtering, IEEE SIGNAL PROCESSING MAGAZINE, 24(2007),no.4, pp.73-85.
[15] L.A.Zadeh. Fuzzy Logic, Information Control, 8(1965),no.3, pp.338-353.
[16] Christopher J. White , Heba Lakany. A fuzzy inference system for fault detection and isolation: Application to a fluid system. Expert Systems with Applications. 35(2008),no.3,pp.1021-1033.
[17] Mendonca.L.F, Sousa.J.M, Sa da Costa, J.M.G. Fuzzy model-based fault detection and isolation, Proceedings of IEEE Conference on Emerging Technologies and Factory Automation, 2, pp.768-774. (2003).
[18] T.Kailath, A.H.Sayed, B.Hassibi. Linear estimation, in Information and System Sciences. Upper Saddle River, NJ: Prentice-Hall, (2000).
[19] Peng Wang, Xiao Ming Chen, Tunc Aldemir. DSD: a generic software for model-based fault diagnosis in dynamic systems, Reliability Engineering and System Safety. 75(2002), pp.31-39.
[20] Marzio Marseguerra, Enrico Zio. Monte Carlo simulation for model-based fault diagnosis in dynamic systems, Reliability Engineering and System Safety. 94(2009), pp.180-186.

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