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Reconstructing wide-band system response from full-wave electromagnetic simulation by adaptive frequency sampling and implicit rational macromodeling

Abstract. *System performance evaluation over a broad frequency range is a crucial issue in many electromagnetic engineering problems. In this paper, a simple adaptive frequency sampling scheme employing interval bisection technique combined with the Cauchy method supported by Stöer–Bulirsch algorithm is examined as a tool for interpolating wide-band system responses from electromagnetic (EM) simulation. Emphasis is placed on examining the convergence of the method. The overall approach does not require matrix inversion, is derivative-free and capable to perform interpolation over a wide frequency band with a single implicitly constructed rational interpolant. The relevant algorithms are easily interfaced with existing EM simulators. Good performance of the method in terms of computational efficiency and accuracy is demonstrated in the context of the method of moments on a sample numerical example involving broadband evaluation of the impedance of a wire radiator.*

Streszczenie. *Określenie właściwości systemu (obiektu) w szerokim zakresie częstotliwości jest kluczowym problemem w wielu zagadnieniach inżynierii pola elektromagnetycznego. W tym artykule omawiamy prosty algorytm częstotliwościowego próbkowania adaptacyjnego, oparty na połowieniu przedziału i połączony z metodą Cauchy'ego interpolacji wymiernej, jako skuteczną metodę szerokopasmowej interpolacji/ekstrapolacji wyników pełnofalowej analizy elektromagnetycznej obiektu promieniującego. Makromodel wymierny wysokiego rzędu jest konstruowany implicite za pomocą algorytmu Stöera-Bulirscha bez konieczności rozwiązywania układu równań na współczynniki funkcji interpolującej i obliczania pochodnych. Wszystkie cząstkowe algorytmy interpolacji łatwo dołączyć do istniejących software'owych symulatorów elektromagnetycznych. Skuteczność omawianego podejścia w połączeniu z metodą momentów zademonstrowano na przykładzie szerokopasmowej analizy prostej anteny cienkoprzewodowej.* ***(Rekonstrukcja szerokopasmowej odpowiedzi systemu z wyników pełnofalowej symulacji elektromagnetycznej z zastosowaniem częstotliwościowego próbkowania adaptacyjnego i makromodelowania wymiernego)***

Keywords: wideband analysis, rational interpolation, adaptive frequency sampling, Stöer-Bulirsch algorithm, method of moments
Słowa kluczowe: analiza szerokopasmowa, interpolacja wymierna, czestotliwościowe próbkowanie adaptacyjne, algorytm Stöera-Bulirscha, metoda momentów

Introduction

Prediction of electromagnetic behavior of radiating/scattering structures over a broad frequency range is essential in many electromagnetic engineering problems. Problems of this kind are usually hardly tractable by analytical methods and therefore their solution heavily depends on skilful numerical modeling and efficient wide-band electromagnetic (EM) simulation. The most straightforward approach to the problem formulated in the frequency domain (FD) consists in point-by-point frequency swept computations, i.e., evaluating samples of the required physical observable over a predefined set of uniformly spaced frequency nodes. The approach is considered computationally inefficient, since EM simulation must be performed repeatedly at many frequencies and the computational cost becomes prohibitive for complex structures. Therefore, techniques to minimize the computation time and reduce the overall computational cost of broadband simulation have received considerable attention over the last years. Among the relevant approaches is the Cauchy method or rational-function interpolation based upon the assumption that the system response can be represented by a rational polynomial [1] – [6]. In this paper, the Cauchy method is considered as a derivative-free data-driven rational macromodeling technique. Thus, we assume that the order of the polynomials and their coefficients are evaluated from given values of data samples available at selected frequency points. The method is fundamentally different from model-based approaches, like the one described in [6], in that these latter require availability of the full set of model equations to construct a macromodel. The performance of a macromodel developed via the Cauchy method is sensitive to the selection of data points. Since the frequency behavior of an investigated system is generally not known in advance, the process usually involves adaptive procedures for selecting support points and model order. For higher interpolation orders, the approach suffers from ill-conditioning of a set of equations resulting from interpolatory conditions for the coefficients of the rational polynomial. A reasonable alternative is the use of a more stable and computationally efficient recursive tabular algorithm developed by Stöer and Bulirsch (SB) [7] – [10]. The algorithm can be combined with a simple bisection (or binary search) procedure providing a conceptually simple solid base for adaptive frequency sampling (AFS) over a broad frequency band. A major advantage of the approach is that both, the SB algorithm and bisectional sampling are exceptionally easily interfaced with existing EM simulation programs. Although the approach has proven its usefulness in solving a demanding EMC benchmark problem [11] its weakness can be perceived in relatively slow convergence of the bisection method calling into question the convergence and thus computational cost of the entire AFS-SB approach. This crucial issue is addressed in this paper. The aim of the paper is to investigate the convergence of the bisectional AFS combined with SB algorithm simultaneously with demonstrating usefulness and effectiveness of the approach in the context of the method of moments (MoM) for problems involving wide-band simulation of antennas. The paper is structured as follows. After introductory notes of this Section, the issues specific to the implementation of the interval bisection method, Stöer–Bulirsch algorithm and their combination are addressed. The paper is completed by presenting numerical results obtained from an in-house MoM code for a multi-resonant wire radiator, together with characterization of the performance of the AFS-SB method.

Basic Adaptive Frequency Sampling Scheme

An adaptive frequency sampling (AFS) scheme employed in this study is perhaps the most natural and straightforward one, and consists in performing repetitive bisection (halving) of the frequency interval(s) until a specified convergence criterion is met for the observable, $H(f)$, at the midpoint(s). This sampling strategy, allowing for the frequency step to be locally refined with no limit, is very well-suited to the analysis of highly resonant structures with sharp resonant peaks in their frequency response that could possibly remain uncaptured or resolved inaccurately in the adaptive process in which the location of samples is restricted to a predefined uniform (even fine) grid of testing points.

A preparatory step to the AFS process involves the computation of the observable samples $\{H_i;\ i = 0, 1, \ldots, K\}$ via "rigorous" EM simulation at discrete frequencies $\{f_i;\ i = 0, 1, \ldots, K\}$ uniformly spaced by a coarse initial frequency step Δf over the whole frequency band of interest. The initial frequency step size is determined from the condition that a phase change corresponding to Δf along the largest linear distance within a structure does not exceed π. The frequency intervals are then successively processed as follows. The interval $[f_{i-1}, f_i]$ is halved and a new frequency sample, H_m, is computed via EM simulation at the midpoint f_m. Then the 2-point trapezoidal rule involving the endpoints f_{i-1} and f_i, and the 3-point extended trapezoidal rule employing f_{i-1}, f_m and f_i are applied over the interval $[f_{i-1}, f_i]$. The convergence criterion is based upon the absolute relative difference, ϵ, between the obtained results, and can be expressed in the form

$$\epsilon = \left| \frac{H_m - 0.5(H_{i-1} + H_i)}{H_m + 0.5(H_{i-1} + H_i)} \right| \leq \delta \tag{1}$$

where δ is the required convergence tolerance. The bisection process continues within $[f_{i-1}, f_i]$ until convergence is reached, and then the procedure goes to the next frequency interval. A major drawback of the bisection-based AFS is that the method is relatively slow and often needs a large number of samples to meet the convergence criterion. The efficiency of the technique can be noticeably improved by employing some kind of interpolation of the quantity of interest, and this can be accomplished by employing the Cauchy method and Stöer–Bulirsch algorithm.

Cauchy Method and Stöer–Bulirsch Algorithm

Application of the Cauchy method in the present context consists in finding a rational function that fits the support points $\{f_i, H_i;\ i = 0, 1, \ldots, N\}$. Such a function corresponds to a general $[L/M]$ rational form defined as a ratio of two polynomials of the orders L and M, respectively, that is [1], [2], [4], [7].

$$R(f) \equiv [L/M] = \frac{p_0 + p_1 f + \cdots + p_L f^L}{1 + q_1 f + \cdots + q_M f^M} \tag{2}$$

The interpolant $R(f)$ can in principle be constructed by solving a set of linear equations for the coefficients $\{p_i; i = 0, \ldots, L\}$ and $\{q_j; j = 1, \ldots, M\}$ resulting from fitting $R(f)$ to the data points provided that $L + M + 1 = N + 1$. However, instead of constructing the interpolant explicitly, it may be reasonable and practical to calculate repeatedly the value of $R(f)$ over a finite set of desired argument values (frequencies). This can be attained by employing a highly efficient recursive tabular Neville–type algorithm developed by Stöer and Bulirsch (SB) [8]. The SB algorithm is pointed at evaluating the value of the interpolating rational function (R) for a single value of its argument (f). The strength of the algorithm follows from that it does not require matrix inversion and can process large number of supporting points without suffering from ill-conditioning. The SB algorithm generates the so-called diagonal rational interpolant with the degree of the numerator and denominator equal (if N is even) or with the degree of the denominator larger by one (if N is odd) [7], [8], [9], and can be summarized by the following recurrence

$$\begin{array}{cccccccc}
 & R_{0,0} & & & & & & \\
 & & \searrow & & & & & \\
R_{0,-1} & \longrightarrow & & R_{1,1} & & & & \\
 & & \nearrow & & \searrow & & & \\
 & R_{1,0} & & \longrightarrow & & R_{2,2} & & \\
 & & \searrow & & \nearrow & & \searrow & \\
R_{1,-1} & \longrightarrow & & R_{2,1} & & \longrightarrow & & R_{3,3} \\
 & & \nearrow & & \searrow & & \nearrow & \\
 & R_{2,0} & & \longrightarrow & & R_{3,2} & & \\
 & & \searrow & & \nearrow & & & \\
R_{2,-1} & \longrightarrow & & R_{3,1} & & & & \\
 & & \nearrow & & & & & \\
 & R_{3,0} & & & & & & \\
\vdots & \vdots & & \vdots & & \vdots & & \ddots
\end{array}$$

Fig. 1. Tabular diagram of Stöer–Bulirsch algorithm

relation [8]:

$$R_{i,k} = R_{i,k-1} + \frac{R_{i,k-1} - R_{i-1,k-1}}{\frac{f - f_{i-k}}{f - f_i}\left[1 - \frac{R_{i,k-1} - R_{i-1,k-1}}{R_{i,k-1} - R_{i-1,k-2}}\right] - 1} \tag{3}$$

starting with the initial conditions

$$R_{i,-1} = 0, \qquad R_{i,0} = H_i \qquad i = 0, 1, \ldots, N \tag{4}$$

A tabular diagram shown in Fig. 1 explains the recursive process. The first two columns of the triangle table correspond to the initial conditions (4) with the data points in the second column. The arrows indicate how the subsequent column elements are constructed recursively according to (3) from their neighbors in the previous columns.

AFS–SB Technique

The basic AFS approach described above is easily combined with SB algorithm to give the AFS-SB technique employing the following convergence criterion:

$$\epsilon = \left| (\widehat{H}_m - H_m)/H_m \right| \leq \delta \tag{5}$$

where H_m and $\widehat{H}_m = [L/M]$ are the sample values at f_m calculated from EM simulation and from (2) via SB algorithm, respectively. To investigate a possibility of reducing the EM simulation overhead, we also examined the AFS-SB scheme in which the AFS process is controlled by error surrogates, i.e., by differences between $\widehat{H}_m$ and another two approximate values for H at f_m derived from two auxiliary rational fitting models constructed over the same set of supporting points as that for $[L/M]$. After performing extensive numerical tests, two auxiliary rational interpolants with the degrees of the numerator and denominator differing by two from those for $[L/M]$ were selected on the basis of a reasonable tradeoff in terms of accuracy and the required number of samples. To be specific, $\widehat{H}^+_m = [L + 2/M - 2]$ and $\widehat{H}^-_m = [L - 2/M + 2]$ are calculated at f_m, and the convergence criterion is taken in the form

$$\epsilon = \max\left(\left| \frac{\widehat{H}^+_m - \widehat{H}_m}{\widehat{H}_m} \right|, \left| \frac{\widehat{H}^-_m - \widehat{H}_m}{\widehat{H}_m} \right| \right) \leq \delta \tag{6}$$

Non-diagonal fitting models $[L+2/M-2]$ and $[L-2/M+2]$ were constructed following the procedure explained in detail in [8] and [9]. It is worth mentioning that the three models, that is, $[L/M]$, $[L + 2/M - 2]$ and $[L - 2/M + 2]$ have the same number of support points, and this reflects the same computational cost of constructing these models.

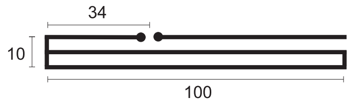

Fig. 2. Wire antenna (dimensions in centimeters; wire radius is 0.5 mm)

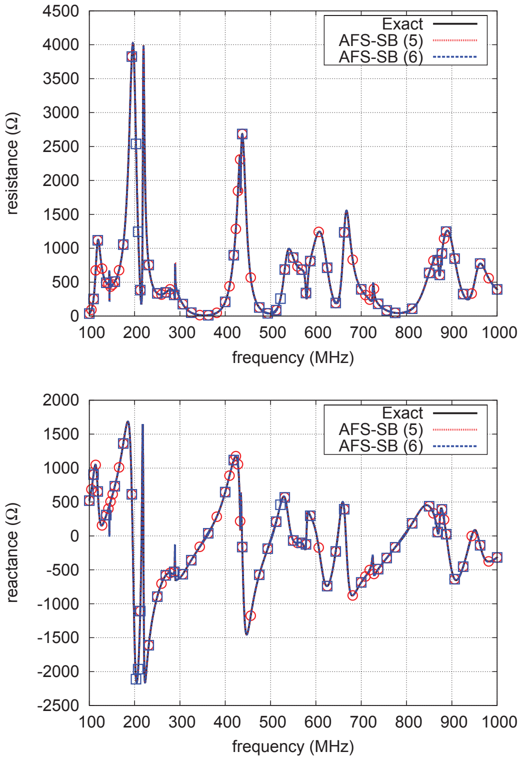

Fig. 3. Driving-point impedance of the antenna depicted in Fig. 2 evaluated via AFS-SB technique employing convergence criteria (5) and (6), respectively, with $\delta = 0.01$; squares and circles indicate locations of support points

Finally, it should be noted that all the samples evaluated via EM simulation are systematically added to the set of support points available for SB interpolation algorithm in successive steps.

Results

As a test example, a simple wire antenna shown in Fig. 2 was considered. For numerical analysis, the structure was discretized into linear segments, and the current on the antenna was expanded in terms of 159 one-dimensional RWG-type basis functions [12]. The MoM was employed as a "rigorous" EM simulation tool to find the antenna current and then the driving-point impedance of the antenna. Numerical experiments were started with evaluating the impedance via basic bisectional adaptive sampling technique already described. The procedure was initiated with the frequency step $\Delta f = 75$ MHz. The key features of the algorithm are summarized in Table 1. As can be seen from the Table, the bisectional AFS algorithm generates a rather large number of samples until it reaches the convergence criterion. For instance, to meet the convergence tolerance of 0.0001, the algorithm generated 7125 samples spaced nonuniformly with the initial frequency resolution $\Delta f = 75$ MHz locally refined by a factor up to 2^{17} resulting in $\Delta f \approx 572$ Hz. The results for $\delta = 0.0001$ are taken in the proceeding as numerically exact MoM reference results.

Fig. 3 shows the AFS-SB approximate results for the input impedance of the antenna over a decade frequency bandwidth from 100 MHz to 1 GHz. The frequency range from 140 MHz to 300 MHz covering three closely spaced sharp resonances of the antenna is magnified in Fig. 4. The results derived from AFS-SB algorithm employing the criteria (5) and (6), respectively, are compared in the Figures against those taken as numerically exact reference. As can be seen, the three sets of results are in excellent agreement, and the corresponding impedance runs are practically indistinguishable.

Table 1. Features of bisectional AFS algorithm with convergence criterion (1)

Convergence tolerance	Number of EM samples	Local refinement factor of initial Δf
0.1	152	2^8
0.01	667	2^{11}
0.001	2273	2^{15}
0.0001	7125	2^{17}

Table 2. Characteristics of numerical solutions obtained via AFS-SB method with convergence criterion (5)

Convergence tolerance	Number of EM samples	Maximum relative interpolation error, dB
0.1	51	-21
0.01	69	-56
0.001	83	-64
0.0001	121	-69

Tables 2 and 3 offer additional insight into properties of numerical solutions obtained with the use of the adaptive sampling schemes for different convergence tolerances. The same reference data were taken for the results in the Tables as those for the results in Figs. 3 and 4. In particular, the relative interpolation error (defined on a logarithmic scale as $20 \log \epsilon$) was checked at each of 7125 frequency nodes generated by the bisectional AFS scheme for $\delta = 0.0001$.

To complement characterization of distinctive features of the AFS-SB approach, Figures 5 and 6 display portions of the impedance curves of Fig. 3 enlarged around the frequencies 144.5 MHz and 725 MHz where the relative error of the interpolation provided by AFS-SB algorithm takes its maximum local values, i.e., -8.6 dB and -20.4 dB, respectively. The inspection of the results clearly indicates the usefulness and efficiency of the considered AFS-SB technique. The accuracy of about 1% (or the relative interpolation error of -40 dB) and better, which seems to be sufficient for most antenna applications, is easily attainable with a number of support points usually several ten times smaller than that required by frequency sampling over a uniform grid. Also, it is worth mentioning that our numerical experiments support the observation reported in [9] that the sub-zoning technique results in increasing the total number of sampling points compared to that required for construction of a single interpolant over the whole frequency band in question.

Table 3. Characteristics of numerical solutions obtained via AFS-SB method with convergence criterion (6)

Convergence tolerance	Number of EM samples	Maximum relative interpolation error, dB
0.1	22	> 0
0.01	46	-8.6
0.001	54	-38
0.0001	67	-75

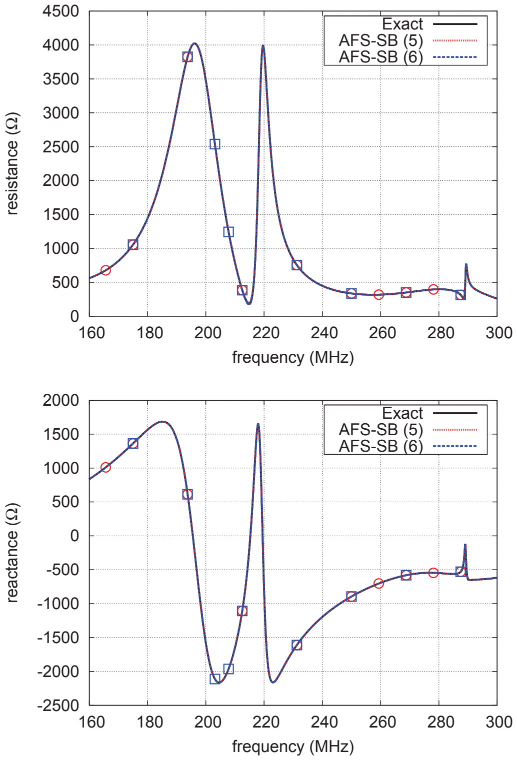

Fig. 4. Driving-point impedance around 230 MHz of the antenna depicted in Fig. 2 evaluated via AFS-SB technique employing convergence criteria (5) and (6), respectively, with $\delta = 0.01$

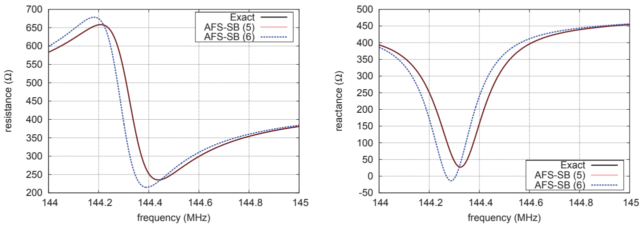

Fig. 5. Driving-point impedance around 144.5 MHz of the antenna of Fig. 2 evaluated via AFS-SB method employing convergence criteria (5) and (6), respectively, with $\delta = 0.01$

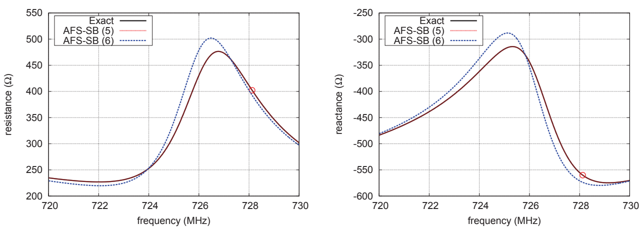

Fig. 6. Driving-point impedance around 725 MHz of the antenna of Fig. 2 evaluated via AFS-SB method employing convergence criteria (5) and (6), respectively, with $\delta = 0.01$

Conclusion

The dynamic adaptive frequency sampling method employing the bisection algorithm (also called the binary search algorithm) supported by the Neville-type recursive tabular Stöer-Bulirsch rational interpolation algorithm was investigated in this paper as a tool for generating wide-band data from the frequency-domain electromagnetic simulation. Despite possible skepticism about the convergence of the method justified by well-known slow convergence of the bisection algorithm, numerical experiments gave evidence for efficiency of the overall approach and its capability of capturing even sharp isolated resonant peaks at substantially reduced computational effort compared to that of conventional uniform sampling. The method belongs to the class of "black-box" techniques, and the only information needed to construct a wide-band approximation of the response of a system is frequency samples of the observable. The approach offers a possibility of controlling the required accuracy of the approximation. Moreover, the approach does not require matrix inversion and is derivative free, and can be easily incorporated into existing computer codes. Therefore, we employ the described technique as the in-house standard in routine wide-band electromagnetic simulations.

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