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Application of State Space Search Method to find a Low Voltage Solution for Ill-Conditioned System

***Abstract**. This paper uses the State Space Search Method (SSSM) in polar coordinate form to obtain low voltage solution and maximum loading point of ill-condition power system. SSSM improves the direction of state variables (buses voltage and phase) of system buses based on optimal multiplier to converge load flow equations in ill-conditioned system. The advantage of SSSM is apparent in constant preservation of dimension of Jacobian matrix in load flow equations. Whereas another approaches such as Homotopy and continuation power flow vary the framework of Jacobian matrix based on predictor and corrector elements during enhancing load demand. The calculation procedure of SSSM is depending on classical Newton-Raphson load flow method. The reliability of SSSM is indicated by IEEE test systems, 14 and 30 buses in well and ill-conditioned at maximum loading point as systems.*

***Streszczenie.** W artykule opisano sposób wykorzystania metody przeszukiwania przestrzeni stanów we współrzędnych biegunowych, w celu uzyskania rozwiązań niskonapięciowych oraz punktu maksymalnego obciążenia w systemach energetycznych oraz źle uwarunkowanych równaniach stanu. Metoda zwiększa poprawność doboru zmiennych stanu systemu poprzez wyznaczenie optymalnego współczynnika skupienia równań przepływu mocy do obciążenia w systemie. Obliczenia oparto na metodzie Newton'a-Raphson'a określania przepływu mocy. (**Implementacja metody przeszukiwania przestrzeni stanów w poszukiwaniu rozwiązań niskonapięciowych w sieciach o źle uwarunkowanych równaniach stanu**)*

Keywords: State Space Search Method, Low Voltage Solution, Optimal Multiplier, Ill-Conditioned System, Maximum Loading Point.
Słowa kluczowe: metoda przeszukiwania przestrzeni stanów, optymalny współczynnik, złe uwarunkowanie równań stanu systemu, punkt maksymalnego obciążenia.

Introduction

Due to the economic and environmental problems power system have been forced to operate near to its maximum capacity [1,2]. These problems are apparent in construction of new generation plants, new transmission lines and the enhancing load demand. These difficulties from available power system viewpoint are emerged in approaching to Maximum Loading Point (MLP), transmission lines with high *R/X* ratio and bus connections have a very high resistance and very low impedance. The power system is become ill conditioned system by aforesaid causes [3]. The effect of the load demand increasing factor is in ill conditioned system is more from another mentioned causes [2,3]. Therefore, the voltage collapse phenomenon is occurred near to maximum loading ability in ill conditioned system [4]. Voltage collapse phenomenon can be considered as static issue Instead of dynamic problem, if the power systems variables change slowly [5]. The slow changing of theses parameter is corresponding to a small load enhancement in power system.

Hence, the study of voltage stability in steady state mode is considered by a set of nonlinear algebraic equations as the power flow equations [6]. In this regards, a individual approaches have been presented based on power flow equations to determine load flow solutions in ill-conditioned system. Under this condition, the power flow equations have Low Voltage Solution (LVS) or solution Type-1 as unstable equilibrium operation point. Whereas, load flow solution is called High Voltage Solution (HVS) as interesting power flow solution [7].

In this context, two main approaches have been introduced to obtain LVS's, the State Space Search Methods (SSSM) and path following methods (PFM) [7, 8]. PFM implements the trajectory of load demand enhancing in PV and QV curves to find MLP and beyond it [9]. This method is robust and exact for calculating LVS at MLP [10]. However, the convergence of PFM needs several iterations iteration and previous information of the direction of loading trajectory [7]. Furthermore, the performance of PFM beyond MLP in unsolvable condition is weak [11]. The PFM is classified as the Homotopy Methods (HM) and the Continuation methods (CM) [9, 11, and 12]. On the other hand, the calculation of LVS in SSSM does not need a large number of iteration and any prior information of the load demand direction in PV or QV curves [13]. The SSSM is based on complete Taylor-series expansions of Newton-Raphson Load Flow Method (NRLFM) that associated with optimal multiplier [13, 14]. The optimal multiplier is used as accelerator damper to improve the convergence of the load flow equations in ill-conditioned systems [15].

In this paper a new approach of SSSM is proposed. Indeed, the closet optimal multiplier to one is selected as the desirable optimal multiplier to detect LVS's at MLP. Based on traditional SSSM, the smallest optimal multiplier has been selected. However, SSSM can be fallen in blind search to find LVS at MLP by choosing the smallest optimal multiplier. The validation of the proposed SSSM is presented by IEEE test systems of 14 and 30 buses in well and maximum loading point as ill-conditioned system.

Improved State Space Search Method

The power flow equations as a set of nonlinear equations can be written as:

$$F(X,H)=0 \tag{1}$$

where: X – Vector of uncontrolled (state) variables, H – Vector of controlled variables (active and reactive power of buses), F – Load flow function.

The equation (1) includes only two variables; bus voltage amplitude (X_a) and its angle (X_b) at buses as independent variables in polar coordinate form. Since, Taylor series, expansion of equation (1) based on these state variables is formulated as follow:

$$F(X_a{}^i+\Delta X_a{}^i, X_b{}^i+\Delta X_b{}^i)-F(X_a{}^i, X_b{}^i)-\left[\Delta_{X_a,X_b}F\right]^i \left[\Delta X_a{}^i,\Delta X_b{}^i\right]-\frac{1}{2}\left[\Delta X_a{}^i,\Delta X_b{}^i\right]^T\left[\Delta^2{}_{X_a,X_b}F\right]^i\left[\Delta X_a{}^i,\Delta X_b{}^i\right]=0 \tag{2}$$

The classical Newton-based methods can solve equation (2) by neglecting the second term equation (2). However, in ill-conditioned system first initial estimation of Newton-Based methods state variables such as Newton Raphson Load Flow Method (NRLFM) is far away from real solution. In this context, the State Space Search Method (SSSM) modifies the direction of state variables in state space from first initial estimation in order to find best stable solution or low voltages solutions [7, 13]. Therefore, the modification of next step of state variables is

(3) $$X^{i+1} = X^i + \lambda \Delta X^i$$

where: λ – is multiplier damper that is used to modify the mismatch vector of state variables.

Based on nonlinear programming technique as in (3), equation (2) can be rewritten by using λ as in equation (4).

(4) $$F(X_a{}^i + \Delta X_a{}^i, X_b{}^i + \Delta X_b{}^i) - F(X_a{}^i, X_b{}^i) - \lambda^i \left[\Delta_{X_a, X_b} F\right]^i \left[\Delta X_a{}^i, \Delta X_b{}^i\right] - \frac{1}{2}\lambda_i{}^2 \left[\Delta X_a{}^i, \Delta X_b{}^i\right]^T \left[\Delta^2{}_{X_a, X_b} F\right]^i \left[\Delta X_a{}^i, \Delta X_b{}^i\right] = 0$$

Under this condition the multiplier cost function in equation (5) is defined as objective function in order to obtain the correction value of state variable

(5) $$C = \frac{1}{2}\Big(F(X_a^i + \Delta X_a^i, X_b^i + \Delta X_b^i) - F(X_a^i, X_b^i) - \lambda^i \left[\Delta_{X_a, X_b} F\right]^i \left[\Delta X_a^i, \Delta X_b^i\right] - \frac{1}{2}\lambda_i{}^2 \left[\Delta X_a{}^i, \Delta X_b{}^i\right]^T \left[\Delta^2{}_{X_a, X_b} F\right]^i \left[\Delta X_a{}^i, \Delta X_b{}^i\right]\Big)^2 = 0$$

Equation (5) can be presented in the following form:

(6) $$C = \frac{1}{2}\left[D + E\lambda + \lambda^2 G\right]^T \left[D + E\lambda + \lambda^2 G\right] \rightarrow$$

$$C = \frac{1}{2}[D^T D + 2D^T E\lambda + (E^T E + 2G^T D)\lambda^2 + 2E^T G\lambda^3 + GG^T \lambda^4]$$

Where

$$D = F(X_a{}^i + \Delta X_a{}^i, X_b{}^i + \Delta X_b{}^i) - F(X_a{}^i, X_b{}^i)$$

$$B = \left[\Delta_{X_a, X_b} F\right]^i \left[\Delta X_a{}^i, \Delta X_b{}^i\right]$$

$$C = \frac{1}{2}\lambda_i{}^2 \left[\Delta X_a^i, \Delta X_b^i\right]^T \left[\Delta^2 X_a, X_b F\right]^i \left[\Delta X_a^i, \Delta X_b^i\right]$$

Iba et al. proved that the optimum points of multiplier cost function as quartic function in equation (6) has a two minimum points and one maximum point [14]. The assumption illustration of equation (6) is depicted in Fig.1. In well conditioned system, the classical newton-based method can only detect on these minimum points as interested solution point called the high voltage solution (HVS). For this case λ is equal to one. In ill-conditioned system, by increasing the load demand of system, two minimum points of equation (6) are forced to get closer to each other and meet at MLP located at maximum point of Fig.1. For this condition the closet value of λ to one is selected as the optimal multiplier to find LVS. Furthermore, it can be observed that equation (6) is based on NRLFM.

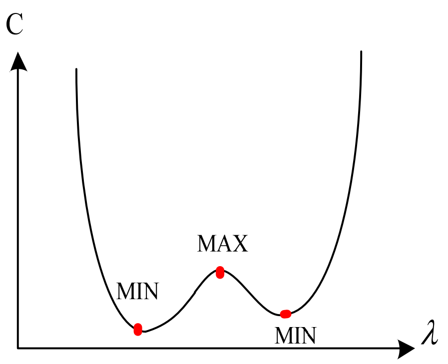

Fig.1. Illustration of assumption multiplier cost function respect λ

However, optimal multiplier λ is computed by minimizing equation (6) respect with λ as

(7) $$\frac{dL}{d\lambda} = D^T E\lambda + (D^T D + 2G^T D)\lambda + 3E^T G\lambda^2 + 2GG^T \lambda^3 0$$

where: $E = -D$ in order to simplify (7) [13].

The Cardan method is used to find possible real roots of this cubic equation. Equation (7) has three real roots in well-conditioned system. The numbers of real solutions of equation (7) are decreased at MLP. Equation (7) does not have any real solution in infeasible operation zone beyond MLP.

The proposed algorithm based on SSSM to calculate LVS in terms of load demand increasing is as follows:

1. Increase predefined load demands based defined step then run NRLFM.
2. If NRLFM converges go to 1 otherwise go to 3.
3. Calculate the closet optimal multiplier to one of equation (6) by using Cardan method due to calculate low voltage solutions.
4. Increase load demands based defined step
5. If that equation (7) has a real solution (as index to approaching to MLP) stop process and calculate the LVS based on this optimal multiplier otherwise go to (3).

Case Study

IEEE 14 and 30- bus test system is tested to show convergence characteristics of SSSM at MLP as ill-conditioned system. The single transmission diagram of IEEE 14 and 30 bus systems are shown in Fig.2 and Fig.3, respectively. The linear active and reactive load demand models are utilized to detect the maximum loading point of these systems as follows [15]:

(8) $$P = P_0 + \alpha P$$

(9) $$Q = Q_0 + \alpha Q$$

In equation (8) and (9), P_0, Q_0 and P, Q are the initial and actual vector of active and reactive powers. Alfa (α) involves the step size and the direction the load demand changing. In this paper the value of step size is equal to 0.001. The calculated MLP based on proposed algorithm for IEEE 14 and 30 bus systems are 3.95135 and 2.93124, respectively. The newton-based methods cannot converge in ill-conditioned system due to the fact the region of LVS if far away from their initial guess.

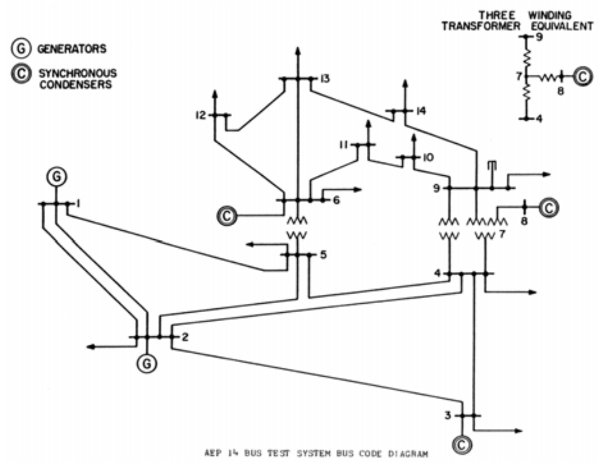

Fig.2. Transmission single line diagram of IEEE 14 bus system

The Table 1 and 2 show the divergence characteristics of NRLFM in calculating LVS's at MLP IEEE 30 and 14 bus systems respectively. Also, the drastic difference between load flow solutions in ill and well-conditioned for both systems confirms that the result of NRLFM is not at vicinity of its initial guess in ill-conditioned system.

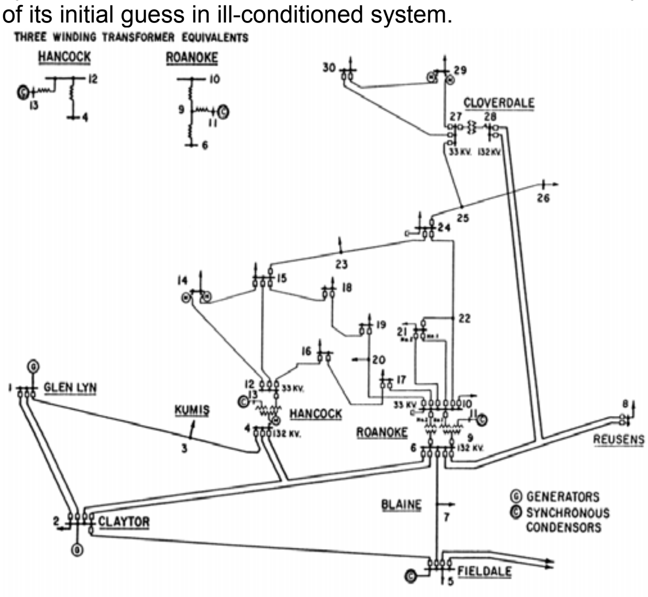

Fig.3. Transmission single line diagram of IEEE 30 bus system

On the other hand, the value of phase angle at MLP in Table 1 and 2 implies the enhancement of buses voltage angle during demand increasing declines the voltage stability margin. Due to voltage stability can be verified as a static issue if the power systems parameters change slowly.

Table 1. The performance of standard Newton-Raphson method for IEEE 30 bus system in well and ill-conditioned system

Bus Number	Well-conditioned		Ill-conditioned (MLP)	
	Voltage (P.U)	Angle (radian)	Voltage (P.U)	Angle (radian)
1	1.06	0	1.06	0
2	1.043	-0.09343	1.043	-0.47751
3	1.02098	-0.13132	0.815706	-0.58657
4	1.01206	-0.16188	0.803409	-0.75188
5	1.01	-0.24748	1.01	-1.08467
6	1.01019	-0.19351	0.852351	-0.90794
7	1.00234	-0.22487	0.885864	-1.01805
8	1.01	-0.20661	1.01	-1.00975
9	1.051	-0.25174	0.757811	-1.27762
10	1.04508	-0.28089	0.576883	-1.53443
11	1.082	-0.25174	1.082	-1.27762
12	1.05563	-0.27122	0.753158	-1.51882
13	1.071	-0.27122	1.071	-1.51882
14	1.04087	-0.28636	0.640906	-1.62638
15	1.03656	-0.28753	0.57752	-1.6264
16	1.04367	-0.27993	0.642334	-1.5502
17	1.0396	-0.28431	0.571194	-1.57242
18	1.02741	-0.29732	0.504248	-1.70584
19	1.02514	-0.29978	0.479262	-1.72491
20	1.02934	-0.29605	0.497643	-1.68255
21	1.03259	-0.28867	0.465421	-1.62291
22	1.0331	-0.28845	0.456753	-1.62505
23	1.0263	-0.29342	0.404919	-1.70571
24	1.02101	-0.29522	0.23136	-1.77448
25	1.01662	-0.28685	-0.052947	-2.0035
26	0.99892	-0.29419	-0.7675	-2.8508
27	1.02243	-0.27713	0.147969	-1.92636
28	1.00676	-0.20450	0.792666	-0.93353
29	1.00257	-0.29864	-0.150049	-10.1672
30	0.99108	-0.31407	-0.399481	-23.1452

Table 2. The performance of standard Newton-Raphson method for IEEE 14 bus system in well and ill-conditioned system

Bus Number	Ill-conditioned (MLP)		Well-conditioned	
	Voltage (P.U)	Angle (radian)	Voltage (P.U)	Angle (radian)
1	1.06	0	1.06	0
2	1.045	11.322	1.045	-0.087009
3	1.01	23.3714	1.01	-0.222258
4	11.0687	22.8167	1.01777	-0.180316
5	12.7693	18.7376	1.02	-0.153031
6	1.07	43.7231	1.07	-0.25882
7	8.19797	34.447	1.06208	-0.238405
8	1.09	34.447	1.09	-0.238405
9	9.19502	39.2377	1.05674	-0.267176
10	8.10829	40.8546	1.05171	-0.270678
11	4.70401	42.4966	1.05733	-0.26702
12	1.81008	43.787	1.05521	-0.273437
13	2.55967	2.55967	1.05055	-0.274556
14	7.44001	43.6162	1.03608	-0.287826

The performance of SSSM to find LVS at MLP is shown in Table 3. On account of value of voltage in Table 3 and 4 can be assessed that SSSM computes the possible lowest LVS. Base on this fact, the role of buses 14 and 30 in IEEE 14 and 30 bus systems as the weakest buses in ill-conditioned are same with their roles in well-conditioned of aforesaid test systems. For instance, bus 30 in IEEE 30bus system is selected as the most sensitive bus for contingency analysis for voltage collapse in ill and well-conditioned simultaneously.

Table 3. The performance of State Space Search Method for IEEE 30 bus system at maximum loading point as ill-conditioned system

Bus number	Voltage (P.U)	Angle (radian)
1	1.06	0
2	1.043	-0.413933
3	0.894945	-0.513603
4	0.889619	-0.647211
5	1.01	-0.961153
6	0.92345	-0.777013
7	0.929161	-0.885209
8	1.01	-0.848162
9	0.915832	-1.00969
10	0.833729	-1.14206
11	1.082	-1.00969
12	0.910295	-1.11679
13	1.071	-1.11679
14	0.846481	-1.18134
15	0.821531	-1.18365
16	0.850475	-1.1454
17	0.819988	-1.16101
18	0.776867	-1.22829
19	0.762978	-1.23909
20	0.777088	-1.21859
21	0.78033	-1.18114
22	0.780842	-1.18037
23	0.7615	-1.21152
24	0.714901	-1.21968
25	0.689427	-1.21004
26	0.603362	-1.26168
27	0.716639	-1.17582
28	0.905938	-0.815416
29	0.595928	-1.32425
30	0.525515	-1.46208

Table 4. The performance of State Space Search Method for IEEE 14 bus system at maximum loading point as ill-conditioned system

Bus number	Voltage (P.U)	Angle (radian)
1	1.06	0
2	1.045	-0.801241
3	1.01	-1.75153
4	0.626277	-1.48106
5	0.585695	-1.25234
6	1.07	-2.3345
7	0.750392	-2.00689
8	1.09	-2.00689
9	0.658537	-2.25809
10	0.687476	-2.31361
11	0.858552	-2.33472
12	0.972703	-2.40185
13	0.919648	-2.39885
14	0.65116	-2.47655

Multiplier cost functions in equation (6) for 14 and 30 IEEE bus systems are depicted in Fig.4 and Fig.5 respectively. A single minimum point in Fig.4 remarks that only one optimal multiplier remains for IEEE 14 bus system. The value of single optimal multiplier of IEEE 14 bus system is 0.8279903 calculated by equation (6). Truly, equation (6) for IEEE 14 bus system has an on real root 0.8279903 and two imaginary roots that are 0.349982+i0.212271 and 0.349982-i0.212271. Furthermore, the existence only one optimal multiplier at MLP shows that the LVS's meeting at MLP for IEEE 14 bus system.

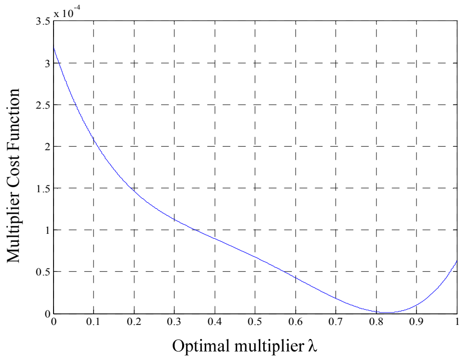

Fig.4 Illustration of multiplier cost function for IEEE 14 bus system

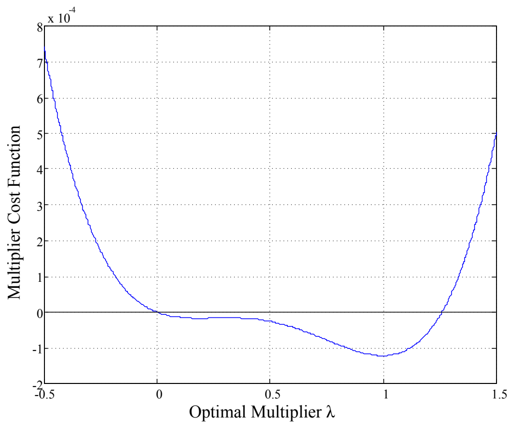

Fig.5 Illustration of multiplier cost function for IEEE 30 bus system

The close being of 0.8279903 to 1 also is shown. IEEE 30 bus system has the three optimal multiplier at MLP consist, 0.117464, 0.381862 and 0.998313. Indeed, the values of multipliers at minimum points of IEEE 30 bus system multiplier cost function are the optimal multipliers. For this case, the closest optimal multiplier to one is chosen as the desirable optimal multiplier. It can be seen the existence of multi-optimal multiplies at MLP. The important point in this context is that the value of their corresponding multiplier cost functions is very near to each other. This is because the scale of 14 and 30 IEEE bus systems multiplier cost functions are based on 0.0001.

Conclusion

This paper shows that the proposed state space search method (SSSM) is a simple, reliable and robust method to obtain Low Voltage solution (LVS) at Maximum Loading Point (MLP) in ill-conditioned system. Furthermore, it has been shown that the proposed algorithms based on SSSM can calculate LVS's and MLP simultaneously. Under this condition, the closet optimal multiplier to one is selected as the desirable optimal multiplier to detect LVS's at MLP. The efficiency of SSSM has been demonstrated by IEEE test systems of 14 and 30 buses in well and maximum loading point as ill-conditioned system.

REFERENCES

[1] B. Stott, "Review of load-flow calculation methods", *Proc. IEEE*, *vol. 62 n. 1*, , pp. 916–929.July 1974.

[2] Wang. Y, da Silva. L.C.P and Wilsun Xu, Investigation of the relationship between ill-conditioned power flow and voltage collapse, *IEEE Power Engineering Review*, pp. 43-45 *vol.20 n.4*, July 2000.

[3] Felix F. Wu "Theoretical Study Of The Convergence of The Fast Decoupled Load Flow", *IEEE Trans. Power Syst* , *vol. 96 n.2*, January 1977.

[4] Y. Tamura, H. Mori and S. Iwamoto, "Relationship Between Voltage Instability and Multiple Load Flow Solutions in Electric Power Systems", *IEEE Trans. Power App .Syst.*, *vol. 102 n. 5* pp.1115 – 1125, May 1983.

[5] A. Z. de Souza, C. A. Cañizares, and V. H. Quintana, "New Techniques to Speed Up Voltage Collapse Computations Using Tangent Vector", *IEEE Trans. Power Syst*, *vol. 12 n.3*, pp, 1380-1387August 1997.

[6] Ning Xie, Ettore Bompard, Roberto, Napoli Franco Torelli "Widely convergent method for finding solutions of simultaneous nonlinear equations". *Electric Power Systems Research journal Volume 83, Issue 1, Pages 1-266* February 2012.

[7] R. B. L. Guedes, L. F. C. Alberto and N. G. Bretas, Power system low-voltage solutions using an auxiliary gradient system for voltage collapse purposes, IEEE Trans. Power Syst, vol. 20 n.3 pp1528-1537, August 2005.

[8] F. Milano, Continuous Newton's Method for Power Flow analysis, *IEEE Trans. Power Syst*, *Vol. 24 n.1*, pp. 50-57, February 2009.

[9] Shahriari, A. ; Bakar, A.H.A. ; Mokhlis, H. "Comparative studies on Non-Divergent Load flow methods in well, ill and unsolvable condition" *IEEE Conference Power System Technology (POWERCON),* 2010.

[10] Shao-Hua Li and Hsiao-Dong Chiang, **"**Continuation Power Flow With Nonlinear Power Injection Variations: A Piecewise Linear Approximation", *IEEE Trans. Power Syst*, *vol. 23 n.4* pp. 1637 – 1643., November 2008.

[11] Y. Chen and C. Shen, " A Jacobian-free Mewton-GMRES(m) method with adaptive preconditioner and its application for power flow calculations" IEEE Trans. Power Syst., vol. 21 n. 3, August .2006, pp. 1096–1103.

[12] Yorino, N. Hua-Qiang Li Sasaki, H, A predictor/corrector scheme for obtaining Q-limit points for power flow studies, *IEEE Trans. Power Syst, vol.20 n. 2*, F, pp.130-137, February 2005.

[13] S. Iwamoto and Y. Tamura, A load flow calculation method for ill conditioned power systems, *IEEE Trans. Power App. Syst.*, *vol. 100 n .3*, , pp. 1736–1743,April1981.

[14] K. Iba, H. Suzuki, M. Egawa, and T.Watanabe, A method for finding a pair of multiple load flow solutions in bulk power systems, *IEEE Trans. Power Syst.*, *vol. 5 n.2*, pp. 582–591, May 1990.

[15] T. J. Overbye and R. P. Klump, Effective calculation of power system low -voltage solutions, *IEEE Trans. Power Syst.*, *vol. 11 n.1* pp.75-82 ., February 1996.

[16] M. D. Schaffer and D. J. Tylavsky, A nondiverging polar form Newton-based power flow, *IEEE Trans. Ind. App.*, *vol. 24 n.1*, pp.870–877.September/October 1988.

[17] L. M. C. Braz, C. A. Castro, and C. A. F. Murari , A critical evaluation of step size optimization based load flow methods, *IEEE Trans. Power Syst.*, *vol. 15 n.1*, , pp. 202–207, February 2000

[18] J. E. Tate and T. J. Overbye, A comparison of the optimal multiplier in polar and rectangular coordinates, *IEEE Trans. Power Syst.*, *vol. 20 n 4.* , pp. 1667–1674, November 2005.

[19] Huang,W.-T.;Yao,K.-C.; "New network sensitivity-based approach for real time complex power flow calculation" IET *Generation, Transmission & Distribution* 2011 Vol. 6, Iss. 2, pp. 109– 12. 2012.

[20] Available: www.ee.washington.edu/research/pstca.

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