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Equivalent Multi-Surge Impedance Model of Transmission Line Towers

Abstract: To analyze exactly the wave propagation processes of a transmission tower when lightning strikes, an equivalent model of a transmission tower was studied in this paper. First, based on the correlative theory of tapered antenna, an equivalent surge impedance model of a single stanchion of tower was made. This tower was regarded as a four-conductor system according to the real configuration of a tower, and the equivalent impedance of the four-conductor system was obtained by multiplying the single pillar impedance with the compensation factor. Combined with the definition of wave impedance regarded with inductance and capacitance, the compensation factor of the four-conductor system was introduced. Then, the bracings equivalent model was obtained based on the analysis of the equivalent model of parallel multi-conductors. In modularizing the tower model, the equivalent impedance of the tower was found. Finally, a 500 kV tower was calculated in this paper using the equivalent impedance of the tower.

Streszczenie. W artykule przedstawiono badania propagacji fali w energetycznej wieży transmisyjnej, po uderzeniu pioruna. Analizę przeprowadzono na podstawie modułowego modelu anteny w dwóch etapach. Najpierw stworzono model impedancyjny przepięcia w wierzy jednopodporowej, a następnie uwzględniono elementy usztywniające. Wykonano przykładowe obliczenia dla wieży pracującej przy napięciu 500 kV(**Model impedancyjny energetycznej wieży przesyłowej w warunkach wielokrotnych przepięć**).

Keywords: Transmission tower, Surge impedance model, Electric and Magnetic energy, Compensation factor. Słowa kluczowe: wieża przesyłowa, impedancyjny model przepięć, energia elektryczna i magnetyczna, wskaźnik kompensacji.

Introduction

The rate of a lightning stroke on a tower increases with the height of the tower. With the requirements for large bulk of power energy, the voltage of the transmission line level increases as the height of the tower increases, thereby the probability of lightning to strike the tower is increased. Thus, strengthening the transmission line's tower lightning protection is necessary for safe and reliable operation of transmission lines [1-3]. The study of a surge impedance model of transmission line tower is important for wave propagation process analysis and lightning strike protection.

Many scholars have studied in depth the tower equivalent model focused on the surge impedance model, which has two representatives. The first involves the measurement method. This method includes the Breuer reflection measurement [4], Kawai direct measurement [5], multistory transmission tower model [6], and Hara model [7]. However, as the voltage level increases, the height of the transmission line tower also increases, and it is not easy to do field measurements for a full-scale tower. The second is the calculation of theoretical modelling, such as the Jordan model [8], Wagner model [9], Surgent model [10], and Yamada model [11]. However, these modelling methods oversimplify the complex structure of the tower, which is not satisfactory for the actual situation of the tower. Therefore, a new method is introduced in this paper to study the tower model.

In this paper, the tower is modularized into three parts, the main body, the crossarms and the bracings of tower. Each part of the tower has an equivalent model, the method to establish equivalent model of the tower will be introduced.

Equivalent model of the main body

The single stanchion of a tower can be considered as a straight cylinder, and if the height of the tower is big enough relative to its equivalent section radius, the process of lightning current can be regard as a spherical wave spreading from top to bottom [12]. This process is similar to the wave process of a tapered antenna (seen in Fig. 1). By borrowing the calculation formula of a tapered antenna in

electric and magnetic fields, the straight cylinder equivalent impedance is obtained [13-15].

(1)
$$Z_e = \frac{U}{I} = \frac{1}{2\pi} \sqrt{\mu/\varepsilon} \ln(\frac{r}{\sqrt{r^2 + h^2} - h})$$

where: μ -magnetic conductivity, ε -dielectric constant, *r*-equivalent section radius, *h*-the height of a tiny segment.



Fig.1.Simplified model of the tower body and tapered antenna

The main part of the actual tower has four pillars. The spatial geometry of the four pillars is similar to the parallel multi-conductor in a short vertical distance. Based on the straight cylinder equivalent impedance, a compensation factor K to obtain the equivalent impedance of four parallel conductors is introduced, as seen in Fig. 2.



Fig.2.Schematic diagram of the parallel four-conductor system equivalent to a single conductor

Based on the definition of surge impedance $Z=(L/C)^{1/2}$, compensation factor *K* is composed of the compensation factor of inductance k_l and the compensation factor of capacitance k_c . Thus, the real surge impedance Z_{eq} of the main body can be obtained as follows, and $K=(k_l/k_c)^{1/2}$,

(2)
$$Z_{eq} = \sqrt{k_l L / k_c C} = KZ$$

(A) Compensation factor of inductance

The compensation factor of inductance k_l can be obtained by calculating the magnetic field deposited by the inductance. According to the law of conservation of energy, if the current flow of the single cylinder and four parallel cylinder conductors are the same, the magnetic field energy deposited in the inductance of the single cylinder is equal to the magnetic field energy deposited in the four parallel cylinder conductors. In considering the self-inductance of each conductor and the mutual-inductance of the four conductor system can be obtained by the law of conservation of energy. After comparing the equivalent inductance and inductance of the single conductor, the compensation factor of inductance k_l is obtained.

The self-inductance of each conductor and the mutualinductance of the four conductors are calculated with the Riemann formula [15].

According to Riemann formula and Fig. 2, the selfinductance of conductor L_i , the mutual-inductance of the adjacent conductors M and the mutual-inductance of the catercorner conductors M_1 can be calculated as follows:

(3)
$$L_{i} = \frac{\mu_{0}}{4\pi} (2r - 2\sqrt{l^{2} + R^{2}} + 2l \ln \frac{l + \sqrt{l^{2} + R^{2}}}{r})$$

(4)
$$M = \frac{\mu_0}{4\pi} [2d - 2\sqrt{l^2 + d^2} - l\ln(-l + \sqrt{l^2 + d^2})]$$

(5)
$$M_1 = \frac{\mu_0}{4\pi} [2\sqrt{2d} - 2\sqrt{l^2 + 2d^2} - l\ln(l + \sqrt{l^2 + 2d^2})]$$

Where: *l*- length of the cylinder, *R*- radius of the cylinder, *d* - adjacent conductors. Assuming that the total magnetic field energy of the conductor is *W*, the equivalent inductance is L_{eq} , and the configuration parameter of each conductor are the same, the energy of each conductor is W_1 , the magnetic field energy of mutual-inductance among the adjacent conductors is W_2 , and the magnetic field energy of mutual inductance among catercorner conductors is W_3 . According to the law of conservation of energy, the relation is as follows:

2

(6)
$$W = 4W_1 + 4W_2 + 2W_3$$

(7)
$$W = \frac{1}{2} L_{\rm eq} I_{\rm ir}$$

(8)
$$W_1 = \frac{1}{2} L_i (\frac{1}{4} I_{in})^2$$

Where: I_{in} – total current flow in the system, and each conductor is assumed to average total current following the conductor system. The expression of W_2 and W_3 can be obtained by replacing the L_i with M and M_1 in Formula (8). Formula (9) is the expression of the equivalent inductance of the four-conductor system.

(9)
$$L_{\rm eq} = \frac{1}{4} \left(L_i + M + \frac{1}{2} M_1 \right)$$

(10)

$$k_{l} = L_{kq}/L_{l} = \frac{1}{4} \{ (2r - 2\sqrt{l^{2} + r^{2}} + 2l \ln \frac{l + \sqrt{l^{2} + r^{2}}}{r}) + \frac{l}{2} (2d - 2\sqrt{l^{2} + d^{2}} - l \ln(-l + \sqrt{l^{2} + d^{2}})) + \frac{1}{2} [2\sqrt{2}d - 2\sqrt{l^{2} + 2d^{2}} - l \ln(-l + \sqrt{l^{2} + 2d^{2}})] \}$$

$$/(2r - 2\sqrt{l^{2} + r^{2}} + 2l \ln \frac{l + \sqrt{l^{2} + r^{2}}}{r})$$

The compensation factor of inductance k_l can be obtained by comparing L_{eq} with L_i . As seen in Formula (10), k_l can be determined by the geometric relations of the four conductors.

(B)Compensation factor of the capacitance

According to the theory of electric and magnetic fields [15, 16], the relationship between electric field energy W_e and capacitance is in Formula (11).

(11)
$$W_e = \frac{1}{2} \int_V (\vec{E} \bullet \vec{D}) \, dV = \frac{1}{2} C U^2$$

where: E - electric field intensity, D - electric flux. According to Formulas (11), the change of capacitance in the same voltage can be expressed by the change of electric field energy. Thus, the compensation factor of capacitance k_c can be expressed as Formula (12).

(12)
$$k_c = \sqrt{C_d / C_0} = \sqrt{W_{ed} / W_{e0}}$$

where: W_{ed} - electric field energy of the four-conductor system in different spaces, W_{e0^-} electric field energy of a single conductor, C_d - equivalent capacitance of the four-conductor system, C_{0^-} equivalent capacitance of a single conductor.



Fig.3. Distribution of the electric field

Fig. 3 a) shows the distribution of an electric field of a single electrification cylinder in a horizontal plane. Fig. 3 b) and c) shows the distribution of electric field in different spaces among the conductors. As seen in Fig. 3, the red areas show that the electric field intensity is strong and the blue areas show weak intensity, and the distribution of four conductors is different from that of a single conductor. Meanwhile, with the change of distance among the four conductors, the distribution of the electric field also changes. The distribution and numerical value of electric field show osculation correlation with the comparative an

contraposition of conductors, Therefore, compensation factor k_c between single and four conductors can be analyzed by the electric field distribution.

The compensation factor of capacitance can be deduced as described in the succeeding paragraph. The electric field distribution and intensity of a column conductor can be emulated via finite element calculation. The radius of the cylinder conductor is *r*, the infinite distance is zero potential, and 500*r* is the emulation boundary. First, the electric intensity of a single conductor with a radius of *r* is calculated and set as the benchmark value W_{e0} . Next, the electric intensity of a four-conductor system is calculated with the radius of each conductor in a four-conductor system still *r*. However, the distance *d* of conductors should be changed with different electric intensities W_{ed} (*d*=2, 3,

...*n*).Then, compared with the electric field intensity specific value of four conductors system in different space length to single conductor.

Take the SZC3 model tower on the 500 kV transmission line of Zhangjiaba-Changshou for example, the fitting curve of W_{ed} which has a different space length compared with W_{e0} is in Fig. 4, the fitting multinomial (13) is the expression of the compensation factors of capacitance k_c .



Fig.4. Graph of polynomial fitting data

Thus, the equivalent model of a tower main body can be obtained through the method mentioned before. The impedance of main body Z_{Tk} is calculated by Formula (1) multiplied by *K*, and *K* is obtained by calculating Formulas (10) and (13).

(14)
$$Z_{Tk} = KZ = \sqrt{k_l/k_c}Z$$

Equivalent model of the bracings

The influence of bracings to the main body is considered by analyzing energy distribution. In physics, a change of configuration will cause a change of inductance and capacitance, and eventually cause energy change in the electric and magnetic fields. This paper considers the main body of a tower as multiple regular cubes, as Fig. 5 show. After emulating the electric and magnetic field of this model and analyzing the change of electric and magnetic field energies around the model with or without bracings, then the effect of bracings on the main body impedance can be obtained indirectly.



Fig.5. Equivalent model of the main body of the tower

(A) Relationship of electric field energy

According to the Formula (11), the change of capacitance in the same voltage can be expressed by the change of electric field energy. Assuming the electric field energy of the tower equivalent solid model without bracings is W_{e1} , and with bracings is W_{e2} , the equivalent capacitance of the entire model without bracings is C_1 , and C_2 with bracings. The compensation factor of capacitance K_C wherein that the bracings influences the tower main body can be expressed as Formula (15).

(15)
$$K_C = \sqrt{C_1/C_2} = \sqrt{W_{e1}/W_{e2}}$$

Compensation factor K_C is obtained by finite element numerical emulation, which analyzes the electric field energy of the model in Fig. 6. Fig. 7 shows the electric field energy density distribution. In the same voltage and appropriate boundary, it can calculate the electric field energy W_{e1} and W_{e2} , and K_C is determined, with the result shown in Table 2. According to Formula (15) and the value in Table 2, the K_C is 0.9895.

Table 2. Computation value of	the electric field en	ergy
	7	

Electric field energy	W_{e1} [10 ⁻⁷ J]	$W_{e2}[10^{-7}J]$
Value	8.801136	8.989198



(a) Model without the bracings (b) Model with the bracings Fig.6. Configuration of the emulation model



(a) Model without the bracings (b) Model with the bracings Fig.7. Distribution of the electric field energy

(B) Relationship of magnetic field energy

According to the theory of electric and magnetic fields, the relationship between magnetic field energy W_m and inductance is shown in Formula (16).

(16)
$$W_m = \frac{1}{2} \int_V (\vec{H} \bullet \vec{B}) \ dV = \frac{1}{2} L I^2$$

Where: H - magnetic field intensity, B - magnetic flux density. According to Formulas (16), the change of inductance in the same current can be expressed by the change of magnetic field energy.

Assuming the magnetic field energy of the tower equivalent solid model in Fig. 6 without the bracings is W_{m1} , whereas the magnetic field energy of the tower equivalent solid model with the bracings is W_{m2} . The equivalent inductance of the entire model without the bracings is L_1 , and with the bracings is L_2 . The compensation factor of inductance K_L wherein that the bracings influences the tower main body can be expressed as Formula (17).

(17)
$$K_L = \sqrt{L_2/L_1} = \sqrt{W_{m2}/W_{m1}}$$

Compensation factor K_L is obtained via finite element numerical emulation, which analyzes the magnetic field energy of the model in Fig. 6. In the same current and

appropriate boundary, the magnetic field energy W_{m1} and W_{m2} are calculated, and the K_L is obtained, the result of which is in Table 3. According to Formula (17) and the value in Table 3, the K_L is 0.8360.

Table 3. Computation value	of the magnetic field en	ergy

Magnetic field energy	<i>W</i> _{m1} [10 ⁻ ^o J]	W_{m2} [10 ⁻⁶ J]	
Value	1.083503	0.757325	

(C) Influence factor of the bracings on the tower main body According to surge impedance definition expression $Z=(L/C)^{1/2}$, the influence factor K_x of bracings to a tower main body can be obtained. The equivalent impedance of a tower module without the bracings is Z_1 , and with the bracings is Z_2 . The relationship between Z_1 and Z_2 is shown in Formula (18).

(18)
$$K_{\rm x} = \frac{Z_2}{Z_1} = \frac{\sqrt{L_2/C_2}}{\sqrt{L_1/C_1}} = \sqrt{\frac{L_2}{L_1}} \sqrt{\frac{C_1}{C_2}} = K_L K_C$$

According to Formula (18) and the value of K_L , K_C , the value of influence factor K_x is 0.8272, indicating that the equivalent impedance of a tower will be reduced to approximately 17% when considering the bracings. Considering the parallel connection of the bracings to the tower main body, the equivalent impedance Z_{Lk} of the bracings can be expressed as Formula (19).

(19)
$$Z_{Lk} = 4.787 Z_{Tk}$$

Equivalent model of the crossarms

The crossarm is equivalent to a parallel four-conductor system in this article. Thus, the equivalent impedance of the crossarm can be obtained using the correlation theory of the equivalent model of parallel multi-conductors [17].



Fig.8. Schematic diagram of crossarm equivalent to four parallel conductors

The voltage equation of parallel multi-conductors system is shown in Formula (20):

$$(20) U = ZI$$

where: $\boldsymbol{U} = [u_1, u_2 \cdots u_n]^T$ -voltage column vector of

each conductor, $I = [i_1, i_2 \cdots i_n]^T$ -current column vector of each conductor, **Z**-surge impedance matrix, composed of self surge impedance and mutual surge impedance.

Meanwhile, the self surge impedance of conductor k is presented in Formula (21), and the mutual surge impedance of conductor k and m is shown in Formula (22):

(21)
$$Z_{kk} = \frac{1}{2\pi} \sqrt{\mu_0 / \varepsilon_0} \ln(h_{kk} / r_k)$$

(22)
$$Z_{km} = \frac{1}{2\pi} \sqrt{\mu_0 / \varepsilon_0} \ln(h_{km} / d_{km})$$

where: h_{kk} -distance between the conductor and the image of conductor, h_{km} -distance between conductor k and the image of conductor m, d_{km} -distance among adjacent conductors, r_k -radius of conductor, as seen in Fig. 9.

For a parallel four-conductor system, the potential equation of parallel multi-conductors is in Formula (23), in which the potential of each conductor is the same: $u_1=u_2=u_3=u_4=u$. Supposing the overall current *i* followed in the conductor system is averaged by each conductor, the current in each conductor is $i_1=i_2=i_3=i_4=i/4$. Formula (23)

can be simplified to Formula (24), and Z is the surge impedance matrix of the crossarms, as seen in Formula (25).

(23)
$$\begin{cases}
u_1 = Z_{11}i_1 + Z_{12}i_2 + Z_{13}i_3 + Z_{14}i_4 \\
u_2 = Z_{12}i_1 + Z_{22}i_2 + Z_{23}i_3 + Z_{24}i_4 \\
u_3 = Z_{13}i_1 + Z_{23}i_2 + Z_{33}i_3 + Z_{34}i_4 \\
u_4 = Z_{14}i_1 + Z_{24}i_2 + Z_{34}i_3 + Z_{44}i_4
\end{cases}$$

(24)





Fig.9. Geometrical relationships of parallel conductors

Therefore, the equivalent impedance of the crossarm Z_{Ak} can be expressed as Formula (26). In substituting the data of Table 4 into Formula (25), the equivalent impedance of the crossarm is obtained.

$$Z_{Ak} = \boldsymbol{U}\boldsymbol{I}^{-1} = \frac{1}{4}\boldsymbol{Z}$$

Table 4 is the parameter of the crossarms of a 500 kV SZC3 tower on Changshou-Zhangjiaba transmission line.

The values of the equivalent impedance of the crossarms are listed in Table 5. The 1, 2, and 3 in Table 4 corresponds with the top, middle, and bottom crossarms.

Table 4.	Parameters	of the crossarm	bracket	

crossarm	$h_{11(22)}[m]$	h ₃₃₍₄₄₎ [m]	$d_{12(34)}$ [m]	<i>d</i> ₁₃₍₂₄₎ [m]	$d_{14(23)}$ [m]
1	140.16	137	2.165	1.58	2.9812
2	115.35	112.4	2.2575	1.475	2.6967
3	92.85	90	2.475	1.425	2.8559

Table 5. Equivalent impedance of the crossarm

Crossarm	1	2	3		
$Z_{Ak}[\Omega]$	291	281	266		

Model validation

(A) Computation Example

According to the calculation methods for tower surge impedance introduced in this paper, the building of an equivalent impedance model of a tower, the SZC3 tower on the 500 kV transmission line of Zhangjiaba-Changshou, is proposed. The structure of SZC3 tower is shown in Fig. 10 a), the computation result of SZC3 tower surge impedance is in Table 6, and the impedance model is built in Fig. 10 b).

Table 6. Computations of surge impedance of SZC3 tower

k	k_l	k _c	K	Ζ[Ω]	$Z_{Tk}[\Omega]$	$Z_{Lk}[\Omega]$	$Z_{Ak}[\Omega]$
1	0.213	1.572	0.368	413	152	728	291
2	0.199	1.640	0.348	396	138	661	281
3	0.162	1.694	0.309	377	117	560	266
4	0.116	1.778	0.255	362	92	440	/
5	0.146	1.901	0.277	347	96	460	/
6	0.146	2.045	0.267	321	86	412	/

(B) EMTP simulation

The accuracy and efficiency of the equivalent multi-surge impedance model proposed in this paper can be verified by comparing with Hara model through EMTP simulation. The lightning current source is standard wave (2.6/50 μ s, 100kA). Jmarti line model is used with system model, and the phase conductor is four bundled conductors LGJ-400/50.The grounding resistance is 15 Ω .



Fig.10. Schematic diagram of 500 kV SZC3 tower and its wave impedance model



Fig.11. Comparison of the top crossarm's voltage

Fig.11 shows the comparison of the top crossarm's voltage between thesis model and Hara model struck by the same lightning current. From Fig.11, the voltage waveform on the top crossarm of this thesis model and Hara model are extremely similar, which verifies the accuracy and efficiency of the equivalent multi-surge impedance model.

Conclusions

1) The equivalent impedance of a single stanchion can be obtained based on the tapered antenna theory. According to the accurate structure of the transmission line tower, the four stanchions of a tower main body are considered as a four-parallel cylinder conductor system. The compensation factor of a four-conductor system equivalent to a single conductor is introduced, finally the equivalent impedance of the tower main body is obtained.

2) Considering the influence of the bracings on the tower main body, the equivalent impedance expression of the bracings is obtained by electric and magnetic field energy analysis.

3) The crossarms of a tower are considered as a four parallel conductor system, based on the theory of the

equivalent model of parallel multi-conductors, the value of the equivalent impedance of the crossarms is obtained.

4) With a modularized tower to the main body, crossarms, and bracings, the equivalent multi-surge impedance model of SZC3 tower is built, and verified the accuracy and efficiency of model by EMTP simulation test.

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