

## Detection of ECG T-wave Alternans Using Maxima of Continuous-Time Wavelet Transform Ridges

**Abstract.** Prognostic utility of microvolt T-wave alternans (TWA) has been established since its clinical acceptance as marker for malignant ventricular arrhythmias, leading to sudden cardiac death. Accurate detection of TWA from surface ECG is a challenge because of invisible nature of the phenomenon. A novel TWA detection scheme based upon analysis of continuous time wavelet ridges (CTWR) of consecutive ventricular repolarization complexes is presented. The CTWR is computed using maxima of wavelet energy coefficients of continuous wavelet transform. Variety of simulated alternans waveforms, wavelet functions, frequency bands and noise levels are used to test the algorithm. The study concludes that CTWR can successfully characterize the alternation of cardiac repolarization and detect TWA phenomenon.

**Streszczenie.** Diagnostyka sygnału TWA odgrywa dużą rolę w badaniach jako marker arytmii powodującej zawał serca. Sygnał TWA jest wykrywany jako składowa sygnału elektrokardiogramu. W artykule opisano wykorzystanie ciągłej transformaty falkowej do analizy tego sygnału. (Detekcja składowej TWA sygnału EKG bazująca na wykorzystaniu ciągłej transformaty falkowej)

**Keywords:** Continuous-Time Wavelet Transform, Detection, Electrocardiography (ECG), T-wave Alternans  
**Słowa kluczowe:** transformata falkowa, elektrokardiogram, sygnał TWA

### Introduction

Electrocardiography (ECG) is an important clinical tool for diagnosis of cardiac diseases. It is commonly measured by placing the surface electrodes on a subject's chest and recording the cardiac cellular potential variations. The measured voltages, graphically plotted as a function of time, represent sequential depolarization (P-wave and QRS complex) and repolarization (T-wave) of cardiac chambers. A single such heart beat is shown in Fig. 1(a).

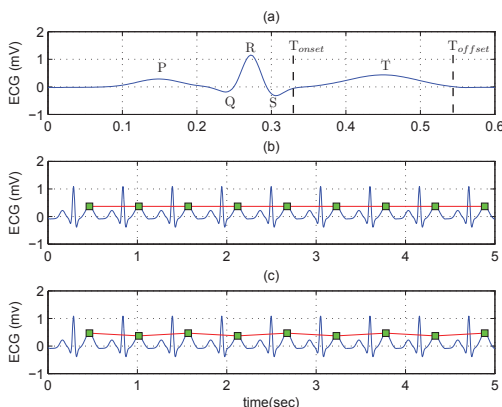


Fig. 1. ECG morphology and TWA presence; (a) a single heart beat, (b) a normal sinus rhythm ECG segment and (c) simulating TWA presence. Red line traces the alternans waveform and green squares mark the peak of T-wave where alternans amplitude is at maximum.

Microvolt T-wave alternans (TWA) is a phenomenon in which ECG T-wave morphology periodically fluctuates in every other heartbeat, as shown in Fig. 1(c). Being peak to peak amplitudes in microvolts, this alternation is not visible through manual examination of surface ECGs. Pathological significance of the phenomenon is established since first observations in 1910 [1], however, the prognosis value is considerably enhanced since the detection and estimation has become possible through advanced signal processing techniques [2]. Since then, TWA has been increasingly linked with malignant ventricular arrhythmias and recently included among the risk stratifiers for sudden cardiac death (SCD) [3, 4].

Wavelet transform has been extensively applied for segmentation and analysis of ECG [5, 6, 7]. In this paper, a new

TWA detection technique is proposed using dynamic analysis of ridges of beat to beat continuous-time wavelet transform (CWT) energy coefficients of respective repolarization segments (T-waves). Detection is performed using different wavelets families to find the wavelet basis giving optimal detection performance.

In the following three sections, we establish theoretical basis of our proposed detection scheme. Section 4 and 5 describes the detection scheme and dataset used for the validation. Detection performance results are presented in the Section 6 with an important conclusions drawn in the last.

### ECG Signal Preprocessing

Block diagram of the proposed detection scheme is shown in Fig. 2. An  $M$  beat digitized ECG signal is fed as input to the analysis system, whose outputs include a decision statistics regarding the TWA presence or absence. The aim of the preprocessing stage is to condition the digitized ECG for subsequent analysis [2].

As TWA is a localized phenomenon only manifested during the repolarization phase of beat to beat ECG signal, it is convenient to adopt a segmentation procedure to extract the relevant portion from the complete ECG signal before analysis. The steps performed in preprocessing and segmentation stages are as follows.

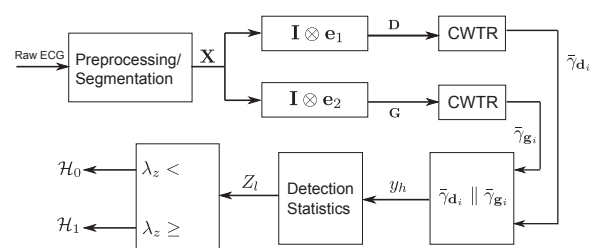


Fig. 2. Block diagram of the proposed detection scheme.

For preliminary analysis, CWT of the whole ECG signal is computed with a fine fractional dyadic discretization of 200 scales ranging from  $a = 2^{a_{\min}}$  to  $2^{a_{\max}}$  and  $a_{\min}$  and  $a_{\max}$  are computed as [8].

$$(1) \quad a_{\min} = \log_2 \left( \frac{F_c F_s}{F_{\max}} \right)$$

$$(2) \quad a_{\max} = \log_2 \left( \frac{F_c F_s}{F_{\min}} \right)$$

where  $F_s$  is the sampling frequency of the ECG signal,  $F_c$  is the center frequency of the mother wavelet and  $F_{\min}$ ,  $F_{\max}$  are the pseudo-frequencies corresponding to the respective CWT scales and chosen according to the band of interest. Frequency contents of the T-wave normally lie between 0.5 to 10 Hz [9] but to cater for the possibility of high frequency components, the repolarization segment is spectrally localized within 0.5 to 15 Hz, which corresponds to a temporal interval of 180 ms from the onset of ST segment. After localizing the repolarization segment, QRS and T-wave peak detection is performed using the waveform locator of Laguna *et al* [10], available at Physionet [11]. Fig. 3 shows preprocessing steps performed on a real ECG signal taken from TWA challenge database sampled at 500 Hz [11, 12].

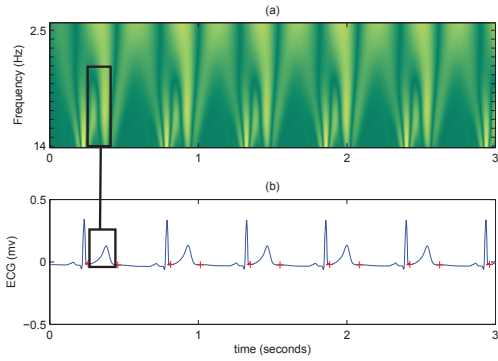


Fig. 3. ECG preprocessing; (a) Localization of ventricular repolarization in time-frequency plane, a portion from 2.5 to 14 Hz is shown for better visualization, and (b) QRS detection and T-wave segmentation in time domain,  $T_{\text{onset}}$  and  $T_{\text{offset}}$  are marked with '+'.

### Signal Model

The output  $\mathbf{X} \in \mathbb{R}^{N \times M}$  of the preprocessing stage contains preprocessed ECG repolarization segments  $\mathbf{X} = [\mathbf{x}_0 \dots \mathbf{x}_{M-1}]$ , where  $\mathbf{x}_l = [x_l[0], \dots, x_l[N-1]]^T$  is the segment corresponding to  $l$ -th beat, having  $N$  samples. TWA is essentially a non-stationary phenomenon and analysis has to be performed on a limited number of beats, therefore, an analysis window  $\mathbf{B} \in \mathbb{R}^{L \times N}$  comprising of  $L$  segments is defined which must be shifted  $r$  times with  $L$  beats to cover the length of whole signal length  $M$ . At  $r = 1$ , the analysis window is considered to be centered at beat  $\frac{l}{2} - 1$ , such that  $\mathbf{B} = [\mathbf{b}_0 \dots \mathbf{b}_{\frac{l}{2}-1} \dots \mathbf{b}_{L-1}]$ . The segmented ST-T complexes in the  $L$  beat analysis window can be written in matrix notation as,

$$(3) \quad \mathbf{X} = \mathbf{S} + \mathbf{W}$$

where  $\mathbf{X} \in \mathbb{R}^{N \times L}$  contains the aligned ST-T segments and  $\mathbf{S}, \mathbf{W} \in \mathbb{R}^{N \times L}$  represent signal and noise samples, respectively. The odd and even ST-T complexes, denoted by  $\mathbf{D} \in \mathbb{R}^{N \times L/2}$  and  $\mathbf{G} \in \mathbb{R}^{N \times L/2}$  respectively, are obtained through  $\mathbf{D} = \mathbf{X}(\mathbf{I} \otimes \mathbf{e}_1)$  and  $\mathbf{G} = \mathbf{X}(\mathbf{I} \otimes \mathbf{e}_2)$ , where  $\mathbf{I} \in \mathbb{R}^{L \times L/2}$  is the thin identity matrix,  $\mathbf{e}_1, \mathbf{e}_2 \in \mathbb{R}^2$  are standard basis and  $\otimes$  represents the Kronecker product. If we

denote  $\mathbf{D} = [\mathbf{d}_0 \mathbf{d}_1 \dots \mathbf{d}_{L/2}]$  and  $\mathbf{G} = [\mathbf{g}_0 \mathbf{g}_1 \dots \mathbf{g}_{L/2}]$ , the signal model in (3) can be disintegrated and rewritten as,

$$(4) \quad \mathbf{d}_i = \mathbf{s}_l + \mathbf{w}_l$$

$$(5) \quad \mathbf{g}_i = \mathbf{s}_{l+1} + \mathbf{w}_{l+1}$$

where  $i = 0, \dots, L/2 - 1$ , and  $\mathbf{s}_l = [s_l[0], \dots, s_l[N-1]]^T$  and  $\mathbf{w}_l = [w_l[0], \dots, w_l[N-1]]^T$  are signal and noise samples of ST-T complexes corresponding to  $l$ -th beat within the analysis window and  $(\cdot)^T$  represents the transpose operation.

Alternan waveform, being the beat to beat fluctuation, is the recurring trace of difference between consecutive repolarization complexes. If  $\mathbf{V} \in \mathbb{R}^{N \times L/2}$  is considered a representation of this difference, such that  $\mathbf{V} = [\mathbf{v}_0 \mathbf{v}_1 \dots \mathbf{v}_{L/2}]$  and  $\mathbf{v}_i = |\mathbf{g}_i - \mathbf{d}_i|$ , we have the alternan waveform modeled as,

$$(6) \quad \mathbf{u}_i = \mathbf{v}_i + \eta_i$$

where  $\mathbf{u}_j$  is the observation vector containing the alternan waveform segment  $\mathbf{v}_i = [v_i[0], \dots, v_i[N-1]]^T$ , corresponding to  $i$ -th pair of consecutive ST-T complexes containing  $N$  samples each, and  $\eta_i = [\eta_i[0], \dots, \eta_i[N-1]]^T$  are residue noise and artifact samples  $\eta_i[n] = w_{l+1}[n] - w_l[n]$ .

### Computation of Wavelet Transform Energy Coefficients

The CWT of a continuous time signal  $x(t)$  is defined in [8] as,

$$(7) \quad C(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left( \frac{t-b}{a} \right)$$

where  $\psi^*(t)$  is the complex conjugate of the wavelet function  $\psi(t)$  and  $a, b$  are the scaling and location parameters of the wavelet. The two dimensional energy density function at a specific location and scale is given by

$$(8) \quad \mathcal{E}(a, b) = |C(a, b)|^2$$

In practice, a discretized approximation of (7) is used, which is fundamentally different than discrete wavelet transform (DWT). The preprocessed and aligned repolarization segments  $\mathbf{d}_i$  and  $\mathbf{g}_i$  of length  $N$  came from the ECG signal sampled with  $F_s$ , therefore, the above location-scale representation of CWT must be translated into a notion of time-frequency atoms.

We denote the computed CWT coefficients of a repolarization segment  $\mathbf{x}_l = [x_l[0], \dots, x_l[N-1]]^T$  corresponding to  $l$ -th beat in the frequency interval  $[F_1 F_2]$  by  $\mathbf{C}_l \in \mathbb{R}^{N \times J}$  with  $\mathbf{C}_l = [\mathbf{c}_{l_0} \mathbf{c}_{l_1} \dots \mathbf{c}_{l_j}]^T$  where  $j = 0, 1, \dots, J-1$  are the discretized frequency locations in time-frequency plane with  $J = \frac{F_2 - F_1}{\Delta F}$  and  $\Delta F$  is the step size of the frequency corresponding to the discrete scaling step of the discretized CWT. The vector  $\mathbf{c}_{l_0} = [c_{l_0}[0], \dots, c_{l_0}[N-1]]^T$  contains wavelet coefficients of  $l$ -th repolarization segment corresponding to first frequency location, i.e.,  $j = 0$ .

A wavelet ridge corresponding to each time sample of the repolarization segments can be computed as

$$(9) \quad \gamma_l[k] = \frac{d}{da} \mathcal{E}_l(a, b) = 0$$

where  $\mathcal{E}_l = |c_l[k]|^2$  and  $\gamma_l[k]$  represents the energy and trend of instantaneous frequencies at  $k$ -th time samples of  $l$ -th cardiac repolarization complex, respectively.

### Detection Scheme

Presence of TWA exhibits an electrophysiological state in which ST-T complexes belonging to every other heartbeat are closely homogeneous, as shown in Fig. 1(c). This implies that respective instantaneous frequencies corresponding to repolarization intervals of even and odd cardiac cycles in an  $L$  beat analysis window may indicate the presence or absence of TWA. Departing from the described signal model, wavelet ridges are computed for  $\mathbf{d}_i$  and  $\mathbf{g}_i$  using (9), resulting in sufficient detection statistics given by  $\tilde{\gamma}_{\mathbf{d}_i}$  and  $\tilde{\gamma}_{\mathbf{g}_i}$  which are maximas of the wavelet ridges corresponding to first and second repolarization complex, respectively, of  $i$ -th pair of heartbeats.

To facilitate dynamic tracking of TWA, these values are finally combined as a beat to beat series of coefficients in the  $L$  beat analysis window obtained through  $y_h = \tilde{\gamma}_{\mathbf{d}_i} \parallel \tilde{\gamma}_{\mathbf{g}_i}$ , such that  $y_h = [y_0, y_1, \dots, y_{L-1}]$ , where  $y_0 = \tilde{\gamma}_{\mathbf{d}_0}, y_1 = \tilde{\gamma}_{\mathbf{g}_0}, \dots, y_{L-2} = \tilde{\gamma}_{\mathbf{d}_{L/2-1}}, y_{L-1} = \tilde{\gamma}_{\mathbf{g}_{L/2-1}}$ .

The series  $y_h$  must exhibit a periodicity of order 2 over the length of the analysis window  $L$  in order to decide about the presence of alternan, therefore, the standard sign change counting TWA detection statistics [2] is computed with each shift of the analysis window in the neighbourhood of the  $l$ -th beat ( $l = l_0 + rL$ ), as,

$$(10) \quad Z_l = \frac{1}{2} \left( L + \left| \sum_{h=l-L+1}^l \text{sign}(\Delta y_h) (-1)^h \right| \right) \begin{matrix} \mathcal{H}_1 \\ \geq \\ \mathcal{H}_0 \end{matrix} \lambda_z$$

where  $\mathcal{H}_1$  implies detection of TWA,  $\mathcal{H}_0$  is the null hypothesis and  $\Delta y_h = y_h - y_{h-1}$ . The hypothesis is tested against a fixed value threshold  $\lambda_z$ , which is a predetermined test significance value. In a scenario where signal and noise statistics are assumed to be unknown, it is assumed that  $\lambda_z = L$ .

### Datasets

Signals with *a priori* knowledge of TWA levels are required to test the performance of detection scheme. Like many other ECG problems, there is no gold standard database as it is not possible to manually (and accurately) detect the microvolt TWA. Therefore, the accepted procedure is to devise realistic synthetic datasets for validation experiments. ECG signals are generated using realistic ECG generator model proposed by McSharry *et al* [13].

The model provides various tunable parameters to change the important morphological and signal characteristics of generated waveforms, such as mean and standard deviation of heart rate, morphology of PQRST cycle, amplitudes and sampling frequency. Beat registers are made by  $L$ -fold repetition of a randomly extracted beat from the stream generated by the generator and choice of  $L$  is varied according to the target experiment. A hamming function is added in each alternative beat  $\{l, l+2, \dots, L-1\}$  of the ECG signal so that any alternan waveform segment  $u_i[n]$  contained in observation vector  $\mathbf{u}_i$  in (6) becomes  $u_i[n] = g_1 (0.54 - 0.46 \cos(2\pi \frac{n}{N})) + g_2 (\eta_i[n])$ , where  $g_1$  and  $g_2$  are the scaling parameters to control alternan waveform amplitude and noise power respectively. The noise  $\eta_i[n]$  is modeled as a white gaussian process, i.e.,  $\eta_i[n] \sim \mathcal{N}(0, \sigma^2)$ .

### Detection Performance

Detector performance is evaluated in different scenarios involving variety of TWA distributions and noise realizations. Three different temporal realizations of alternan episodes are devised to simulate variety of stationary and non-stationary characteristics. Stationary TWA (dataset TWA-1) is simulated by equally distributing a single episode in an  $M$  beat signal. For non-stationary TWA, three different datasets are devised to simulate both large and small scale non-stationary characteristics. TWA-2 dataset includes multiple episodes of randomly varying length and TWA amplitude. TWA-3 dataset includes a single TWA episode with increasing magnitudes with  $\tau$  magnitude transition levels.

Inappropriate selection of wavelet basis as well as signal bandwidth of interest will adversely effect the resolution in time and frequency domains, and therefore the performance of algorithm. Therefore, preliminary analysis before algorithm testing is carried out by studying time-frequency energy characteristics of T-wave using a number of wavelet functions which have been validated previously for physiological signal analysis [8].

Fig. 4 shows mean  $\mathcal{E}_{l_j}[k]$  of a T-wave computed using five different mother wavelets for respective scales corresponding to 0.5-15 Hz. It is observed that repolarization morphology is best traced by *Mexican Hat*, *Symlet4* and truncated *Morlet* wavelet (only the real part), whereas, the energies towards the second half of ventricular repolarization were significantly suppressed by *Daubechie4* and *Meyer* wavelets.

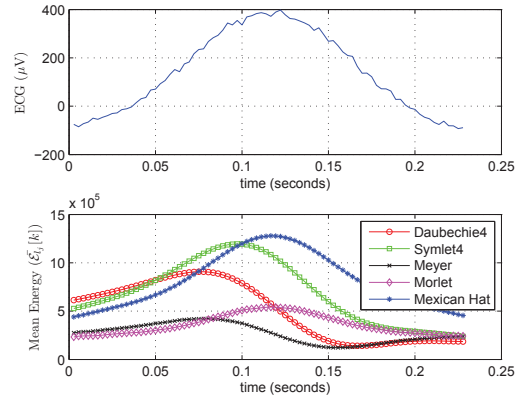


Fig. 4. Applying different mother wavelets to analyze T-wave morphology; (a) T-wave from an ECG signal from TWA challenge database (twa01), and (b) mean energy,  $\bar{\mathcal{E}}_l[k] = \frac{1}{J} \sum_0^{J-1} \mathcal{E}_{l_j}[k]$ , for *Daubechie4*, *Symlet4*, *Meyer*, *Morlet* and *Mexican Hat* wavelet functions for all scales corresponding to 0.5-15 Hz.

Detector output results for a noisy ECG signal with signal-to-noise ratio (SNR) = 20 dB are reported below. Fig. 5 shows detection of a TWA-1 episode of alternan amplitude  $20\mu\text{V}$  in a 128 beat ECG signal ( $M = 128$ ). The detection remains accurate for the complete range of analysis window, i.e.,  $L = 4, \dots, M$ , implying independence of algorithm sensitivity from the length of analysis window  $L$ , assuming TWA stationarity across  $M$  beats. In case of non-stationary TWA, the detection performance obviously depends upon the window length and would significantly degrade if TWA episode is significantly shorter than  $L$ , e.g., detection of an 8 beat alternan episode keeping  $L = 32$ . A simple solution is to keep the length of window as small as possible according to the clinical dictates.

Detection of TWA-2 with three randomly generated alternan episodes is shown in Fig. 6. All the episodes are

accurately detected by keeping minimum length of analysis window, i.e.,  $L = 4$ . TWA-3 is a special case where TWA is present within the whole ECG signal, however, the magnitude is changing. This gradual increase in alternan magnitude is also detected by the algorithm, as shown in Fig. 7.

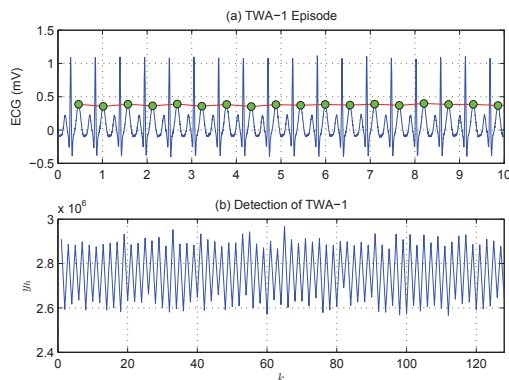


Fig. 5. Detection of stationary T-wave alternans; (a) a single TWA-1 episode with constant alternan magnitude and (b) detector output.

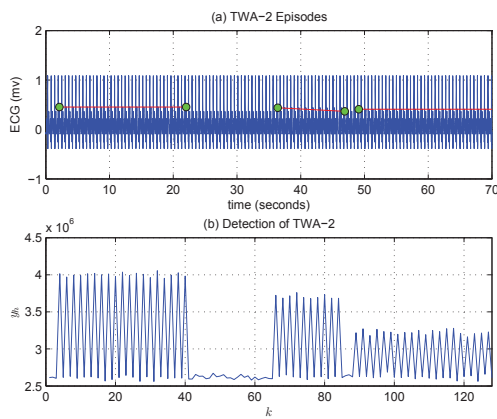


Fig. 6. Detection of non-stationary T-wave alternans; (a) three random episodes of TWA-3 and (b) detector output

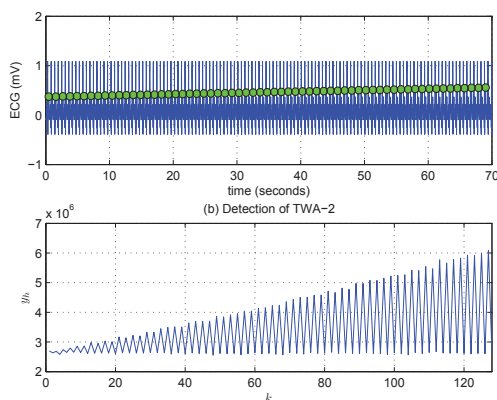


Fig. 7. Detection of non-stationary T-wave alternans; (a) a single TWA-3 episode with increasing magnitude and (b) detector output.

## Conclusion

A new TWA detection algorithm based upon variation of continuous-time wavelet transform ridges (CTWR), computed using maxima of wavelet energy coefficient corresponding to consecutive cardiac repolarization complexes is presented. It is proposed that CTWR, which represents instantaneous frequency of a cardiac repolarization segment

can be successfully used to study the short time dynamical periodicity in cardiac repolarization. The implementation of the algorithm is tested using realistically synthesized ECG signals with variety of alternan distributions and magnitudes. The detection has been tested for different wavelet basis with varying noise levels. The study presents a new application of CTWR as an important analytical tool for detection of TWA which is among the most important marker and risk stratifier of sudden cardiac death.

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