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Higher-Order Modeling of Electrostatic Separator of Plastic Particles

Abstract. *Optimization of a separator of plastic particles is carried out. The objective function maximizes the number of particles falling down to the corresponding bins. Electric field in the system is solved numerically, using the fully adaptive higher-order finite element method. The trajectories of particles are determined by means of an adaptive Runge-Kutta-Fehlberg method with a time varying time step. The shape of the electrodes is performed by a technique based on higher-order conjugate gradients.*

Streszczenie. *W artykule przeprowadzono optymalizację separatora cząstek plastycznych. Funkcja celu maksymalizuje liczę cząstek spadających do odpowiednich pojemników. Pole elektryczne w tym systemie wyznaczane jest numerycznie, poprzez zastosowanie pełno adaptacyjnej metody elementów skończonych wyższego rzędu. Trajektorie ruchu cząstek zostały wyznaczone za pomocą adaptacyjneh metody Runge-Kutta-Fehlberga ze zmiennym krokiem czasowym. Kształt elektrod jest wyznaczany techniką gradientów sprzężonych wyższego rzędu.* ***(Modelowanie wyższego rzędu elektrostatycznego separatora cząstek plastycznych)***

Keywords: electrostatic separator, plastic particles, higher-order finite element method, electric field
Słowa kluczowe: separator elektrostatyczny, cząstki plastikowe, metoda elementów skończonych wyższego rzędu, pole elektryczne

Introduction

The paper deals with the possibility of recycling plastic materials. As their particular levels should be as pure as possible, it is crucially important to have a sufficiently powerful technique for their mutual separation. One of the promising techniques of this kind is based on the triboelectric effect consisting in the fact that small particles of a mixture of plastics are able to accept electric charge whose value depends on the type of plastic [1–2]. And after charging, the particles are transported to a stronger electric field where they freely fall down and their movement is driven by the local field strength.

An appropriate device (separator) of this kind is indicated in Fig. 1.

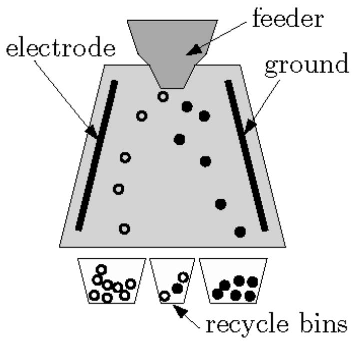

Fig. 1. Typical separator of plastic particles based on the triboelectric effect

It consists of a vessel with two electrodes, one of them being grounded. The electrodes are supposed to be covered by a suitable insulating material that prevents recharging of the particles in case of direct impacts with them. The mixture of charged particles is delivered by the feeder. At the bottom of the device there are recycle bins used for accumulating of particular levels of plastics. One of the principal demands is to tune the shape of the electrodes and widths of the bins so that the particles of different levels fall down exactly to the corresponding bin.

The aim of the paper is to model the dynamics of the particles in the device determined for separation of two levels (polyethylene – PET and polyvinylchloride – PVC) and optimize the shape of both electrodes in order to satisfy the demands concerning the purity of particular levels. The mass and charge of the mixed particles obey the Gauss distribution.

Continuous mathematical model

The mathematical model of the process of separation is described by a linear partial differential equation describing the electric field in the system and one ordinary strongly nonlinear differential equation describing the movement of the particle.

Electric field in the space between the electrodes in the separator is described by the equation for the electric potential φ [3]

(1) $$\mathrm{div}\left(\varepsilon\,\mathrm{grad}\varphi\right) = 0\,,$$

where ε is the dielectric permittivity. The boundary conditions are given by the known values of the electric potential on the electrodes and Neumann condition along the artificial boundary placed at a sufficiently distance from the device.

The movement of the particle obeys the equations for its velocity $\boldsymbol{v}$ and trajectory $\boldsymbol{s}$ in the forms

(2) $$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{F}_{\mathrm{e}} + \boldsymbol{F}_{\mathrm{a}} + \boldsymbol{F}_{\mathrm{g}}\,,\quad \boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}\,.$$

Here, $\boldsymbol{F}_{\mathrm{e}}$ denotes the total Coulomb force acting on the particle, $\boldsymbol{F}_{\mathrm{a}}$ represents the aerodynamic resistance and $\boldsymbol{F}_{\mathrm{g}}$ is the gravitational force. These forces are given by relations

(3) $$\boldsymbol{F}_{\mathrm{e}} = Q\boldsymbol{E} = -Q\,\mathrm{grad}\,\varphi\,,$$
$$\boldsymbol{F}_{\mathrm{a}} = -\boldsymbol{v}\,\frac{1}{2}\rho c S v\,,$$
$$\boldsymbol{F}_{\mathrm{g}} = m\boldsymbol{g}\,,$$

where $\boldsymbol{E}$ denotes the local value of the electric field strength, ρ is the density of ambient air, c stands for the friction coefficient (depending on geometry of the particle), S is the characteristic surface of the particle, v is the module of its velocity, m denotes the mass of the particle and $\boldsymbol{g}$ is the gravitational acceleration.

Generally, the Coulomb force acting on the particle is produced not only by the electric field generated by the electrodes, but also by other charged particles present in the space. But as these charges are rather small, these additional force effects can be neglected without any significant error

The corresponding initial conditions read

$$\boldsymbol{v}(0) = \boldsymbol{v}_0, \quad \boldsymbol{s}(0) = \boldsymbol{s}_0 , \tag{4}$$

where $\boldsymbol{s}_0$ is the entry position of the particle in the separator.

Numerical solution

The above model (equations (1) and (2)) was solved numerically by our own code Agros2D [4]. Agros2D is a powerful user's interface serving for pre-processing and post-processing of the problems solved. The code collaborates with the library Hermes2D [5] containing the most advanced fully adaptive algorithms for solution of systems of generally nonlinear and nonstationary partial differential equations (PDEs) based on the finite element method of higher order of accuracy.

Both above codes are freely distributable and in 2D version they exhibit a lot of unique features, such as fully automatic *hp*-adaptivity, work with hanging nodes of any level, multimesh technology (every field can be calculated on a different mesh generally varying in time) and a possibility of combining triangular, quadrilateral and curved elements.

Example of computation

The basic dimensions and other data concerning the investigated separator can be found in Fig. 2.

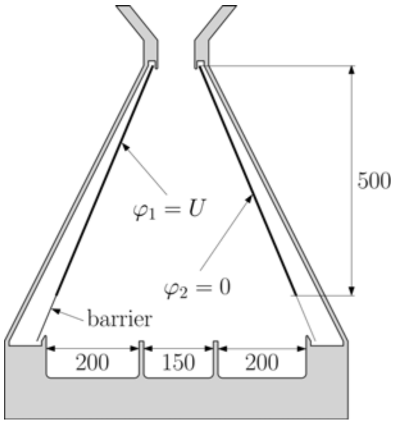

Fig. 2. Arrangement of the investigated device (dimensions in mm)

Two types of particles were considered for the computations: positively charged polyethylene (PET) particles and negatively charged polyvinylchloride (PVC) particles. Their charges and dimensions are characterized by the normal distribution whose parameters are listed in Tab. 1. The numbers of particles $n = 250$ for each kind. The voltage U between the electrodes is variable.

Tab. 1: Selected parameters of the particles

type	density	radius		charge	
	ρ (kg/m^3)	μ (mm)	σ (mm)	μ (C)	σ (C)
PET	1330	2	0.25	+0.25E–9	0.8E–10
PVC	1370	2	0.25	–0. 5E–9	0.8E–10

μ denotes the median, σ is the variance

The distributions of the radii and charges of the particles are depicted graphically in Figs. 3–5, symbol N expressing the corresponding frequency.

The optimization process (finding the most favourable shape of the electrodes whose initial arrangement is shown in Fig. 2) is based on the minimization of two functionals F and G representing the accuracy of impact of PVC and PET particles, respectively.

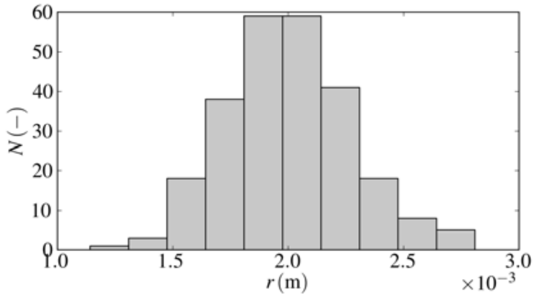

Fig. 3. Distribution of radii of the PET and PVC particles

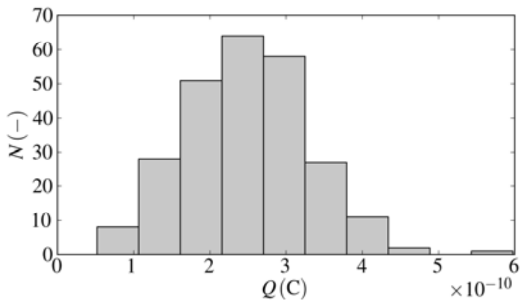

Fig. 4. Distribution of charges of the PET particles

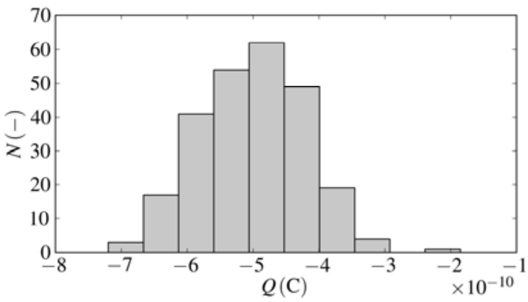

Fig. 5. Distribution of charges of the PVC particles

Functionals F and G are calculated as average distances between the real and desired positions of impact on each side of the bottom of the separator model. In the case of the multi-criteria optimization, more than one optimal design can be found. But this also means that any improvement of one functional inevitably leads to worsening of the other one. A set of such optimal solutions form the so-called Pareto front (see Fig. 6). Actual optimization has been carried out using one-criterion optimization by the conjugate-gradient method. We performed multiple optimizations using criteria function of the form

$$\alpha F + (1-\alpha) G \tag{5}$$

for different values $\alpha \in (0,1)$. This is one of the simplest methods for obtaining the Pareto front.

The conjugate-gradient method is a step method based on the evaluation of gradients of the steepest descend, that are in each step used to calculate direction, where a 1-dimensional optimization (so-called line search) is performed. In our calculations, we encountered problems with the gradient calculations. Although we tried to design our criteria functions to be smooth, we experienced problems calculating gradient components using standard formula. We had to use higher-order differentiation formula

$$(6) \qquad f(x) = \frac{1}{12h}\begin{pmatrix} -f(x+2h)+8f(x+h) \\ -8f(x-h)+f(x-2h) \end{pmatrix},$$

so we could use large enough h to achieve correct results. The values of gradients were not reliable for too small values of h. As can be seen from Fig. 6, no such front is formed here. We can deduce that two selected functionals do not move against each other.

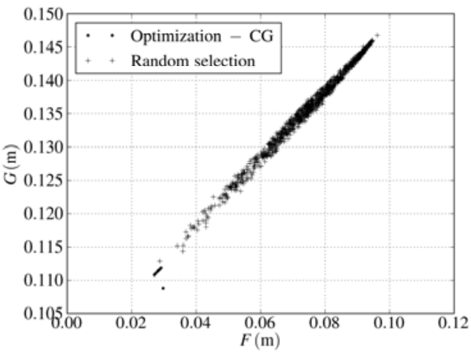

Fig. 6. The Pareto front of the optimization problem

The optimization process provided new shapes of electrodes whose electric field makes more charged particles fall down to the correct bins. This shape is indicated in Fig. 7.

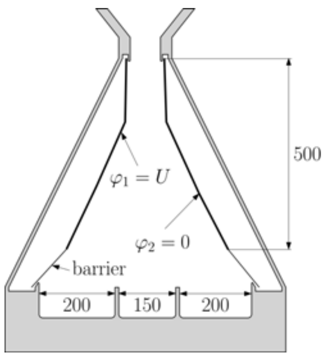

Fig. 7.Optimized arrangement of the electrodes

Figures 8 and 9 show, for illustration, selected trajectories of the PET and PVC particles in the electric field generated by the optimized electrodes.

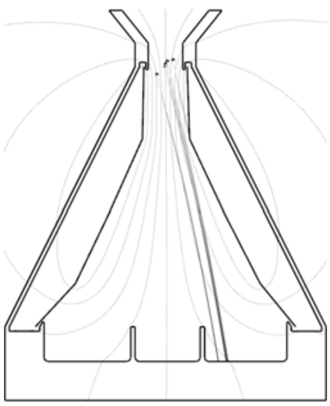

Fig. 8. Trajectories of the PVC particles

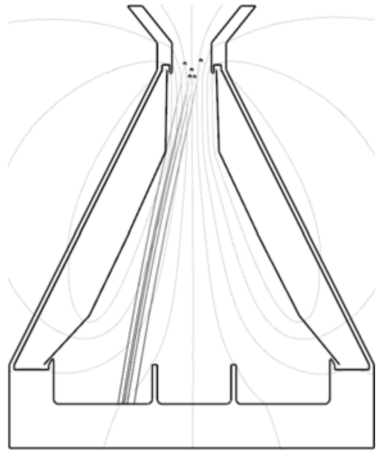

Fig. 9. Trajectories of the PET particles

Important is the efficiency of the separation process expressing how many particles fall down to the appropriate bins. This quantity strongly depends on the voltage U. Its curves for both the basic and optimized arrangements are depicted in Fig. 10. The efficiency of the optimized variant is higher by about 20 %.

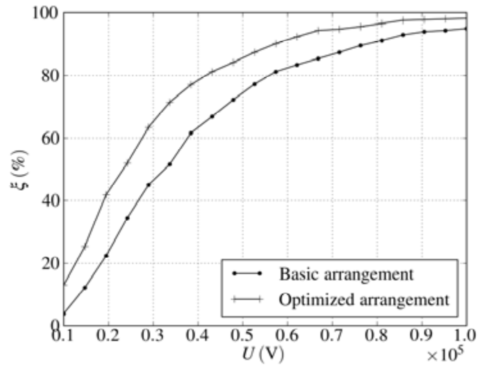

Fig. 10. Efficiency of the separation process as a function of voltage for the basic and optimized arrangements

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