Andrzej POPENDA

Czestochowa University of Technology, Faculty of Electrical Engineering, Institute of Industrial Electrical Engineering

Model-simulation investigations of induction motor with the consideration of skin effect in rotor bars

Abstract. *In this paper the mathematical model of induction motor with the consideration of the skin effect in squirrel-cage rotor bars is proposed. This phenomenon is taken into account by making variable both rotor resistance and rotor leakage inductance, being the parameters of circuit mathematical model of induction motor. The proposed mathematical model is compared to the other model with the variable parameters, known from the literature sources, as well as this model is compared to the simplest circuit model with the constant parameters. The experimental verification of the proposed mathematical model of induction motor was made in order to find out if the applied solutions are proper.*

Streszczenie*. W pracy zaproponowano model matematyczny silnika indukcyjnego z uwzględnieniem zjawiska naskórkowości w prętach klatki wirnika poprzez uzmiennienie rezystancji i reaktancji rozproszenia wirnika w obwodowym modelu matematyczny silnika indukcyjnego, odnosząc zaproponowane modyfikacje modelu matematycznego do innego, znanego z literatury sposobu uzmiennienia parametrów silnika oraz do najprostszego modelu obwodowego o stałych parametrach. Dla wykazania poprawności zastosowanych rozwiązań przeprowadzono eksperymentalną weryfikację otrzymanych wyników badań modelowo-symulacyjnych proponowanego modelu matematycznego silnika indukcyjnego. (****Badania modelowo-symulacyjne silnika indukcyjnego z uwzględnieniem zjawiska naskórkowości w prętach wirnika****).*

Keywords: modelling and simulation, induction motor, squirrel-cage rotor, skin effect.
Słowa kluczowe: modelowanie i symulacja, silnik indukcyjny, wirnik klatkowy, efekt naskórkowości.

Introduction

The operation of squirrel-cage induction motor when the slip is higher than several percent, especially during direct starting a motor, causes one-sided displacement of current flowing in rotor bars. This phenomenon is known as a skin effect. The accuracy of model-simulation results is higher if the skin effect in rotor bars is taken into account. Especially important is consideration of the aforementioned phenomenon in the case of induction motors equipped with deep-bar rotor or double-squirrel-cage rotor.

In this paper the simple method of taking into account the abovementioned phenomenon is proposed. This method consists in consideration of varying both rotor resistance and rotor leakage inductance, being the parameters of circuit mathematical model of induction motor. The proposed mathematical model is compared to the other model with the variable parameters, known from the literature sources, as well as this model is compared to the simplest circuit model with the constant parameters.

Variable parameters of circuit mathematical model of induction motor are controversial for some authors of papers, e.g. [1,2], due to the fact, the variable parameters of circuit model are not entirely consistent with the assumptions accepted for the mathematical model formulated on the basis of Lagrange's formalism, by which the parameters of modeled system should be constant. According to these authors the multi-squirrel-cage model based on constant parameters should be used in case of motors equipped with double-squirrel-cage rotor or deep-bar rotor [2]. The wound-rotor induction motors and single-squirrel-cage motors with circular bars may be described by the simplest single-squirrel-cage model.

The mathematical model of induction motor based on variable parameters, presented in this paper, differs from the models formulated on the basis of Lagrange's formalism. The proposed model is based on circuit voltages' balance equations of stator phase windings and rotor equivalent phase windings, transformed from three-phase system to Cartesian coordinate system. The experimental verification of the proposed mathematical model of induction motor was made in order to find out if the applied solutions are proper.

Mathematical analysis

It is known, that the skin depth (magnetic diffusion depth) may be defined by solving the magnetic diffusion equation, that describes the diffusion of magnetic field and current into conductor for quasi-magneto-static field or for low frequencies of flowing current. The magnetic diffusion equation may be derived from Faraday, Ampere and Gauss's laws

$$(1)\qquad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\qquad \nabla\times\mathbf{H}=\mathbf{J}_f,\qquad \nabla\cdot\mathbf{B}=0$$

For linear materials with constant permeability μ and constant Ohmic conductivity γ, moving with velocity $\mathbf{V}$

$$(2)\qquad \mathbf{B}=\mu\mathbf{H},\qquad \mathbf{J}_f=\gamma(\mathbf{E}+\mathbf{V}\times\mu\mathbf{H})$$

The abovegiven equations may by reduced to one equation by calculation of curl of the second equation (1) and by applying the first and the second equations (2): $\nabla\times(\nabla\times\mathbf{H})=$

$$=\nabla\times\mathbf{J}_f=\gamma[\nabla\times\mathbf{E}+\mu\nabla\times(\mathbf{V}\times\mathbf{H})]=\mu\gamma\left[-\frac{\partial\mathbf{H}}{\partial t}+\nabla\times(\mathbf{V}\times\mathbf{H})\right]$$

The double cross product of $\mathbf{H}$ can be simplified using the vector identity $\nabla\times(\nabla\times\mathbf{H})=\nabla(\nabla\cdot\mathbf{H})-\nabla^2\mathbf{H}$, hence $\frac{1}{\mu\gamma}\nabla^2\mathbf{H}=\frac{\partial\mathbf{H}}{\partial t}-\nabla\times(\mathbf{V}\times\mathbf{H})$, where $\mathbf{H}$ has no divergence from the third equation (1).

The following assumptions were accepted: (a) each rotor bar without slot skew conducts current along the rotor shaft axis – *y* (one-dimensional case), (b) the magnetic field is in the *z* direction (along the unrolled rotor circumference) and only depends on the *x* coordinate axis (along bar height), (c) the fringing is neglected, and (d) the reference coordinate system is connected to the rotor. Taking into consideration the abovementioned assumptions it may be written $\frac{\partial^2 H_z}{\partial x^2}-\mu\gamma\frac{\partial H_z}{\partial t}=0$.

There is the sinusoidally varying magnetic field within the conductor if the sinusoidally varying current flows via conductor, so $H_z(x,t)=\mathrm{Re}\left[\underline{H}_z(x)e^{j\omega t}\right]$.

Taking into account the abovegiven dependency the following equation can be derived $\frac{d^2 \underline{H}_z}{dx^2} - j\omega\mu\gamma \underline{H}_z = 0$ with the solution $\underline{H}_z(x) = A_1 e^{(1+j)x/\sigma} + A_2 e^{-(1+j)x/\sigma}$ where the skin depth σ is defined as

$$\sigma = \sqrt{\frac{2}{\omega\mu\gamma}} = \sqrt{\frac{2\rho}{\omega\mu}} = \frac{1}{2\pi}\sqrt{\frac{10^7 \rho}{\mu_r f}} \tag{3}$$

In the abovegiven dependency the variable $f = f_2$ is frequency of rotor current, ρ is Ohmic resistivity of conductor.

The skin depth σ determines active area of bar cross-section penetrated by the current at given frequency f_2, hence, the rotor resistance depends on σ

$$R_r = R_{rn} \frac{S_p}{\sigma b_{r1}} \approx R_{rn} \frac{h_r}{\sigma} = k_r R_{rn} \tag{4}$$

where R_{rn} is rotor resistance transformed to the stator side with the neglect of skin effect, S_p is area of bar cross-section, b_{r1} is width of rotor slot equal approximately to the width of bar. Variable k_r is the function of square root of flowing current's frequency, what may be expressed as follows

$$k_r = \sqrt{\frac{f_2}{f_{2gr}}} \tag{5}$$

where f_{2gr} is frequency of rotor current for which the skin depth is equal to the height of rotor bar.

In some literature sources, e.g. [3,4], the dependency describing variable k_r is given as a complicated function of current's frequency and conductor's Ohmic resistivity

$$k_r = \xi \frac{\operatorname{sh} 2\xi + \sin 2\xi}{\operatorname{ch} 2\xi - \cos 2\xi}, \quad \xi = 2\pi h_r \sqrt{\frac{b_r}{b_s}\frac{f_2}{\rho_r} \cdot 10^{-7}} \tag{6}$$

where h_r is height of rotor bar, b_r is width of rotor bar, b_s is width of rotor slot, f_2 is frequency of rotor current, ρ_r is resistivity of rotor bars. Both functions i.e. the proposed one (5) and the function known from the literature sources (6) are presented in Fig. 1, whereupon the parameters of the exemplary induction motor were accepted.

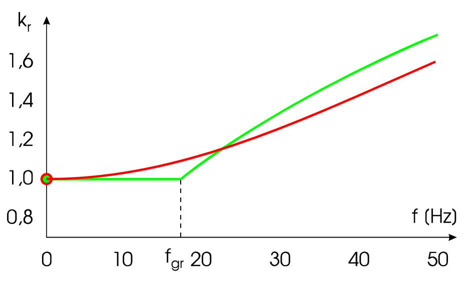

Fig. 1 Variable k_r describing variation of rotor resistance as a consequence of skin effect; the broken line is the proposed approximation, whereas the smooth line is the approximation known from the literature sources

The advantage of approximation known from the literature sources is higher accuracy whereas its disadvantage is complication and discontinuity of function describing this approximation at current's frequency equal to zero.

The variable k_l describing variation of rotor leakage inductance, known from the literature sources, is also defined as a function of flowing current's frequency

$$k_l = \frac{3}{2\xi}\frac{\operatorname{sh} 2\xi + \sin 2\xi}{\operatorname{ch} 2\xi - \cos 2\xi} \tag{7}$$

Comparing the dependencies (6) and (7), by analogy to (4) the simplified dependency describing varying rotor leakage inductance can be defined

$$L_{\sigma r} \approx \frac{L_{\sigma rn}}{k_r} \tag{8}$$

where $L_{\sigma rn}$ is rotor leakage inductance transformed to the stator side with the neglect of skin effect.

The above modifications, that consist in making variable the parameters of rotor winding, are introduced into circuit mathematical model of induction motor consisted of voltage equations describing stator and rotor windings and expressed by spatial vectors

$$\underline{u}_s = R_s \underline{i}_s + \frac{d}{dt}\underline{\psi}_s + j\omega_a \underline{\psi}_s \tag{9}$$

$$\underline{u}_r = R_r \underline{i}_r + \frac{d}{dt}\underline{\psi}_r + j(\omega_a - p_b\omega_m)\underline{\psi}_r$$

where $\underline{u}_s$, $\underline{u}_r$, $\underline{\psi}_s$, $\underline{\psi}_r$, $\underline{i}_s$, $\underline{i}_r$ are spatial vectors of voltages, fluxes and currents of stator and rotor, R_s, R_r are resistances of stator and rotor windings, ω_m is angular velocity of rotor, p_b is number of pole pairs. Equations (9) should be completed with dependencies (10) and equation of rotor motion (11)

$$\underline{\psi}_s = L_{\sigma s}\underline{i}_s + L_m(\underline{i}_s + \underline{i}_r), \quad \underline{\psi}_r = L_{\sigma r}\underline{i}_r + L_m(\underline{i}_s + \underline{i}_r) \tag{10}$$

$$J_m \frac{d\omega_m}{dt} = m_e - m_m, \qquad m_e = p_b \operatorname{Im}(\underline{\psi}_s^* \cdot \underline{i}_s) \tag{11}$$

where $L_{\sigma s}$, $L_{\sigma r}$, L_m are stator and rotor leakage inductances and magnetization inductance, respectively, J_m is moment of rotor inertia, m_e is electromagnetic torque of motor, m_m is load torque coming from working machine (external load) and motor losses (internal load).

Comparison of model-simulation results and experimental results

Measurements of direct starting a motor were made using the motor-generator set including: (a) squirrel-cage induction motor with the following rated parameters read from a data plate: type SZJe 34b; 1430 rpm; 4 kW; 220/380 V; 14,6/8,5 A; power factor 0,85 and (b) braking generator with the appropriate parameters. The motor was connected directly to the power grid of 400 V, then the phase current and phase voltage varying with time were registered as well as the instant power absorbed by motor phase winding was calculated on the basis of measured values of voltage and current.

Measurements of motor working with blocked rotor (short-circuit measurements) and measurements of loaded motor with rated load torque on motor shaft were also made in order to determine values of short-circuit parameters of motor (i.e. resistance and inductance of stator, resistance and inductance of rotor for short-circuit state and load state). Measurements of motor idle running were made in order to determine values of no-load parameters of motor.

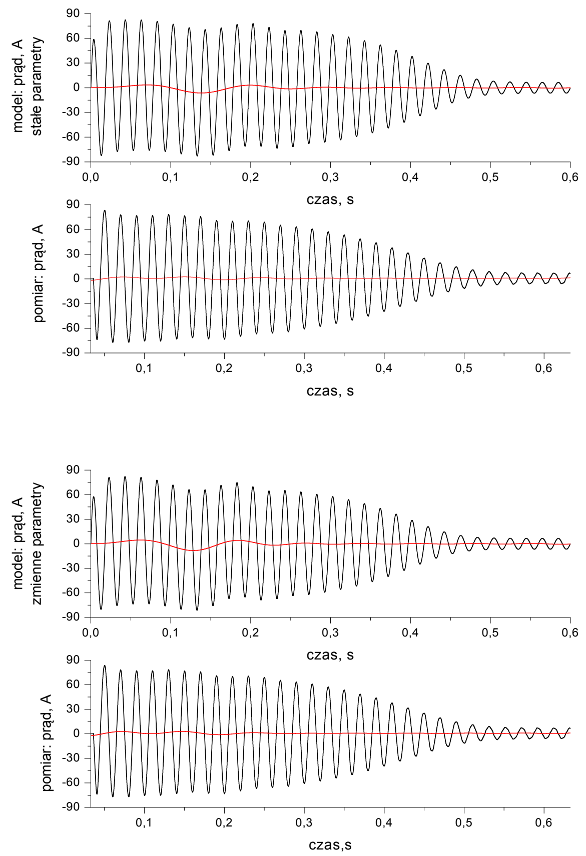

Fig. 2 Measured and simulated phase current versus time during starting the motor, where “prąd” is current, “stałe parametry” are constant parameters, “zmienne parametry” are variable parameters, “pomiar” is measurement, “czas” is time

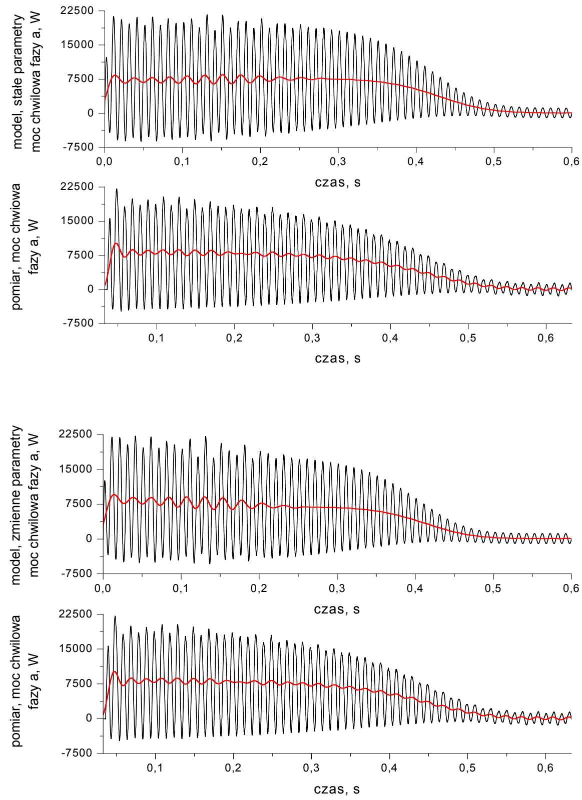

Fig. 3 Measured and simulated instant power absorbed by motor phase winding versus time during starting the motor, where “moc chwilowa fazy a” is instant power of a-phase winding

Final correction of motor parameters was made on the basis of measurements using test stand.

Comparison of time changes of measured and simulated variables using: (a) test stand and (b) models based on constant and variable parameters, is presented in Fig. 2 and 3, respectively; the filtered variables using low-pass filter FFT – 50 Hz are also presented.

The presented patterns of phase current contain fundamental-frequency component (50 Hz) and low-frequency component but magnitude of low-frequency component in both mathematical models is higher than in the case of measurement.

In the case of the mathematical model based on constant parameters the process of starting the motor is a bit longer in comparison with: (a) measurement and (b) mathematical model based on variable parameters. It can be concluded on the basis of duration of magnitude’s decreasing of current and instant power’s oscillations. The difference in duration is small because the squirrel-cage motor tested experimentally was a standard motor (without deep-bars). Thus, the increase of rotor resistance caused by the skin effect during starting the motor is small in comparison with the rotor resistance at rated slip. In case of deep-bar motor, much higher increase of rotor resistance caused by skin effect during starting the motor is expected. Consideration of variable parameters in mathematical model results in the faster decrease of magnitude of instant power’s oscillation similarly to the case of measured power.

Conclusions

As a result of carried out investigations it was concluded that in the case of circuit model of induction model based on constant parameters the process of starting a motor was longer than the process of staring a motor in the cases of measurements and proposed mathematical model based on the variable parameters. Duration of magnitude’s decreasing of phase current and instant power’s oscillations is similar in the cases of measurement and mathematical model of induction motor based on variable parameters and longer in the case of mathematical model of induction motor based on constant parameters.

REFERENCES

[1] Puchała A., *Dynamika maszyn i układów elektromechanicznych*, PWN Warszawa, 1977.

[2] Macek-Kamińska K., *Estymacja parametrów modeli matematycznych silników indukcyjnych dwuklatkowych i głębokożłobkowych*, Wyższa Szkoła Inżynierska w Opolu, Opole 1992.

[3] Rusek A., Model symulacyjno-komputerowy asynchronicznego silnika indukcyjnego z uwzględnieniem w modelu matematycznym nasycenia i zjawiska wypierania prądu, *Przegląd Elektrotechniczny*, 86 (2010), no. 12, 127-130

[4] Sudhoff S.D., Aliprantis D.C., Kuhn B.T., Chapman P.L., An induction machine model for predicting inverter-machine interaction, *IEEE Trans. on Energy Conver.*, 17 (2002) no. 2, 203–210

[5] Czaban A., Lis M., Mathematical model and analysis of the induction motor with a long elastic element considering distributed parameters, *Prace Naukowe Instytutu Maszyn, Napędów i Pomiarów Elektrycznych Politechniki Wrocławskiej, Zagadnienia maszyn, napędów i pomiarów elektrycznych*, T.1, Wrocław 2012, 224-230

***Author**: prof. ndzw. dr hab. inż. Andrzej Popenda, Politechnika Częstochowska, Instytut Elektrotechniki Przemysłowej, Al. Armii Krajowej 17, 42-200 Częstochowa, E-mail: popenda@el.pcz.czest.pl*