Evaluation of the boundaries of the solutions space for the Harmonic Cancellation Technique

Abstract. The Harmonic Cancellation Technique (HCT) is a modulation technique able to reduce the output voltage THD in Auxiliary Railway Power Supplies which supply both a linear and a well-known non linear load, helping the manufacturer to achieve the output voltage THD specifications. It is based on the pre-distortion of the inverter output voltage, through the analytical determination of the IGBTs switching events. So, along this paper the limitations of the available solutions space for the HCT is presented, along with some design parameters which may help to enlarge it.

Streszczenie. Technika redukcji harmonicznych (HCT) jest metodą modulacji umożliwiającą zmniejszenie wartości współczynnika THD w napięciu wyjściowym trakcyjnego źródła potrzeb własnych, zasilającego zarówno odbiorniki liniowe jak i dobrze znane odbiornik nieliniowe. Metoda ta, umożliwiająca producentom osiągnięcie wymaganego w specyfikacji współczynnika THD, bazuje na ocenie odkształcenia napięcia wyjściowego falownika, poprzez analityczne wyznaczenie chwil przełączania IGBT. W artykule przedstawiono ograniczenia techniki HCT oraz kilka parametrów konstrukcyjnych, umożliwiających rozszerzenie możliwości jej zastosowania. (Ocena możliwych obszarów zastosowania techniki modulacji HCT).

Keywords: Auxiliary Railway Power Supplies, inverters, low THD, inverters modulation.

Introduction

Auxiliary Railway Power Supplies (ARPS) implemented by means of an inverter is a widespread application of the power electronics. The interface characteristics that must be satisfied by an ARPS are detailed in [1]. Regarding the ARPS output voltage characteristics, it is said that the maximum allowable THD at its output shall be established as a trade-off between the car builder and the converter manufacturer; since it is a compromise between the cost of the inverter filter, maintainability and the harmonic content that the loads are able to accommodate.

Therefore, depending on the loads connected to the inverter output, the maximum allowable THD may be different. As an example, three commonly used maximum THD values are: 4%, 7% and 14%.

Keeping the output voltage THD within the limits is an important challenge, especially when the inverter must feed simultaneously linear and nonlinear loads, since the nonlinear loads operation provokes additional low frequency harmonic content which distort the power supply output voltage and so increases significantly the THD.

Along the work presented in this paper, a typical auxiliary railway power supply (ARPS) has been considered in order to develop and check the proposed modulation technique. The auxiliary power supply is used to feed low and medium voltage equipment from the catenary (lightning, air conditioning, battery charger, etc.).

The typical ARPS is formed by an input filter which provides overvoltage protection to the converter placed downstream, a Voltage Source Inverter (VSI), an AC output filter (L\(_\text{F}\), C\(_\text{F}\)) which attenuates the switching ripple and finally the auxiliary services of the train (modelled as an AC linear load) and a battery charger. See Fig. 1.

Although some other options can be considered for the battery charger (PWM rectifiers [2], [3]), it is usually implemented by means of a three phase full bridge thyristor rectifier due to its robustness and cost effective characteristics, despite its non linear nature.

The presence of this nonlinear load provokes additional low frequency harmonics content which flow back through the impedances of the system and distort the AC filter output voltage. Thus the output voltage THD increases and doesn’t comply with the specifications.

State of the art solutions reduce the output voltage THD by means of either multi-loop strategies ([4]-[6]) or active filtering ([8]-[10]). However neither multi-loop strategies nor active filtering are applicable to the case of an ARPS due to stability constraints or unaffordable increase of cost, size and complexity, respectively [11].

Therefore, the pre-distortion of the inverter output voltage is a convenient strategy which is able to cancel out the low frequency content due to the non linear load without any significant increment of the size and cost of the system. The switching events are calculated analytically as it is summarized in Section II.

The present paper deals with the boundaries that limit the feasible solutions space of the Harmonic Cancellation Technique. The boundaries for a given design have been explored and the critical operating point, in terms of existence of solution, is identified. It should be remarked that the feasible solutions space is not only determined by the operating conditions but it is also influenced by the converter design parameters. So, an insight of the converter design parameters which may help to enlarge the ranges of operating conditions for which HCT can be applied are also considered in this paper.

Along Sections IV and V the limitations of the feasible solutions space and the design parameters which might help to enlarge the solutions space are described. In section VI the experimental performance of the Harmonic Cancellation Technique is shown. Finally some conclusions are extracted.
Operation Principle

The Harmonic Cancellation Technique (hereinafter HCT) aims to reduce the output voltage THD through the proper pre-distortion of the inverter output voltage. Since the pre-distortion of the inverter output voltage is calculated analytically for each given set of operating conditions, the HCT is a technique suitable in those cases in which the non linear load is concentrated and well known.

Let’s consider the per phase equivalent circuit (Fig. 2) of the AC power supply in Fig. 1. The current drawn by the non linear load, referred to the primary side of the transformer is represented by means of a current source. The low frequency current harmonics drawn by the nonlinear load flow through the filter inductance and provoke a voltage drop across it ($v_f$) which distorts the filter output voltage ($v_o$).

The proposed modulation technique is able to generate an inverter output voltage formed by a controlled fundamental component as well as a set of low frequency harmonics which added to the ones across the filter inductance, they cancel each other out. Thus, the inverter output voltage has been represented qualitatively through a inductance, they cancel each other out. Thus, the inverter output voltage ($v_o$) can be described by a Fourier series formed by odd terms in sine and cosine.

Additionally, it should be taken into account the following considerations in order to determine the harmonic content of $v_{AB}$:

- The required fundamental component is given as an specification. So, applying this condition on (1), equation (2) is obtained.

$$v_{AB}\text{-spec} = v_{AB\text{-}1}G_F\left(j\omega_1\right) - i_{3\text{-}1}G_F\left(j\omega_3\right)X_F\left(j\omega_1\right)$$

In order to cancel out the low frequency harmonics, the voltage at the filter output for each of the low frequency harmonics considered must be zero. So, to achieve a successful cancelation, each harmonic component must satisfy (3).

$$0 = v_{AB\text{-}n} - i_{3\text{-}n}X_F\left(jn\omega_1\right) \text{ for } n\neq1$$

Analytical determination of the switching events

Although the development of the HCT has been already published in [11], along this section a brief summary has been included in order to improve the understanding of the following ones.

The key idea of the modulation technique is grounded on the generation at the inverter output of a set of low frequency harmonics which cancel out those across the filter inductance due to the current drawn by the non linear load. Therefore, control over both the phase and module of each harmonic is needed. Additionally, in order to prevent the presence of even harmonics in the system, half wave symmetry is required. Thus, the inverter output voltage ($v_o$) can be described by a Fourier series formed by odd terms in sine and cosine.

The Fourier coefficients of $v_o$ as a function of the switching angles ($\alpha_p$) are known [12] and provided in (4) and (5).

$$A_{n\text{-}vd} = \frac{4}{\pi} \frac{V_o}{2n} \sum_{p=1}^{\infty} (-1)^p \sin(n\alpha_p)$$

$$B_{n\text{-}vd} = \frac{4}{\pi} \frac{V_o}{2n} \sum_{p=1}^{\infty} (-1)^{p+1} \cos(n\alpha_p)$$

where: N - Number of switching angles to be determined, $V_{in}$ - Inverter DC input voltage, $\alpha_0$ - Switching angles, n - Harmonic order.

As in any other inverter, the voltage of the middle point of any leg reproduces the switching pattern of its upper switch. So, if the desired pre-distorted voltage at the middle point of a leg A ($v_{AB}$) is determined, the switching patterns of the six switches can be derived.

Therefore, the first step to determine the switching events of the inverter must be the characterization of the desired inverter output voltage ($v_{AB}$). In order to determine the inverter output voltage, it has been used the basic equations which define the system operation: (2) and (3).

After the characterization of the current drawn by the non linear load and some algebraic manipulation of the equations, the Fourier coefficients of $v_{AB}$ are obtained as a function of the operating point of the converter and its physical parameters. The complete theoretical development is already published in [11].

Finally, replacing the obtained Fourier coefficients in (4) and (5), the equation system in (6) and (7) is obtained. Solving this system of equations for a given set of operating conditions will lead to a switching pattern able to modulate the inverter output and cancel out the low frequency content due to the non linear load operation.

The switching patterns calculated for different sets of operating conditions are stored within a look-up table (control table) in order to provide harmonic cancellation at any time during the regular operation of the converter. The control table will be calculated taking into account the ranges of variation of the operating conditions involved.
Component and 6 additional non triplen odd harmonics: 5th, 7th, 11th, 13th, 17th and 19th). In Fig. 3 the evolution of the 14 switching events evolution vs. inverter supply voltage for four different conditions at the battery charger output.

Thus, it can be said that the limiting factor is the voltage drop across the filter inductance due to the current drawn by the non linear load. As the current drawn by the battery charger increases, the voltage drop across the filter inductance increases and the minimum required input voltage to achieve a feasible solution increases.

Then, part of the available energy at the inverter input will be used to generate those low frequency harmonics. And so, as the low frequency harmonic content of the current drawn by the battery charger increases, the voltage across the filter inductance increases, and the low frequency harmonics at the inverter output will also be bigger. So a bigger amount of the available energy at the input will be injected at low frequencies.

Therefore, the worst case for the generation of the switching events will be the following combination of operating conditions:

- Minimum input voltage
- Maximum power delivered by the non linear load
- Minimum voltage at the non linear load

However, the feasible solutions space does not depend exclusively on the operating conditions, but it can be modified if the design of the converter is taken into account. This is, some parameters of the converter may help to enlarge the feasible solutions space. They are explained along the following section.

**Design parameters which enlarge the feasible solutions space**

Take into account the simplified per-phase equivalent of the converter shown in Fig. 4. The current drawn by the non linear load is represented by means of a current source (i'3). Where, i'3 is the current drawn by the battery charger seen at the primary side of the transformer. This current flows through the filter inductance provoking a voltage drop.
across that inductance ($V_F$). Therefore, it is easily observed that the size of the filter inductance directly affects the size of the low frequency harmonics which the inverter must generate to cancel out those due to the non linear load. Then, the effect of the size of this inductance is shown in Fig. 5.

![Fig. 4 Simplified per-phase equivalent of the Auxiliary Railway Supply which supplies a linear and a non linear load](image)

On Fig. 5 the minimum input voltage required to achieve a feasible solutions is represented against the power delivered by the battery charger. Additionally, in order to illustrate the influence of the filter inductance on the feasible solutions space, three different values of filter inductance have been considered, as well as two different DC output voltages. It can be observed that the use of lower DC output voltages penalizes the feasible solutions space, as it has been previously established.

Regarding the influence of the filter inductance, the tendency is the same regardless the DC output voltage. This is, as the filter inductance value decreases the feasible solutions space is increased; since the minimum required voltage is lower.

Another design parameter involved, is the transformer ratio between its primary and tertiary sides ($n_{13}$). Since the modules of the harmonic components which flow through the filter inductance are affected by this transformer ratio. As it has been illustrated previously, the larger the non linear load current, the bigger the minimum required input voltage. Then, if the number of turns of the transformer tertiary side is reduced, then the feasible solutions space will be enlarged.

Actual converters are subject to parasitic voltage drops such as parasitic resistances of the filter inductances, etc. Even though they must be taken into account within the calculation of the switching events; their effect on the feasible solutions space is negligible compared to the effect of the operating conditions and the aforementioned design parameters.

**HCT Experimental Performance**

The experimental performance of the Harmonic Cancellation Technique regarding the output voltage THD (9) has been carried out on a scaled-down prototype which characteristics are summarized in Table II. Considering an inverter input voltage range from 325V to 475V and a DC output power from 5% to 40%, the HCT is able to provide a successful performance in the whole range.

$$THD = \sqrt{\sum_{n=2}^{40} \frac{V_o^2}{V_{o,n}^2}}$$

In order to illustrate the HCT performance, in Fig. 6 a comparison between the well known Selective Harmonic Elimination (SHE) versus HCT has been carried out. As it can be seen, the THD is reduced when the inverter is modulated with the HCT.

![Fig. 6: Comparison of SHE vs. HCT for I=5.2A (CCM). vA voltage in the middle point of the branch A of the inverter referred to ground, vo line to line voltage on the secondary side of the transformer, i3 current drawn by the nonlinear load](image)

![Fig. 7: Output voltage THD vs. %Pac for an inverter input voltage of 325V](image)

Fig. 6 only shows a trial for an inverter input voltage of 375V, but it has been carried out a sweep regarding the
input voltage and the DC output power. The obtained results are shown in Fig. 7 and Fig. 8 for an inverter input voltage of 325V and 475V, respectively. Three different DC output powers have been considered for each voltage. It can be observed that the THD is reduced in all cases. Since the high frequency THD is higher for higher input voltages, the THD reduction is slightly better for low input voltages than for high input voltages.

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Fig. 8: Output voltage THD vs. %Pac for an inverter input voltage of 475V

Conclusions

Along the present paper, the feasible solutions space for the Harmonic Cancellation Technique (HCT) has been studied. The HCT is able to pre-distort the inverter output in order to cancel out the harmonic content due to a non linear load performance. And so reduce the THD at the output voltage of the auxiliary railway power supply. For each set of operating conditions (V_in, V_DC, P_DC) the proper switching pattern is determined analytically. However, not all the combinations of the aforementioned variables lead to a feasible solution. Thus, the determination of the feasible solutions space is needed to guarantee a successful cancellation of the low frequency content along the whole range of variation of the converter operating conditions.

For any converter the most restrictive operating conditions in terms of achieving a feasible solution, are: minimum input voltage, maximum power delivered by the non linear load and minimum voltage at the non linear load. So, if a solution is found in these conditions then the HCT can be used for the whole converter range of operation.

However the feasible solutions space does not depend only on the operating conditions, some physical parameters of the converter must be taken also into account. In order to enlarge the feasible solutions space, the filter inductance and the primary-tertiary transformer ratio should be designed as small as possible.

Acknowledgements: The authors would like to thank SEPSA for they support along the development of this work. This work has been also supported by Ministerio de Educación y Ciencia (Spain), by means of the research projects FLAME (DPI: 2010:121110-C02-02) and SAUCE (DPI: 2009-12501).

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