Multiple co-channel interferers influence on the selection diversity system over Rayleigh fading channels

Abstract. Selection combining (SC) diversity receiver experiencing an arbitrary number of multiple, independent, equal power co-channel interferers, in the presence of Rayleigh fading channels was analyzed. Closed form expressions are obtained for the output SIR’s probability density function (PDF) and cumulative distribution function (CDF). In order to show the effects of the number of multiple interferers, diversity order and input SIR unbalance to the system performances, an outage probability (OP) analysis is derived. Another important measure of the system’s performances, an average bit error probability (ABER) is efficiently evaluated for non-coherent modulation schemes such as binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK).

Keywords: co-channel interference, correlated fading channels, probability distribution, Rayleigh fading, selection diversity combining

Introduction
In cellular communication systems, usually a large number of low-power transmitters broadcast a signal in relatively small geographic areas - cells. Commercial and military cellular systems tend to conserve the available spectrum by reusing allocated frequency channels in areas that are geographically located as close to each other as possible. Unfortunately, due to frequency reuse, signals from two or more channels operating at the same frequency, but from different locations, interfere. Amount of co-channel interference determines limitation in distance for reusing frequency channels. In general, the power of any interfering signal diminishes with increasing distance between interfering users. A carrier frequency can be reused if the interference level is reduced sufficiently by separation between the co-channel calls. In order to determine the practical system implementation which satisfies the predetermined minimum performance levels, it is necessary to analyze how the interference as a general distortion affects well-accepted criterions of performance of wireless systems, such as outage probability and average bit error probability [1]. The interference level can be measured through the signal-to-interference power ratio (SIR). The SIR ratio is the primary criteria used in designing frequency reuse plans.

Multipath fading can also seriously degrade performances of wireless communication systems. In cellular radio systems, various techniques for reducing fading effects and influence of co-channel interference are used. When multiple receiver antennas are used space diversity is an efficient method for amelioration system’s quality of service (QoS). One of the least complicated space combining methods is selection combining (SC). Other space combining techniques like equal gain combining (EGC) and maximal ratio combining (MRC) require all or some of the amount of the channel state information of received signal, which increases its complexity. SC receiver process only one of the diversity branches, and is much simpler for practical realization, in opposition to these combining techniques. In fading environments as in cellular systems where the level of the co-channel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [2]. This type of SC in which the branch with the highest SIR is selected, can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54). Most of the recently published papers assume independent fading between the diversity branches and also between the co-channel interferers. The effect of co-channel interference on the performance metrics of wireless communication system has been extensively analyzed [3-5]. In [3] performance analysis of optimum combining with multiple co-channel interferers over Rayleigh fading channels were presented. In [4] closed-form expressions for out-age probabilities of mobile radio channels experiencing multiple co-channel, independent Nakagami-m interferers are derived. SIR based analysis of dual branch SC was presented in [5], but for the case of single interferer over each channel.

System model
Assume that there are $M_i$ independent equal power Rayleigh distributed interferers over $i$-th branch of the SC diversity system with arbitrary number of branches. The latter assumption is a reasonable one when all interfering signals are at approximately the same distance from the mobile station. The independent instantaneous interfering signals are added together to produce the resultant instantaneous interfering signal at the $i$-th branch of diversity system can be written as:

$$I_i = I_{i1} + I_{i2} + I_{i3} + \ldots + I_{iM_i}$$

with the total probability density function given by [6] as:

$$p_{y_i}(y_i) = \frac{y_i^{M_i-1} \exp(-\frac{y_i}{\Omega_{yi}})}{\Omega_{yi}^M (M_i - 1)!}$$

where $\Omega_{yi}$ is the total interference power at the $i$-th branch of the diversity system, given in the function of the average power of each interferer $\Omega_{yi}$ as:

$$\Omega_{yi} = \frac{2M_i \Gamma(M_i + \frac{1}{2})}{\Gamma(M_i)}$$

and $\Gamma(x)$ denotes the Gamma function [7]. The desired signal envelopes on the $i$-th diversity branch also follow the Rayleigh fading distribution, whose probability density func-
tion is given by:

$$p_{x_i}(x_i) = \frac{\exp\left(-\frac{x_i}{\Omega x_i}\right) I_{x_i}}{\Omega x_i}$$  \hspace{1cm} (4)$$

Let $\lambda_i = x_i/y_i$ be the SIR at the $i$-th ($i = 1, 2, \ldots, N$) diversity branch of the SC receiver. The joint probability density function of independent random variables $\lambda_1, \lambda_2, \ldots, \lambda_N$ (since branches are not correlated), can be written as:

$$p_{\lambda_1,\ldots,\lambda_N}(\lambda_1, \ldots, \lambda_N) = p_{\lambda_1}(\lambda_1) \ldots p_{\lambda_N}(\lambda_N)$$  \hspace{1cm} (5)$$

After substituting (2) and (4) into (5) and some mathematical manipulation pervious expression can be written in the form of:

$$p_{\lambda_i}(\lambda_i) = \left(\frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)^{\lambda_i} \frac{\Gamma(M_i+1)}{(\Gamma(1/2))^{M_i+1}} (M_i - 1)!$$  \hspace{1cm} (6)$$

with $S_i = \frac{\Omega x_i}{\lambda_i}$ being the average SIR’s at the $i$-th input branch of the selection combiner system. Similarly, joint cumulative distribution function can be written in the form of:

$$F_{\lambda_1,\ldots,\lambda_N}(\lambda_1, \ldots, \lambda_N) = F_{\lambda_1}(\lambda_1) \ldots F_{\lambda_N}(\lambda_N)$$  \hspace{1cm} (7)$$

$$F_{\lambda_i}(\lambda_i) = \int_0^{\lambda_i} p_{\lambda_i}(\xi_i) d\xi_i$$

After substituting (6) into (7) and some mathematical manipulation pervious expression can be written in the form of:

$$F_{\lambda_i}(\lambda_i) = \left(\frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)^{\lambda_i} \frac{\Gamma(M_i+1/2)}{(\Gamma(1/2))^{M_i+1}} (M_i - 1)!$$  \hspace{1cm} (8)$$

$$\times 2F_1\left(1, 2; 1 - M_i; \lambda_i + \frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)$$

with $2F_1\left(u_1, u_2; u_3; x\right)$, being the Gaussian hyper-geometric function [7]. The selection combiner chooses and outputs the branch with the largest SIR:

$$\lambda = \lambda_{\text{out}} = \max (\lambda_1, \lambda_2, \ldots, \lambda_N)$$

The CDF of multibranch SIR-based SC output could be derived from (7) by equating the arguments $\lambda_1 = \lambda_2 = \ldots = \lambda_N$ as:

$$F_{\lambda}(\lambda) = F_{\lambda_1}(\lambda) \ldots F_{\lambda_N}(\lambda) = \prod_{i=1}^{N} F_{\lambda_i}(\lambda);$$  \hspace{1cm} (9)$$

$$\times 2F_1\left(1, 2; 1 - M_i; \lambda + \frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)$$

The probability density function at the output of the SC can be found as:

$$p_{\lambda}(\lambda) = \frac{d}{d\lambda} F_{\lambda}(\lambda) = \sum_{i=1}^{N} p_{\lambda_i}(\lambda_i) \prod_{j=1, j \neq i}^{N} F_{\lambda_j}(\lambda);$$  \hspace{1cm} (10)$$

$$p_{\lambda}(\lambda) = \sum_{i=1}^{N} \left(\frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)^{\lambda_i} \frac{\Gamma(M_i+1)}{(\Gamma(1/2))^{M_i+1}} (M_i - 1)! \times 2F_1\left(1, 2; 1 - M_i; \lambda + \frac{S_i M_i}{2(\Gamma(M_i+1/2))^{M_i}}\right)$$

Fig. 1 shows the PDF of output SIR for various values of the number of multiple interferers and diversity branches.

Outage probability

Outage probability $P_{\text{out}}$ is one of the accepted performance measure for diversity systems operating in fading environments. Pout is a measure of the system performance, used to control the co-channel interference level, helping the designers of wireless communications systems in order to meet the QoS and grade of service (GoS) demands. In the interference limited environment, $P_{\text{out}}$ is defined as the probability that the output SIR of the SC falls below a given outage threshold $\gamma_{\text{th}}$, also known as a protection ratio [5]. Protection ratio depends on modulation technique and expected QoS.

$$P_{\text{out}} = P(t < \gamma) = \int_0^{\gamma} p_{\epsilon}(t) dt = F_{\epsilon}(t)$$  \hspace{1cm} (12)$$

Outage probability versus normalized parameter $S_1/\gamma$ for balanced and unbalanced ratio of SIR at the input of the branches and various values of the number of multiple interferers, diversity order is shown on Fig. 2.

From Fig. 2, we can see how the outage probability increases due to growth of interference domination when the number of multiple independent co-channel interferers.
Average error probability

The average error probability at the SC output is derived for non-coherent and binary signaling according to following expressions:

$$P_e = \int_0^\infty p_\varepsilon(t) \frac{1}{2} \exp(-gt)$$

Where $g$ denotes modulation constant, i.e., $g = 1$ for BDPSK and $g = 1/2$ for BFSK. Substituting (11) in (13) numerically obtained average error probability is shown on Figs. 3 and 4 for balanced and unbalanced ratio of SIR at the input of the branches and various values of diversity order and the number of multiple interferers [5].

A comparison of curves shows better performance of BDPSK modulation scheme against BFSK modulation scheme. Also is more evident how ABER increases at both figures when the number of multiple independent co-channel interferers increases due to growth of interference domination. ABER performance behavior improves for the constant number of co-channel interferers as the diversity order increases. If we want to achieve the same quality of the transmission (for example outage probability of $10^{-4}$), we need higher level of average input SIR for dual branch case then for the triple branch case (for example of above 5 dB for BDPSK and 6dB for BFSK). Input SIR unbalance at the diversity branches deteriorates system performances.

Conclusions

Performance analysis of a multibranch SIR-based SC system operating over Rayleigh fading channels where each channel experiences an arbitrary number of multiple, independent, Rayleigh co-channel interferers with equal average powers, was derived. Closed form expressions for the SIR's PDF and CDF at the output of the combiner are derived. Capitalizing on this, important performance measures like OP and average BER, for some non-coherent modulation techniques, are graphically analyzed, in order to show the effects of the number of multiple interferers, diversity order, and input SIR unbalance to the system performances.

REFERENCES


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