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Multiple co-channel interferers influence on the selection diversity system over Rayleigh fading channels

Abstract. *Selection combining (SC) diversity receiver experiencing an arbitrary number of multiple, independent, equal power co-channel interferers, in the presence of Rayleigh fading channels was analyzed. Closed form expressions are obtained for the output SIR's probability density function (PDF) and cumulative distribution function (CDF). In order to show the effects of the number of multiple interferers, diversity order and input SIR unbalance to the system performances, an outage probability (OP) analysis is derived. Another important measure of the system's performances, an average bit error probability (ABER) is efficiently evaluated for non-coherent modulation schemes such as binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK).*

Streszczenie. *Przeanalizowano wpływ interferencji między wieloma niezależnymi kanałami(w obecności kanału Rayleigha z zanikami) na wybór najlepszego sygnału. Przedstawiono wyrażenie na prawdopodobieństwo parametru SIR (signal to interference ratio) oraz średnie prawdopodobieństwo bitu błędu (ABER).(**Wpływ interferencji między wieloma niezależnymi kanałami(w obecności kanału Rayleigha z zanikami) na wybór najlepszego sygnału**)*

Keywords: co-channel interference, correlated fading channels, probability distribution, Rayleigh fading, selection diversity combining
Słowa kluczowe: interferencja między kanałami, korelacja kanałów z zanikami.

Introduction

In cellular communication systems, usually a large number of low-power transmitters broadcast a signal in relatively small geographic areas - cells. Commercial and military cellular systems tend to conserve the available spectrum by reusing allocated frequency channels in areas that are geographically located as close to each other as possible. Unfortunately, due to frequency reuse, signals from two or more channels operating at the same frequency, but from different locations, interfere. Amount of co-channel interference determines limitation in distance for reusing frequency channels. In general, the power of any interfering signal diminishes with increasing distance between interfering users. A carrier frequency can be reused if the interference level is reduced sufficiently by separation between the co-channel calls. In order to determine the practical system implementation which satisfies the predetermined minimum performance levels, it is necessary to analyze how the interference as a general distortion affects well-accepted criterions of performance of wireless systems, such as outage probability and average bit error probability [1]. The interference level can be measured through the signal-to-interference power ratio (SIR). The SIR ratio is the primary criteria used in designing frequency reuse plans.

Multipath fading can also seriously degrade performances of wireless communication systems. In cellular radio systems, various techniques for reducing fading effects and influence of co-channel interference are used. When multiple receiver antennas are used space diversity is an efficient method for amelioration system's quality of service (QoS). One of the least complicated space combining methods is selection combining (SC). Other space combining techniques like equal gain combining (EGC) and maximal ratio combining (MRC) require all or some of the amount of the channel state information of received signal, which increase its complexity. SC receiver process only one of the diversity branches, and is much simpler for practical realization, in opposition to these combining techniques. In fading environments as in cellular systems where the level of the co-channel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [2]. This type of SC in which the branch with the highest SIR is selected, can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54). Most of the recently the published papers assume independent fading between the diversity branches and also between the co-channel interferers. The effect of co-channel interference on the performance metrics of wireless communication system has been extensively analyzed [3-5]. In [3] performance analysis of optimum combining with multiple co-channel interferers over Rayleigh fading channels were presented. In [4] closed-form expressions for outage probabilities of mobile radio channels experiencing multiple co-channel, independent Nakagami-m interferers are derived. SIR based analysis of dual branch SC was presented in [5], but for the case of single interferer over each channel.

System model

Assume that there are M_i independent equal power Rayleigh distributed interferers over i-th branch of the SC diversity system with arbitrary number of branches. The latter assumption is a reasonable one when all interfering signals are at approximately the same distance from the mobile station. The independent instantaneous interfering signals are added together to produce the resultant instantaneous interfering signal at the i-th branch of diversity system can be written as:

$$I_i = I_{1i} + I_{2i} + I_{3i} + ... + I_{Mi} \qquad (1)$$

with the total probability density function given by [6] as:

$$p_{y_i}(y_i) = \frac{{y_i}^{M_i-1}\exp(-\frac{y_i}{\Omega_{y_i}})}{\Omega_{y_i}^{M_i}(M_i-1)!} \qquad (2)$$

where Ω_{y_i} is the total interference power at the i-th branch of the diversity system, given in the function of the average power of each interferer Ω_{0y_i} as:

$$\Omega_{y_i} = \frac{2\Omega_{0y_i}}{M_i}\left(\frac{\Gamma(M_i+\frac{1}{2})}{\Gamma(\frac{1}{2})}\right)^{\frac{1}{M_i}} \qquad (3)$$

and $\Gamma(x)$ denotes the Gamma function [7]. The desired signal envelopes on the i-th diversity branch also follow the Rayleigh fading distribution, whose probability density function is given by:

$$p_{x_i}(x_i) = \frac{\exp(-\frac{x_i}{\Omega_{x_i}})}{\Omega_{x_i}} \tag{4}$$

Let λ_i = x_i/y_i be the SIR at the i-th ($i = 1, 2...N$) diversity branch of the SC receiver. The joint probability density function of independent random variables $\lambda_1,\ \lambda_2,\ ...,\lambda_N$(since branches are not correlated), can be written as:

$$\begin{aligned} p_{\lambda_1,...,\lambda_N}(\lambda_1, ..., \lambda_N) &= p_{\lambda_1}(\lambda_1) \ldots p_{\lambda_N}(\lambda_N) \\ p_{\lambda_i}(\lambda_i) &= \int\limits_0^\infty y_i p_{x_i}(y_i\lambda_i) p_{y_i}(y_i) dy_i \end{aligned} \tag{5}$$

After substituting (2) and (4) into (5) and some mathematical manipulation pervious expression can be written in the form of:

$$p_{\lambda_i}(\lambda_i) = \frac{\left(\frac{S_i M_i}{2\left(\frac{\Gamma(M_1+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right)^{M_i} \Gamma(M_i+1)}{\left(\lambda_i + \frac{S_i M_i}{2\left(\frac{\Gamma(M_1+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right)^{M_i+1} (M_i-1)!} \tag{6}$$

with $S_i = \frac{\Omega_{x_i}}{\Omega_{0y_i}}$ being the average SIR's at the i-th input branch of the selection combiner system. Similarly, joint cumulative distribution function can be written in the form of:

$$\begin{aligned} F_{\lambda_1,...,\lambda_N}(\lambda_1, ..., \lambda_N) &= F_{\lambda_1}(\lambda_1) \ldots F_{\lambda_N}(\lambda_N) \\ F_{\lambda_i}(\lambda_i) &= \int\limits_0^{\lambda_i} p_{t_i}(t_i) dt_i \end{aligned} \tag{7}$$

After substituting (6) into (7) and some mathematical manipulation pervious expression can be written in the form of:

$$\begin{aligned} F_{\lambda_i}(\lambda_i) = &\left(\lambda_i + \frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right) \frac{\Gamma(M_i+1/2)}{\Gamma(1/2)} \\ &\times {}_2F_1\left(1, 2; 1-M_i; \lambda_i + \frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right) \end{aligned} \tag{8}$$

with ${}_2F_1(u_1, u_2; u_3; x)$, being the Gaussian hyper-geometric function [7] The selection combiner chooses and outputs the branch with the largest SIR:

$$\lambda = \lambda_{out} = max\,(\lambda_1, \lambda_2, ..., \lambda_N) \tag{9}$$

The CDF of multibranch SIR-based SC output could be derived from (7) by equating the arguments $\lambda_1 = \lambda_2 = ... = \lambda_N$ as:

$$\begin{aligned} F_\lambda(\lambda) &= F_{\lambda_1}(\lambda) \ldots F_{\lambda_N}(\lambda) = \prod_{i=1}^N F_{\lambda_i}(\lambda); \\ F_\lambda(\lambda) &= \prod_{i=1}^N \left(\lambda + \frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right) \frac{\Gamma(M_i+1/2)}{\Gamma(1/2)} \\ &\times {}_2F_1\left(1, 2; 1-M_i; \lambda + \frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right) \end{aligned} \tag{10}$$

The probability density function at the output of the SC can be found as:

$$\begin{aligned} p_\lambda(\lambda) &= \frac{d}{d\lambda} F_\lambda(\lambda) = \sum_{i=1}^N p_{\lambda_i}(\lambda) \prod_{j=1, j\neq i}^N F_{\lambda_j}(\lambda); \\ p_\lambda(\lambda) &= \sum_{i=1}^N \frac{\left(\frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right)^{M_i} \Gamma(M_i+1)}{\left(\lambda_i + \frac{S_i M_i}{2\left(\frac{\Gamma(M_i+1/2)}{\Gamma(1/2)}\right)^{M_i}}\right)^{M_i+1} (M_i-1)!} \\ &\prod_{j=1, j\neq i}^N \left(\lambda + \frac{S_j M_j}{2\left(\frac{\Gamma(M_j+1/2)}{\Gamma(1/2)}\right)^{M_j}}\right) \frac{\Gamma(M_j+1/2)}{\Gamma(1/2)} \\ &\times {}_2F_1\left(1, 2; 1-M_j; \lambda + \frac{S_j M_j}{2\left(\frac{\Gamma(M_j+1/2)}{\Gamma(1/2)}\right)^{M_j}}\right) \end{aligned} \tag{11}$$

Fig. 1 shows the PDF of output SIR for various values of the number of multiple interferers and diversity branches.

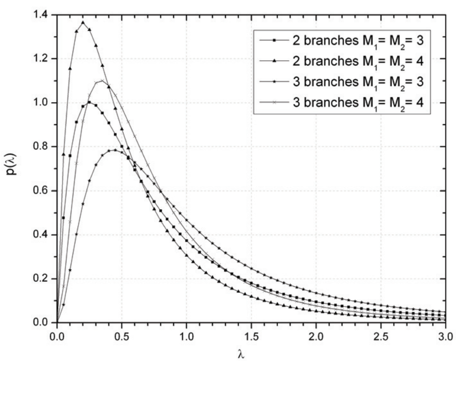

Fig. 1. PDF of output SIR for various values of the number of multiple interferers and diversity branches.

Outage probability

Outage probability P_{out} is one of the accepted performance measure for diversity systems operating in fading environments. Pout is a measure of the system performance, used to control the co-channel interference level, helping the designers of wireless communications system's in order to meet the QoS and grade of service (GoS) demands. In the interference limited environment, P_{out} is defined as the probability that the output SIR of the SC falls below a given outage threshold γ_{th} also known as a protection ratio [5]. Protection ratio depends on modulation technique and expected QoS.

$$P_{out} = P_R(\varepsilon < \gamma) = \int\limits_0^\gamma p_\varepsilon(t) dt = F_\varepsilon(t) \tag{12}$$

Outage probability versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches and various values of the number of multiple interferers, diversity order is shown on Fig. 2.

From Fig. 2. we can see how the outage probability increases due to growth of interference domination when the number of multiple independent co-channel interferers.

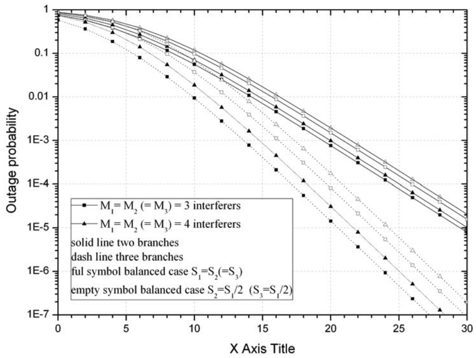

Fig. 2. Outage probability versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches and various numbers of diversity order and multiple interferers.

Average error probability

The average error probability at the SC output is derived for non-coherent and binary signaling according to following expressions:

$$P_e = \int_0^\infty p_\varepsilon(t)\frac{1}{2}\exp(-gt) \tag{13}$$

Where g denotes modulation constant, i.e., $g = 1$ for BDPSK and $g = 1/2$ for BFSK. Substituting (11) in (13) numerically obtained average error probability is shown on Figs. 3 and 4 for balanced and unbalanced ratio of SIR at the input of the branches and various values of diversity order and the number of multiple interferers [5].

A comparison of curves shows better performance of BDPSK modulation scheme against BFSK modulation scheme. Also is more evident how ABER increases at both figures when the number of multiple independent co-channel interferers increases due to growth of interference domination. ABER performance behavior improves for the constant number of co-channel interferers as the diversity order increases. If we want to achieve the same quality of the transmission (for example outage probability of 10^{-4}), we need higher level of average input SIR for dual branch case then for the triple branch case (for example of above 5 dB for BDPSK and 6dB for BFSK). Input SIR unbalance at the diversity branches deteriorates system performances.

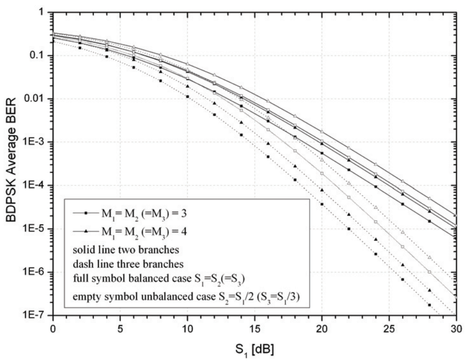

Fig. 3. Average BER versus S_1 in non-coherent BDPSK for balanced and unbalanced ratio of SIR at the input of the branches and various numbers of diversity order and multiple interferers.

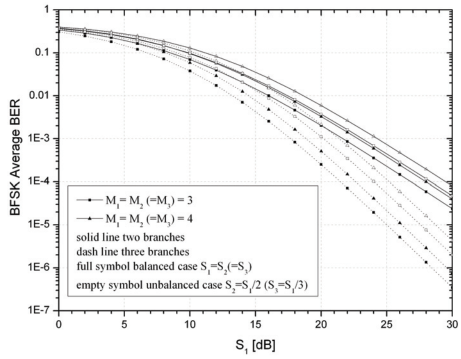

Fig. 4. Average BER versus S_1 in non-coherent BFSK for balanced and unbalanced ratio of SIR at the input of the branches and various numbers of diversity order and multiple interferers.

Conclusions

Performance analysis of a multibranch SIR-based SC system operating over Rayleigh fading channels where each channel experiences an arbitrary number of multiple, independent, Rayleigh co-channel interferers with equal average powers, was derived. Closed form expressions for the SIR's PDF and CDF at the output of the combiner are derived. Capitalizing on this, important performance measures like OP and average BER, for some non-coherent modulation techniques, are graphically analyzed, in order to show the effects of the number of multiple interferers, diversity order, and input SIR unbalance to the system performances.

REFERENCES

[1] P. Stavroulakis, "'Interference analysis and reduction for wireless systems," *Artech House, INC, London*, May 2003.

[2] W. Lee„ "Mobile communications engineering," *New York, McGraw-Hill*, June 2001.

[3] A. Shah, A. Haimovich, "Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference," in *IEEE Transactions on Communications vol. 46, no.4 pp.473-479*, April 1998.

[4] A. Abu-Dayya, N. Beaulieu, "Outage probabilities of celular mobile radio systems with multiple Nakagami interferers," *IEEE Transactions on Vehicular Technology, vol. 40, no. 4, pp. 757 - 768*, November 1991.

[5] S. Panic, M. Stefanovic and A. Mosic, "Performance Analysis of Selection Combining Diversity Receiver over $\alpha - \mu$ Fading Channels in the Presence of CCI.," in *IET Communications, vol. 3 no. 11, pp. 1769-1777*, November 2009.

[6] V. Bhaskar, "Capacity evaluation for equal gain diversity schemes over Rayleigh fading channels.," in *AEU - International Journal of Electronics and Communications Volume 63, Issue 4, Pages 235-240*, April 2009.

[7] I. Gradshteyn, I. Ryzhik, , "Tables of Integrals, Series, and products, 7^{th} edition," in *Academic Press, New Yourk*, February 2007.

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