I₂-l∞ Filtering for Discrete Time-Delay Markovian Jump Neural Networks

Abstract. This paper considers the I₂-l∞ filter problem for discrete time-delay Markovian jump neural networks. Attention is focused on the design of a reduced-order filter to guarantee stochastic stability and a prescribed l∞ performance for the filtering error system. In terms of linear matrix inequalities (LMIs), a delay-dependent sufficient condition for the solvability of the addressed problem is presented. When these LMIs are feasible, an explicit expression for the desired reduced-order filter is given. A numerical example is provided to show the effectiveness of the proposed results.

Streszczenie. W artykule analizuje się problem filtru I₂-l∞ dla dyskretnego opóźnienia czasowego sieci neuronowej ze skokiem Markova. Szczególną uwagę zwrócono na projekt filtru zredukowanego rządu dla zagwarantowania stochastycznej stabilności. Zaproponowano wystarczające warunki dla rozwiązywalności układy przy liniiowej macierzy nierówności LMIs. (Filtrowanie I₂-l∞ dla dyskretnego opóźnienia czasowego sieci neuronowej ze skokiem Markova)

Keywords: Neural networks, I₂-l∞ filtering, Time-varying delays, Transition probabilities.

Słowa kluczowe: sieci neuronowe, filtrowanie, skok Markova

Introduction
Time delays are often unavoidable in many practical engineering systems, such as communication systems, electrical networks, manufacturing systems, and chemical processing systems. The presence of delays may induce undesirable effects such as performance degradation or even loss of stability. In the past few decades, the study of time-delay systems has received considerable attention, and a great number of results on this topic have been reported in the literature; see, e.g., [1-3] and references therein. It should be pointed out that these results can be classified into two categories, namely delay-dependent results and delay-independent results. Usually, delay-dependent results are less conservative than delay-independent ones, especially when the time delay is comparatively small [1].

On the other hand, Markovian jump systems, which are modelled by a set of subsystems with transitions among the models governed by a Markov chain taking values in a finite set, have been extensively investigated in recent years [4, 5]. It has been shown that Markovian jump systems are appropriate models to represent various practical systems, which experience abrupt changes in their structure, caused by component failures or repairs, changing subsystem interconnections, and sudden environmental disturbances. Usually, the transition probabilities of Markovian jump systems are assumed to be completely known in order to facilitate research. As noted by Zhang and Boukas [6], however, obtaining the ideal information on all transition probabilities is questionable or generally expensive. Thus, rather than measure or estimate all transition probabilities with great complexity, it is much better to investigate more general Markovian jump systems with partially unknown transition probabilities.

In this paper we consider the problem of I₂-l∞ filter design for a class of time-delay Markovian jump systems, namely discrete time-delay Markovian jump neural networks. It is worth noting that for neural networks, the study of filter analysis and synthesis is still in the early stages of development [7], and the reduced-order filter design problem has not yet been investigated. In the networks considered here, the time delays are assumed to be time-varying, and the transition probabilities are allowed to be partially unknown. The problem we address is to design a reduced-order filter such that the filtering error system not only is stochastic stable but also satisfies a prescribed I₂-l∞ performance. In terms of linear matrix inequalities (LMIs), a delay-dependent sufficient condition for the solvability of this problem is proposed by using the Lyapunov-functional method and some inequality techniques. When these LMIs are feasible, an explicit expression for the desired reduced-order filter is also presented. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.

Throughout this paper, ε(·) denotes the expectation operator with respect to some probability measure, * denotes the symmetric block in a symmetric matrix, and I represents the identity matrix with appropriate dimension.

Problem Formulation
Consider a discrete time-delay Markovian jump neural network described by:

\[ x_{k+1} = A(r_k)x_k + W_1(r_k)g(x_k) + W_2(r_k)g(x_{k-	au}) + D_1(r_k)ε_k \]

\[ y_k = B(r_k)x_k + C(r_k)x_{k-	au} + D_2(r_k)ε_k, \]

\[ z_k = E(r_k)x_k, \]

\[ x_i = \theta_i, \quad i = -d_2, \ldots, 0 \]

where \( x_1 = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \) is the neuron state vector; \( ε_k \in \mathbb{R}^m \) is the exogenous disturbance input; \( y_k \in \mathbb{R}^r \) is the output or measurement; \( z_k \in \mathbb{R}^s \) is the signal to be estimated; \( d_i \leq \tau_i < d_i \) denote the transmission delay that satisfies \( d_i \leq \tau_i < d_i \), where \( d_i \) and \( d_i \) are prescribed positive integers corresponding to the lower and upper bounds of the time delays; \( r_k \) represents a discrete-time, discrete-state Markov chain taking values in a finite set \( S = \{1, \ldots, s\} \) with transition probabilities \( P_i[r_{k+1} = j] = \pi_{ij} \), where \( \pi_{ij} \geq 0 \), and for any \( i \in \mathbb{S} \), \( \sum_{i=1}^{s} \pi_{ij} = 1 \); \( \theta_i \), \( i = -d_2, \ldots, 0 \), are real-valued continuous functions, which are assumed to be independent of the process \( \{ε_k\} \); \( A(r_k) = \text{diag}(a_1(r_k), \ldots, a_n(r_k)) \) with \( \|a_i(r_k)\|<1 \) describes the rate with which the ith neuron will reset its potential to the resting state in isolation; \( W_1(r_k) = [W_{11}(r_k), W_{12}(r_k)] \) is the connection weight matrix; \( W_2(r_k) = [W_{12}(r_k), W_{22}(r_k)] \) is the delayed connection weight matrix; \( B(r_k), C(r_k), D_1(r_k), D_2(r_k), D_3(r_k), E(r_k) \) are known matrixes with appropriate dimensions; \( g(x_k) = [g_1(x_1) \ldots g_n(x_n)]^T \) is the neuron activation function vector, which are assumed to satisfy:

\[ G_{i}^0 \leq g_i(ε)/ε \leq G_{i}^+ \]

\[ g_i(ε) = 0, \quad i = 1, \ldots, n \]

for any \( ε \in \mathbb{R}, ε > 0 \); where \( G_{i}^0 \) and \( G_{i}^+ \) are constants.
Lemma 1

Main results

\[ (4) \text{ is stochastically stable, and under zero initial conditions,} \]

\[ (6) \text{ holds for any non-zero function} \]

\[ \|z\|_{\infty} \leq \gamma \|\omega\|_{2} \]

holds for any non-zero function \( \omega \in L_{2}[0, \infty) \), where \( \|\cdot\|_{\infty} = \sup_{x} \|e_{k}(x_{k})\|^{2} / 2 \).

Main results

First, we introduce the following lemmas.

Lemma 1 [8]
follow ing stochastic Lyapunov functional: 

\[ V(x_k,r_k) = \sum_{i,j} \pi^i \gamma_{ij} + \sum_{i,k} \eta_i \xi_{jk} + \sum_{i,j,k} \eta_{ijk} U_i \eta_j, \]

where \( \eta_i, \xi_{jk}, \eta_{ijk} \) are defined as in (13)-(16) that

\[ \Delta V(x_k,r_k) \leq \frac{\gamma^2}{2} P_i \]

Next, we will establish the reduced-order \( l_2, l_\infty \) performance for filtering system (4). To this end, consider the following index:

\[ H(x_k) = \sum_{i=0}^{k-1} \alpha_i \theta_i \]

where \( \theta_i = [\gamma^2 \omega_i^T \omega_i] \). With this and (12), we obtain

\[ e \{ V(x_k,r_k) \} \leq \sum_{i=0}^{k-1} \omega_i \theta_i \]

On the other hand, by Schur complement, (8) guarantees that

\[ \alpha_i \theta_i \leq \gamma^2 P_i \]

Now, from (4), (17)-(19) we can conclude that \( e \{ e_i e_i^T \} \leq \gamma^2 \sum_{i=0}^{k-1} \omega_i \theta_i \), which implies (6). This completes the proof.

Finally, we deal with the reduced-order \( l_2, l_\infty \) filter design problem. The following result can be easily accessible from Theorem 1, thus the proof is omitted.
Remark 2

Theorem 2 provides a sufficient condition for the solvability of the reduced-order \( l_1 \)-filter design problem. Desired gain matrices can be constructed through the solution of LMIs, which can be solved efficiently by the Matlab LMI toolbox.

Numerical example

Consider the discrete time-delay Markovian jump neural network (1) with four jumping modes:

\[
A_i = \text{diag}(0.83, 0.75, 0.79), A_j = \text{diag}(0.85, 0.77, 0.72),
\]

\[
A_3 = \text{diag}(0.83, 0.77, 0.79), A_4 = \text{diag}(0.83, 0.77, 0.72),
\]

\[
B_j = [0.18, 0.15, 0.17], B_k = [0.15, 0.16, 0.22],
\]

\[
B_j = [0.15, 0.16, 0.19], B_k = [0.18, 0.16, 0.16],
\]

\[
C_i = [0.21, 0.16, 0.13], C_j = [0.17, 0.25, 0.08],
\]

\[
C_j = [0.17, 0.16, 0.17], C_k = [0.21, 0.12, 0.17],
\]

\[
D_1 = [0.05, 0.11, 0.08], D_2 = [-0.03, -0.09, -0.06],
\]

\[
D_3 = [-0.03, 0.11, 0.08], D_4 = [-0.03, 0.11, -0.06],
\]

\[
D_1 = 0.03, D_2 = 0.02, D_3 = 0.05, D_4 = 0.07,
\]

\[
\theta_1 = [-0.5, -0.2, 0.2], \theta_2 = [0.18, 0.33, 0.2],
\]

\[
\theta_4 = [-0.25, 0.5, -0.3], \theta_5 = [-0.32, -0.2, 0.3],
\]

\[
\alpha_k = e^{-0.2k}, \tau_k = 2L_1(0.5, \sigma), g_i(x_k) = \text{tanh}(x_k), i = 1, 2, 3,
\]

where \( ? \) represents the unmeasurable elements. It can be verified that \( g_i \) satisfies (2) with \( G_i = 0 \) and \( G_k = 1 \). In addition, it can be calculated that \( \|\alpha\|_{\infty} = 1.7416 \).

The disturbance attenuation level in this example is taken as \( \gamma = 0.3 \). By using the Matlab LMI toolbox, we solve the LMIs in (20)-(22) and can obtain a feasible solution (omitted here for brevity). Therefore, by Theorem 1, the reduced-order \( l_1 \)-filter design problem is solvable, and the gains of an admissible reduced-order filter (3) can be designed as

\[
A_{F_i} = [0.1685, 0.0481, 0.0240, 0.2063], B_{F_i} = [-0.0533, -0.1166], E_{F_i} = [-0.2813, -0.2388],
\]

\[
A_{F_j} = [0.1878, 0.0359, 0.0345, 0.2130], B_{F_j} = [-0.0202, -0.0539], E_{F_j} = [-0.2895, -0.1967],
\]

Then reduced-order \( l_1 \)-filter design problem is solvable, and the gains of an admissible filter (3) are given by

\[
A_{Fi} = M_i^{-1}A_i, B_{Fi} = M_i^{-1}B_i, E_{Fi} = E_i.
\]
Under the evolution of the jumping mode depicted in Fig.1, the state response of the filtering error system (4) without disturbance is given in Fig.2, while the error response of the filtering error system (4) with zero initial conditions is shown in Fig.3. It can be seen from Fig.2 that applying the designed reduced-order filter makes the filtering error system (4) stochastically stable. Furthermore, from Fig.3 we can see that \( ||e||_{\infty}=0.0325 \), and thus
\[
\gamma = 0.0325/1.7416 = 0.0187 < \gamma
\]
which confirms the effectiveness of filter design procedure.

### Conclusions

The \( l_2-l_\infty \) filter design problem for discrete time-delay Markovian jump neural networks is investigated in the paper. A delay-dependent sufficient condition for the solvability of this problem is proposed. An explicit expression for the desired reduced-order filter can be constructed through the numerical solutions of linear matrix inequalities. The results obtained in the paper can be extend further to deal with more complex systems, for instance, systems with parameter uncertainties and distributed delays.

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