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l_2-l_∞ Filtering for Discrete Time-Delay Markovian Jump Neural Networks

***Abstract**. This paper considers the l_2-l_∞ filter problem for discrete time-delay Markovian jump neural networks. Attention is focused on the design of a reduced-order filter to guarantee stochastic stability and a prescribed l_2-l_∞ performance for the filtering error system. In terms of linear matrix inequalities (LMIs), a delay-dependent sufficient condition for the solvability of the addressed problem is presented. When these LMIs are feasible, an explicit expression for the desired reduced-order filter is given. A numerical example is provided to show the effectiveness of the proposed results.*

***Streszczenie.** W artykule analizuje się problem filtru l_2-l_∞ dla dyskretnego opóźnienia czasowego sieci neuronowej ze skokiem Markova. Szczególną uwagę zwrócono na projekt filtru zredukowanego rzędu dla zagwarantowania stochastycznej stabilności. Zaprezentowano wystarczające warunki dla rozwiązywalności układu przy liniowej macierzy nierówności LMIs. (**Filtrowanie l_2-l_∞ dla dyskretnego opóźnienia czasowego sieci neuronowej ze skokiem Markova**)*

Keywords: Neural networks, l_2-l_∞ filtering, Time-varying delays, Transition probabilities.
Słowa kluczowe: sieci neuronowe, filtrowanie, skok Markova

Introduction

Time delays are often unavoidable in many practical engineering systems, such as communication systems, electrical networks, manufacturing systems, and chemical processing systems. The presence of delays may induce undesirable effects such as performance degradation or even loss of stability. In the past few decades, the study of time-delay systems has received considerable attention, and a great number of results on this topic have been reported in the literature; see, e.g., [1-3] and references therein. It should be pointed out that these results can be classified into two categories, namely delay-dependent results and delay-independent results. Usually, delay-dependent results are less conservative than delay-independent ones, especially when the time delay is comparatively small [1].

On the other hand, Markovian jump systems, which are modelled by a set of subsystems with transitions among the models governed by a Markov chain taking values in a finite set, have been extensively investigated in recent years [4, 5]. It has been shown that Markovian jump systems are appropriate models to represent various practical systems, which experience abrupt changes in their structure, caused by component failures or repairs, changing subsystem interconnections, and sudden environmental disturbances. Usually, the transition probabilities of Markovian jump systems are assumed to be completely known in order to facilitate research. As noted by Zhang and Boukas [6], however, obtaining the ideal information on all transition probabilities is questionable or generally expensive. Thus, rather than measure or estimate all transition probabilities with great complexity, it is much better to investigate more general Markovian jump systems with partially unknown transition probabilities.

In this paper we consider the problem of l_2-l_∞ filter design for a class of time-delay Markovian jump systems, namely discrete time-delay Markovian jump neural networks. It is worth noting that for neural networks, the study of filter analysis and synthesis is still in the early stages of development [7], and the reduced-order filter design problem has not yet been investigated. In the networks considered here, the time delays are assumed to be time-varying, and the transition probabilities are allowed to be partially unknown. The problem we address is to design a reduced-order filter such that the filtering error system not only is stochastic stable but also satisfies a prescribed l_2-l_∞ performance. In terms of linear matrix inequalities (LMIs), a delay-dependent sufficient condition for the solvability of this problem is proposed by using the Lyapunov-functional method and some inequality techniques. When these LMIs are feasible, an explicit expression for the desired reduced-order filter is also presented. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.

Throughout this paper, $\varepsilon\{\cdot\}$ denotes the expectation operator with respect to some probability measure, $\star$ denotes the symmetric block in a symmetric matrix, and I represents the identity matrix with appropriate dimension.

Problem Formulation

Consider a discrete time-delay Markovian jump neural network described by:

$$\text{(1)}\quad \begin{aligned} x_{k+1} &= A(r_k)x_k + W_1(r_k)g(x_k) + W_2(r_k)g(x_{k-\tau_k}) + D_1(r_k)\omega_k \\ y_k &= B(r_k)x_k + C(r_k)x_{k-\tau_k} + D_2(r_k)\omega_k, \\ z_k &= E(r_k)x_k \\ x_i &= \theta_i, \quad i = -d_2, \cdots, 0 \end{aligned}$$

where $x_k=[x_{1k} \ldots x_{nk}]^T \in R^n$ is the neuron state vector; $\omega_k \in R^q$is the exogenous disturbance input; $y_k \in R^m$ is the output or measurement; $z_k \in R^p$ is the signal to be estimated; τ_k denotes the transmission delay that satisfies $d_1 \leqslant \tau_k \leqslant d_2$, where d_1 and d_2 are prescribed positive integers corresponding to the lower and upper bounds of the time delays; r_k represents a discrete-time, discrete-state Markov chain taking values in a finite set $S=\{1,\ldots,s\}$ with transition probabilities $P_r\{r_{k+1}=j|r_k=i\}=\pi_{ij}$, where $\pi_{ij}\geq 0$, and for any $i\in S$, $\pi_{i1}+\ldots+\pi_{is}=1$; θ_i, $i=-d_2,\ldots,0$, are real-valued continuous functions, which are assumed to be independent of the process $\{\omega_k\}$; $A(r_k)=\text{diag}(a_1(r_k),\ldots,a_n(r_k))$ with $|a_i(r_k)|<1$ describes the rate with which the ith neuron will reset its potential to the resting state in isolation; $W_1(r_k) =(W^1_{ij}(r_k))_{n\times n}$ is the connection weight matrix; $W_2(r_k) =(W^2_{ij}(r_k))_{n\times n}$ is the delayed connection weight matrix; $B(r_k)$, $C(r_k)$, $D_1(r_k)$, $D_2(r_k)$, $D_3(r_k)$, and $E(r_k)$ are known real matrices with appropriate dimensions; $g(x_k)=[g_1(x_{1k})\ldots g_n(x_{nk})]^T$ is the neuron activation function vector, which are assumed to satisfy:

$$\text{(2)}\quad G_i^- \le g_i(\varepsilon)/\varepsilon \le G_i^+,\ g_i(0)=0, \quad i=1,\cdots,n$$

for any $\varepsilon\in R$, $\varepsilon\neq 0$; where G_k^- and G_k^+ are constants.

In addition, the transition probabilities of the jumping process $\{r_k, k\geq 0\}$ are assumed to be partly available as in [6], i.e., some elements in the transition probability matrix $\Pi = (\pi_{ij})_{s\times s}$ are unmeasurable. For notational clarity, for any $i\in S$, we denote $S_K^i=\{j: \pi_{ij}$ is known$\}$, $S_U^i=\{j: \pi_{ij}$ is unknown$\}$.

In order to estimate the signal Z_k, we consider the following lth-order filter ($l<n$):

(3)
$$\begin{aligned}\hat{x}_{k+1} &= A_F(r_k)\hat{x}_k + B_F(r_k)y_k, \quad \hat{x}_0 = 0\\ \hat{z}_k &= E_F(r_k)\hat{x}_k\end{aligned}$$

where $\hat{x}_j = [\hat{x}_{1j} \dots \hat{x}_{nj}]^T\in R^l$ is the filter state vector, $\hat{z}_k \in R^p$ is the estimate of Z_k, $A_F(r_k) \in R^{l\times l}$, $B_F(r_k) \in R^{l\times m}$ and $E_F(r_k) \in R^{p\times n}$ are filter gain matrices to be determined.

Let $e_k=z_k-\hat{z}_k$. Then, from systems (1) and (3), the filtering error dynamics can be described by:

(4)
$$\begin{aligned}\xi_{k+1} &= \overline{A}(r_k)\xi_k + \overline{C}(r_k)J\xi_{k-\tau_k} + \overline{W}_1(r_k)Jf(\xi_k)\\ &\quad + \overline{W}_2(r_k)Jf(\xi_{k-\tau_k}) + \overline{D}(r_k)\omega_k\\ e_k &= \overline{E}(r_k)\xi_k\\ \xi_i &= \overline{\theta}_i, \quad i=-d_2,\cdots,0\end{aligned}$$

where

$$f(\xi_j)=\left[g_1(x_{1j})\cdots g_n(x_{nj})\ g_1(\hat{x}_{1j})\cdots g_l(\hat{x}_{lj})\right]^T$$
$$\xi_j=[x_{1j}\cdots x_{nj}\ \hat{x}_{1j}\cdots \hat{x}_{lj}]^T, j=k, k-\tau_k$$
$$\overline{A}(r_k)=\begin{bmatrix}A(r_k) & 0\\ B_F(r_k)B(r_k) & A_F(r_k)\end{bmatrix}, \overline{C}(r_k)=\begin{bmatrix}0\\ B_F(r_k)C(r_k)\end{bmatrix}$$
$$\overline{D}(r_k)=\begin{bmatrix}D_1(r_k)\\ B_F(r_k)D_2(r_k)\end{bmatrix}, \overline{W}_1(r_k)=\begin{bmatrix}W_1(r_k)\\ 0\end{bmatrix}, \overline{W}_2(r_k)=\begin{bmatrix}W_2(r_k)\\ 0\end{bmatrix}$$
$$\overline{E}(r_k)=\left[E(r_k)\ \ -E_F(r_k)\right], J=\left[I\ \ 0\right], \overline{\theta}_i=\left[\theta_i^T\ \ 0\right]^T$$

Now we introduce the following definition.

Definition 1 [6]
The filtering error system (4) is said to be stochastically stable if, when $\omega_k=0$,

(5)
$$E\left\{\sum_{k=0}^{\infty}\|\xi_k\|_2^2 \Big/ \overline{\theta}_{-d_2},\cdots,\overline{\theta}_0, r_0\right\}<\infty$$

holds for any initial state $\overline{\theta}_i \in R^n$, $i = -d_2, \dots, 0$, and initial mode $r_0\in S$, where $\|\bullet\|_2$ represents the usual l_2 norm.

The reduced-order l_2-l_∞ filter design problem to be addressed in the paper can be formulated as follows: given the discrete time-delay Markovian jump neural network (1) and a prescribed disturbance attenuation level $\gamma>0$, design a reduced-order filter (3) such that: the filtering error system (4) is stochastically stable, and under zero initial conditions,

(6)
$$\|e\|_{\varepsilon\infty} \leq \gamma\|\omega\|_2$$

holds for any non-zero function $\omega_k\in l_2[0, +\infty)$, where $\|e\|_{\varepsilon\infty} = (\sup_k \varepsilon\{e_k^T e_k\})^{1/2}$.

Main results

First, we introduce the following lemmas.

Lemma 1 [8]
For any real matrices U, V, X with appropriate dimensions such that $X>0$, $U^TV+V^TU\leqslant U^TXU+V^TX^{-1}V$.

Lemma 2
For any integers l and m $(l<m)$, vectors $\rho_1, \dots, \rho_n, \in R^n$, and matrices $X, Y_1, Y_2 \in R^{n\times n}$ with $X>0$, we have

(7)
$$-\sum_{i=l}^{m-1}\sigma_i^T X\sigma_i \leq \upsilon^T\left\{\begin{bmatrix}Y_1+Y_1^T & -Y_1+Y_2^T\\ -Y_1^T+Y_2 & -Y_2-Y_2^T\end{bmatrix}+(m-l)\begin{bmatrix}Y_1^T\\ Y_2^T\end{bmatrix}X^{-1}\begin{bmatrix}Y_1 & Y_2\end{bmatrix}\right\}\upsilon$$

where $\sigma_i = \rho_{i+1} - \rho_i$, and $\upsilon = [\rho_m^T\ \rho_l^T]^T$.

Remark 1
The inequality (7) given in Lemma 2 can be viewed as a discrete version of the integral inequality (12) in [2].

Then, we consider the problem of reduced-order l_2-l_∞ filter analysis. The following theorem provides a sufficient condition guaranteeing that the filtering error system (4) not only is stochastic stable but also satisfies a prescribed l_2-l_∞ performance.

Theorem 1
Given a scalar $\gamma>0$, the filtering error system (4) is stochastically stable, and under zero initial conditions, (6) holds for any non-zero function $\omega_k\in l_2[0, +\infty)$ if for each mode $i\in S$, there exist matrices Y_1, Y_2, Z_1, $Z_2 \in R^{n\times n}$, $M_i \in R^{(n+l)\times(n+l)}$, positive diagonal matrices L_1, $L_2 \in R^{(n+l)\times(n+l)}$, and positive definite matrices U_1, $U_2 \in R^{n\times n}$, P_i, Q_1, Q_2, $Q_3 \in R^{(n+l)\times(n+l)}$, such that the following LMIs hold:

(8)
$$\begin{bmatrix}-P_i & -\overline{E}_i^T\\ * & -\gamma^2 I\end{bmatrix}<0$$

(9)
$$\sum\nolimits_{j\in S_K^i}\pi_{ij}\Theta_{ij}<0$$

(10)
$$\Theta_{ij}<0, \quad j\in S_U{}^i$$

where

$$\Theta_{ij}=\begin{bmatrix}\Lambda_i & -d_1Y & -(d_2-d_1)Z & -\Phi_iU & -\Psi_iM_i\\ * & -d_1U_1 & 0 & 0 & 0\\ * & * & -(d_2-d_1)U_2 & 0 & 0\\ * & * & * & -U & 0\\ * & * & * & * & -M_i-M_i^T+P_j\end{bmatrix}$$
$$Y=\left[Y_1J\ Y_2J\ 0\ 0\ 0\ 0\ 0\right]^T, Z=\left[0\ Z_1J\ Z_2J\ 0\ 0\ 0\ 0\right]^T$$
$$U=d_1U_1+(d_2-d_1)U_2, \Psi_i=\left[\overline{A}_i\ 0\ 0\ \overline{C}_iJ\ \overline{W}_{1i}J\ \overline{W}_{2i}J\ \overline{D}_i\right]^T$$
$$\Phi_i=\left[J(\overline{A}_i-I)\ 0\ 0\ J\overline{C}_iJ\ J\overline{W}_{1i}J\ J\overline{W}_{2i}J\ J\overline{D}_i\right]^T$$
$$\Lambda_i=\begin{bmatrix}\Lambda_{i11} & \Lambda_{i12} & 0 & 0 & \Lambda_{i15} & 0 & 0\\ * & \Lambda_{i22} & \Lambda_{i23} & 0 & 0 & 0 & 0\\ * & * & \Lambda_{i33} & 0 & 0 & 0 & 0\\ * & * & * & \Lambda_{i44} & 0 & \Lambda_{i46} & 0\\ * & * & * & * & -2L_1 & 0 & 0\\ * & * & * & * & * & -2L_2 & 0\\ * & * & * & * & * & * & -I\end{bmatrix}$$
$$\Lambda_{i11}=-P_i+(d_2-d_1+1)Q_1+Q_2+J^TY_1J+J^TY_1^TJ-2G_\alpha L_1G_\beta$$
$$\Lambda_{i12}=-J^TY_1^TJ+J^TY_2J,\ \Lambda_{i15}=L_1\left(G_\alpha+G_\beta\right)$$
$$\Lambda_{i22}=-Q_2+Q_3-J^TY_2J-J^TY_2^TJ+J^TZ_1J+J^TZ_1^TJ$$
$$\Lambda_{i23}=-J^TZ_1^TJ+J^TZ_2J,\ \Lambda_{i33}=-Q_3-J^TZ_2J-J^TZ_2^TJ$$

$$\Lambda_{44} = -Q_l - 2G_\alpha L_2 G_\beta,\ \Lambda_{46} = L_2\left(G_\alpha + G_\beta\right)$$
$$G_\alpha = diag\left(G_1^-,\cdots,G_n^-,G_1^-,\cdots,G_l^- \right)$$
$$G_\beta = diag\left(G_1^+,\cdots,G_n^+,G_1^+,\cdots,G_l^+ \right)$$

Proof
Define

$$\overline{P}_i = \sum_{j\in S}\pi_{ij}P_j$$

$$\overline{\Theta}_i = \begin{bmatrix} \Lambda_i & -d_1 Y & -(d_2-d_1)Z & -\Phi_i U & -\Psi_i M_i \\ * & -d_1 U_1 & 0 & 0 & 0 \\ * & * & -(d_2-d_1)U_2 & 0 & 0 \\ * & * & * & -U & 0 \\ * & * & * & * & -M_i - M_i^T + \overline{P}_i \end{bmatrix}$$

Then, from (9)-(10) we have

$$\overline{\Theta}_i = \sum_{j\in S_K^i}\pi_{ij}\Theta_{ij} + \sum_{j\in S_U^i}\pi_{ij}\Theta_{ij} < 0$$

which, by Lemma 1, implies

(11) $$\begin{bmatrix} \Lambda_i & -d_1 Y & -(d_2-d_1)Z & -\Phi_i U & -\Psi_i M_i \\ * & -d_1 U_1 & 0 & 0 & 0 \\ * & * & -(d_2-d_1)U_2 & 0 & 0 \\ * & * & * & -U & 0 \\ * & * & * & * & -\overline{P}_i \end{bmatrix} < 0$$

By Schur complement, (11) is equivalent to

(12)
$$(\mu_{i\alpha\beta})_{\alpha=1,\cdots,7;\,\beta=1,\cdots,7} = \Lambda_i + \Psi_i \overline{P}_i \Psi_i^T + \Phi_i U \Phi_i^T + d_1 Y U_1^{-1} Y^T + (d_2-d_1) Z U_2^{-1} Z^T < 0$$

By Lemma 2, the following inequalities hold true:

(13) $$-\sum_{i=k-d_1}^{k-1}\eta_i^T U_1 \eta_i \le \upsilon_1^T\left\{\begin{bmatrix} Y_1+Y_1^T & -Y_1+Y_2^T \\ -Y_1+Y_2^T & -Y_2-Y_2^T\end{bmatrix} + d_1\begin{bmatrix} Y_1^T \\ Y_2^T\end{bmatrix} U_1^{-1}\begin{bmatrix} Y_1 & Y_2\end{bmatrix}\right\}\upsilon_1$$

(14)
$$-\sum_{i=k-d_2}^{k-d_1-1}\eta_i^T U_2 \eta_i \le \upsilon_2^T\left\{\begin{bmatrix} Z_1+Z_1^T & -Z_1+Z_2^T \\ -Z_1+Z_2^T & -Z_2-Z_2^T\end{bmatrix} + (d_2-d_1)\begin{bmatrix} Z_1^T \\ Z_2^T\end{bmatrix} U_2^{-1}\begin{bmatrix} Z_1 & Z_2\end{bmatrix}\right\}\upsilon_2$$

where $\eta_i = J\xi_{i+1} - J\xi_i$, $\upsilon_1 = [\xi_k^T J^T \ \ \xi_{k-d_1}^T J^T]^T$ and $\upsilon_2 = [\xi_{k-d_1}^T J^T \ \ \xi_{k-d_2}^T J^T]^T$.
Moreover, in view of (2), we obtain

(15)
$$2\left\{\xi_k^T L_1(G_\alpha + G_\beta) f(\xi_k) - \xi_k^T G_\alpha L_1 G_\beta \xi_k - f^T(\xi_k) L_1 f(\xi_k)\right\} \ge 0$$

(16)
$$2\left\{\xi_{k-\tau_k}^T L_2(G_\alpha + G_\beta) f(\xi_{k-\tau_k}) - \xi_{k-\tau_k}^T G_\alpha L_2 G_\beta \xi_{k-\tau_k} - f^T(\xi_{k-\tau_k}) L_2 f(\xi_{k-\tau_k})\right\} \ge 0$$

Now, we are in a position to prove the stochastic stability of the filtering error system (4). To this end, consider the following stochastic Lyapunov functional:

(17)
$$V(\xi_k, r_k) = \xi_k^T P_{r_k}\xi_k + \sum_{i=k-\tau_k}^{k-1}\xi_i^T Q_1 \xi_i + \sum_{i=k-d_1}^{k-1}\xi_i^T Q_2 \xi_i + \sum_{i=k-d_2}^{k-d_1-1}\xi_i^T Q_3 \xi_i + \sum_{i=-d_2+1}^{-d_1}\sum_{j=k+i}^{k-1}\xi_j^T Q_1 \xi_j + \sum_{i=-d_1}^{-1}\sum_{j=k+i}^{k-1}\eta_j^T U_1 \eta_j + \sum_{i=-d_2}^{-d_1-1}\sum_{j=k+i}^{k-1}\eta_j^T U_2 \eta_j$$

where $\eta_j = J\xi_{j+1} - J\xi_j$. Let $\Delta V(\xi_k, r_k) = \varepsilon\{[V(\xi_{k+1}, r_{k+1})/\xi_k, r_k] - V(\xi_k, r_k)\}$. Then for $r_k = i, r_{k+1} = j, \omega_k \equiv 0$, we have by (13)-(16) that

$$\Delta V(\xi_k, r_k) \le \vartheta^T(\xi_k)(\mu_{i\alpha\beta})_{\alpha=1,\cdots,6;\,\beta=1,\cdots,6}\vartheta(\xi_k)$$

where $\vartheta(\xi_k) = [\xi_k^T \ \ \xi_{k-d_1}^T \ \xi_{k-d_2}^T \ \xi_{k-d_{\tau_k}}^T \ f^T(\xi_k) \ f^T(\xi_{k-\tau_k})]^T$. Note that (12) guarantees $(\mu_{ijk})_{j=1,\ldots,6;\,k=1,\ldots,6} < 0$. Following the same lines as in the proof of theorem 1 in [4], we can deduce (5), that is, filtering error system (4) is stochastically stable.

Next, we will establish the l_2-l_∞ performance for filtering error system (4). To this end, consider the following index:

$$H = \varepsilon\{V(\xi_k, r_k)\} - \sum_{l=0}^{k-1}\omega_l^T\omega_l$$

Under zero-initial condition, we have $V(\xi_0, r_0) = 0$, which gives

$$H \le \sum_{l=0}^{k-1}\left(\overline{\vartheta}^T(\xi_l)(\mu_{i\alpha\beta})_{\alpha=1,\cdots,7;\,\beta=1,\cdots,7}\overline{\vartheta}(\xi_l)\right)$$

where $\overline{\vartheta}(\xi_l) = [\ \vartheta^T(\xi_l) \ \ \omega_l^T]$. With this and (12), we obtain

(18) $$\varepsilon\{V(\xi_k, r_k)\} \le \sum_{l=0}^{k-1}\omega_l^T\omega_l$$

On the other hand, by Schur complement, (8) guarantees that

(19) $$\overline{E}_i^T\overline{E}_i \le \gamma^2 P_i$$

Now, from (4), (17)-(19) we can conclude that $\varepsilon\{e_k^T e_k\} \le \gamma^2\sum_{l=0}^{\infty}\omega_l^T\omega_l$, which implies (6). This completes the proof.

Finally, we deal with the reduced-order l_2-l_∞ filter design problem. The following result can be easily accessible from Theorem 1, thus the proof is omitted.

Theorem 2
Given the discrete time-delay Markovian jump neural network (1) and let $\gamma > 0$ be a prescribed disturbance attenuation level. If for each mode $i \in S$, there exist matrices Y_1, Y_2, Z_1, Z_2, $M_{1i} \in R^{n\times n}$, P_{2i}, Q_{21}, Q_{22}, $Q_{23} \in R^{n\times l}$, $E_{fi} \in R^{p\times l}$, M_{2i}, $A_{fi} \in R^{l\times l}$, $B_{fi} \in R^{l\times m}$, $M_{3i} \in R^{l\times n}$, positive diagonal matrices L_{11}, $L_{12} \in R^{n\times n}$, L_{21}, $L_{22} \in R^{l\times l}$, and positive definite matrices P_{1i}, Q_{11}, Q_{12}, Q_{13}, U_1, $U_2 \in R^{n\times n}$, P_{3i}, Q_{31}, Q_{32}, $Q_{33} \in R^{l\times l}$ such that the following LMIs hold

(20) $$\begin{bmatrix} -P_{1i} & -P_{2i} & -E_i^T \\ * & -P_{3i} & E_{fi}^T \\ * & * & -\gamma^2 I\end{bmatrix} < 0$$

(21) $$\sum_{j\in S_K^i}\pi_{ij}\acute{\Upsilon}_{ij} < 0$$

(22) $$\acute{\Upsilon}_{ij} < 0, \quad j \in S_U^i$$

where

$$\acute{\Upsilon}_{ij} = \begin{bmatrix} \bar{\Omega}_i & -d_1 Y & -(d_2-d_1)Z & -\Phi_i U & -\Gamma_i \\ * & -d_1 U_1 & 0 & 0 & 0 \\ * & * & -(d_2-d_1)U_2 & 0 & 0 \\ * & * & * & -U & 0 \\ * & * & * & * & -\Xi_{ij} \end{bmatrix}$$

$$\Phi_i = \begin{bmatrix} A_i - I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_{1i} & 0 & W_{2i} & 0 & D_{1i} \end{bmatrix}^T$$

$$Y = \begin{bmatrix} Y_1 & 0 & Y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$Z = \begin{bmatrix} 0 & 0 & Z_1 & 0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\Xi_{ij} = \begin{bmatrix} M_{1i}^T + M_{1i} - P_{1j} & M_{3i}^T + KM_{2i} - P_{2j} \\ * & M_{2i}^T + M_{2i} - P_{3j} \end{bmatrix}$$

$$\bar{\Omega}_i = \begin{bmatrix} \bar{\Omega}_{i1} & \bar{\Omega}_{i3} \\ * & \bar{\Omega}_{i2} \end{bmatrix}, \ \Gamma_i = \begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} \end{bmatrix}^T$$

$$\Gamma_{1i} = \begin{bmatrix} M_{1i}A_i + KB_{fi}B_i & KA_{fi} & 0 & 0 & 0 & 0 & KB_{fi}C_i \\ M_{3i}A_i + B_{fi}B_i & A_{fi} & 0 & 0 & 0 & 0 & B_{fi}C_i \end{bmatrix}$$

$$\Gamma_{2i} = \begin{bmatrix} 0 & M_{1i}W_{1i} & 0 & M_{1i}W_{2i} & 0 & M_{1i}D_{1i} + KB_{fi}D_i \\ 0 & M_{3i}W_{1i} & 0 & M_{3i}W_{2i} & 0 & M_{3i}D_{1i} + B_{fi}D_{2i} \end{bmatrix}$$

$$\bar{\Omega}_{i1} = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} & 0 & 0 & 0 & 0 & 0 \\ * & \Omega_{i22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{i33} & \Omega_{i34} & \Omega_{i35} & 0 & 0 & 0 \\ * & * & * & \Omega_{i44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{i55} & -Q_{23} & 0 & 0 \\ * & * & * & * & * & -Q_{33} & 0 & 0 \\ * & * & * & * & * & * & \Omega_{i77} & -Q_{21} \\ * & * & * & * & * & * & * & \Omega_{i88} \end{bmatrix}$$

$$\bar{\Omega}_{i2} = diag(-2L_{11}, \ -2L_{21}, \ -2L_{12}, \ -2L_{22}, \ -I)$$

$$\bar{\Omega}_{i3} = \begin{bmatrix} L_{11}(G_{1\alpha}+G_{1\beta}) & 0 & 0 & 0 & 0 \\ 0 & L_{21}(G_{2\alpha}+G_{2\beta}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{12}(G_{1\alpha}+G_{1\beta}) & 0 & 0 \\ 0 & 0 & 0 & L_{22}(G_{2\alpha}+G_{2\beta}) & 0 \end{bmatrix}$$

$$\Omega_{i11} = -P_{1i} + (d_2 - d_1 + 1)Q_{11} + Q_{12} + Y_1 + Y_1^T - 2G_{1\alpha}L_{11}G_{1\beta}$$

$$\Omega_{i12} = -P_{2i} + (d_2 - d_1 + 1)Q_{21} + Q_{22}, \ \Omega_{i13} = -Y_1^T + Y_2$$

$$\Omega_{i22} = -P_{3i} + (d_2 - d_1 + 1)Q_{31} + Q_{32} - 2G_{2\alpha}L_{21}G_{2\beta}$$

$$\Omega_{i33} = -Q_{12} + Q_{13} - Y_2 - Y_2^T + Z_1 + Z_1^T, \ \Omega_{i34} = -Q_{22} + Q_{23}$$

$$\Omega_{i35} = -Z_1^T + Z_2, \ \Omega_{i44} = -Q_{32} + Q_{33}, \ \Omega_{i55} = -Q_{13} - Z_2 - Z_2^T$$

$$\Omega_{i77} = -Q_{11} - 2G_{1\alpha}L_{12}G_{1\beta}, \Omega_{i88} = -Q_{31} - 2G_{2\alpha}L_{22}G_{2\beta}, K = \begin{bmatrix} I & 0 \end{bmatrix}^T$$

$$G_{1\alpha} = diag(G_1^-, \cdots, G_n^-), G_{2\alpha} = diag(G_1^-, \cdots, G_l^-)$$

$$G_{1\beta} = diag(G_1^+, \cdots, G_n^+), G_{2\beta} = diag(G_1^+, \cdots, G_l^+)$$

then reduced-order l_2-l_∞ filter design problem is solvable, and the gains of an admissible filter (3) are given by

$$A_{Fi} = M_{2i}^{-1}A_{fi}, \quad B_{Fi} = M_{2i}^{-1}B_{fi}, \quad E_{Fi} = E_{fi}$$

Remark 2
Theorem 2 provides a sufficient condition for the solvability of the reduced-order l_2-l_∞ filter design problem. Desired gain matrices can be constructed through the solution of LMIs, which can be solved efficiently by the Matlab LMI toolbox.

Numerical example

Consider the discrete time-delay Markovian jump neural network (1) with four jumping modes:

$$W_{11} = \begin{bmatrix} 0.057 & 0.053 & 0.061 \\ 0.033 & 0.024 & 0.036 \\ 0.081 & 0.045 & 0.072 \end{bmatrix}, W_{12} = \begin{bmatrix} 0.057 & 0.047 & 0.065 \\ 0.033 & 0.028 & 0.036 \\ 0.078 & 0.045 & 0.072 \end{bmatrix}$$

$$W_{13} = \begin{bmatrix} 0.051 & 0.053 & 0.065 \\ 0.037 & 0.024 & 0.036 \\ 0.081 & 0.045 & 0.069 \end{bmatrix}, W_{14} = \begin{bmatrix} 0.051 & 0.047 & 0.065 \\ 0.033 & 0.024 & 0.039 \\ 0.081 & 0.049 & 0.072 \end{bmatrix}$$

$$W_{21} = \begin{bmatrix} -0.024 & -0.017 & 0.033 \\ 0.008 & 0.013 & 0.016 \\ 0.025 & 0.009 & -0.027 \end{bmatrix}, W_{22} = \begin{bmatrix} -0.021 & -0.017 & 0.031 \\ 0.008 & 0.013 & 0.016 \\ 0.023 & 0.009 & -0.027 \end{bmatrix}$$

$$W_{23} = \begin{bmatrix} -0.021 & -0.017 & 0.033 \\ 0.015 & 0.013 & 0.016 \\ 0.023 & 0.009 & -0.022 \end{bmatrix}, W_{24} = \begin{bmatrix} -0.021 & -0.017 & 0.033 \\ 0.008 & 0.013 & 0.016 \\ 0.023 & 0.009 & -0.027 \end{bmatrix}$$

$$A_1 = diag(0.83, 0.75, 0.79), A_2 = diag(0.85, 0.77, 0.72)$$

$$A_3 = diag(0.83, 0.77, 0.79), A_4 = diag(0.83, 0.77, 0.72)$$

$$B_1 = [0.18 \ 0.15 \ 0.17], \ B_2 = [0.15 \ 0.13 \ 0.22]$$

$$B_3 = [0.15 \ 0.16 \ 0.19], \ B_4 = [0.18 \ 0.16 \ 0.16]$$

$$C_1 = [0.21 \ 0.16 \ 0.13], \ C_2 = [0.17 \ 0.25 \ 0.08]$$

$$C_3 = [0.17 \ 0.16 \ 0.17], \ C_4 = [0.21 \ 0.12 \ 0.17]$$

$$D_{11} = \begin{bmatrix} 0.05 & 0.11 & 0.08 \end{bmatrix}^T, D_{12} = \begin{bmatrix} -0.03 & -0.09 & -0.06 \end{bmatrix}^T$$

$$D_{13} = \begin{bmatrix} -0.03 & 0.11 & 0.08 \end{bmatrix}^T, D_{14} = \begin{bmatrix} -0.03 & 0.11 & -0.06 \end{bmatrix}^T$$

$$D_{21} = 0.03, \ D_{22} = 0.02, \ D_{23} = 0.05, \ D_{24} = 0.07$$

$$\theta_{-3} = \begin{bmatrix} -0.5 & -0.2 & 0.27 \end{bmatrix}^T, \ \theta_{-2} = \begin{bmatrix} 0.18 & 0.33 & 0.21 \end{bmatrix}^T$$

$$\theta_{-1} = \begin{bmatrix} -0.24 & 0.5 & -0.3 \end{bmatrix}^T, \ \theta_0 = \begin{bmatrix} -0.32 & -0.2 & 0.36 \end{bmatrix}^T$$

$$\omega_k = e^{-0.2k}, \tau_k = 2 - \sin(0.5k\pi), g_i(x_{ik}) = \tanh(x_{ik}), i = 1,2,3$$

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.3 & 0.2 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}, \ \Pi = \begin{bmatrix} 0.5 & 0.2 & ? & ? \\ 0.5 & ? & ? & ? \\ 0.3 & 0.1 & 0.3 & 0.3 \\ 0.5 & ? & ? & 0.3 \end{bmatrix}$$

where '?' represents the unmeasurable elements. It can be verified that g_i satisfies (2) with $G_k^- = 0$ and $G_k^+ = 1$. In addition, it can be calculated that $\|\omega\|_2$=1.7416.

The disturbance attenuation level in this example is taken as $\gamma = 0.3$. By using the Matlab LMI toolbox, we solve the LMIs in (20)-(22) and can obtain a feasible solution (omitted here for brevity). Therefore, by Theorem 1, the reduced-order l_2-l_∞ filter design problem is solvable, and the gains of an admissible reduced-order filter (3) can be designed as

$$A_{F1} = \begin{bmatrix} 0.1685 & 0.0481 \\ 0.0240 & 0.2063 \end{bmatrix}, \ B_{F1} = \begin{bmatrix} -0.0533 \\ -0.1166 \end{bmatrix}, \ E_{F1} = \begin{bmatrix} -0.2813 \\ -0.2388 \end{bmatrix}^T$$

$$A_{F2} = \begin{bmatrix} 0.1878 & 0.0359 \\ 0.0345 & 0.2130 \end{bmatrix}, \ B_{F2} = \begin{bmatrix} -0.0202 \\ -0.0539 \end{bmatrix}, \ E_{F2} = \begin{bmatrix} -0.2895 \\ -0.1967 \end{bmatrix}^T$$

$$A_{F3} = \begin{bmatrix} 0.1631 & 0.0072 \\ -0.0056 & 0.1426 \end{bmatrix},\ B_{F3} = \begin{bmatrix} -0.0059 \\ -0.1286 \end{bmatrix},\ E_{F3} = \begin{bmatrix} -0.2767 \\ -0.2433 \end{bmatrix}^T$$

$$A_{F4} = \begin{bmatrix} 0.1778 & 0.0235 \\ 0.0160 & 0.1870 \end{bmatrix},\ B_{F4} = \begin{bmatrix} -0.0265 \\ -0.1065 \end{bmatrix},\ E_{F4} = \begin{bmatrix} -0.3104 \\ -0.2047 \end{bmatrix}^T$$

Under the evolution of the jumping mode depicted in Fig.1, the state response of the filtering error system (4) without disturbance is given in Fig.2, while the error response of the filtering error system (4) with zero initial conditions is shown in Fig.3. It can be seen from Fig.2 that applying the designed reduced-order filter makes the filtering error system (4) stochastically stable. Furthermore, from Fig.3 we can see that $\|e\|_{\varepsilon\infty}=0.0325$, and thus

$$\|e\|_{\varepsilon\infty} / \|\omega\|_2 = 0.0325/1.7416 = 0.0187 < \gamma$$

which confirms the effectiveness of filter design procedure.

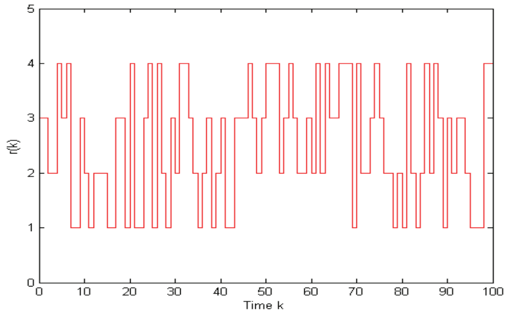

Fig.1. Evolution of the mode

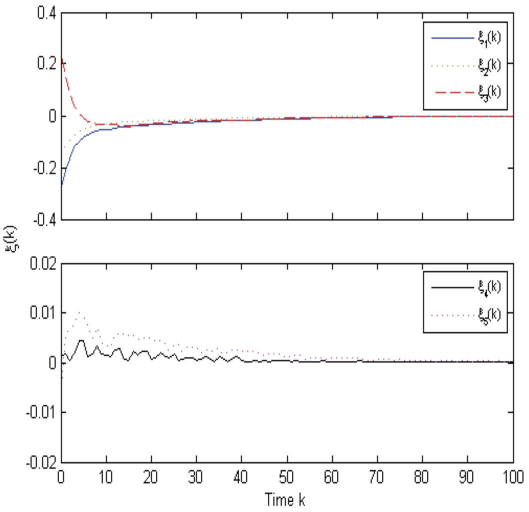

Fig.2. State response of the filtering error system

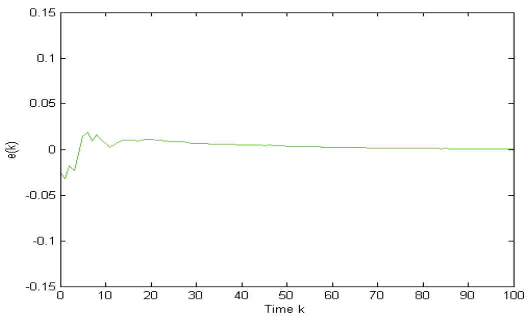

Fig.3. Error response of the filtering error system

Conclusions

The l_2-l_∞ filter design problem for discrete time-delay Markovian jump neural networks is investigated in the paper. A delay-dependent sufficient condition for the solvability of this problem is proposed. An explicit expression for the desired reduced-order filter can be constructed through the numerical solutions of linear matrix inequalities. The results obtained in the paper can be extend further to deal with more complex systems, for instance, systems with parameter uncertainties and distributed delays.

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