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Sliding mode control with a robust observer of induction motor

***Abstract**. The purpose of this paper is to associate a sliding mode control (SMC) with a powerful nonlinear observer robust flux for an induction motor. We implement this design strategy through an extension of a special class of nonlinear multivariable systems satisfying some regularity assumptions. We show by an extensive study that this purpose is completely satisfactory at low and nominal speeds and it is not sensitive to disturbances and parametric errors. It is robust to changes in load torque, rotational speed and rotor resistance.*

***Streszczenie.** Celem niniejszego artykułu jest skojarzenie sterowania ślizgowego z nieliniowym obserwatorem strumienia odpowiednim dla silników indukcyjnych. Realizujemy tę strategię projektowania poprzez rozszerzenie specjalnej klasy układów nieliniowych wielu zmiennych spełniających pewne założenia prawidłowości. Przedstawiono analizę wykazująca, że ten cel jest spełniony dla niskich prędkości i nominalnym, a system nie jest wrażliwy na zakłócenia i błędy parametryczne. Jest odporny na zmiany momentu obciążenia, prędkości obrotowej i oporu wirnika. (**Sterowanie ślizgowe silnikiem indukcyjnym z odpornym oberwatorem**)*

Keywords: Sliding mode control, induction motor, nonlinear observer, rotor resistance
Słowa kluczowe: sterowanie ślizgowe, silnik indukcyjny, nieliniowy obserwator.

Introduction

The induction motor constitutes a theoretically challenging control problem since the dynamical system is nonlinear, the electric rotor variables are not measurable, and the physical parameters are most often imprecisely known. The control of the induction motor has attracted much attention in the past few decades. Especially the speed sensorless control of induction motors (IM) has been a popular area due to its low cost and strong robustness [1],

The new industrial applications necessitate flux and speed variations having high dynamic performances, good precision in permanent regime, and a high capacity of overload over the whole range of position and speed, and robustness to different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories [2].

Among nonlinear control strategies, sliding-mode control is one of the effective control methodologies for IM drive control because of its disturbance rejection, strong robustness subject to system parameter variations and uncertainties and particularly its simplicity of practical implementation. Upon these advantages so far, many research notes have been reported for IM drives control or estimation, using sliding-mode technique [3]. The sliding mode control proposed by [2,4-6] decouples completely the model of IM actually it is not required to establish a decoupling by field oriented control (FOC) as is usually done in vector control.

So the idea is to combinate a sliding mode controller and high gain observer in order to have a strong robustness [7]. This paper is organized as follows. The oriented model of an induction motor is introduced in section 2. In section 3, a robust observer of IM using the high gain is proposed. The sliding mode theory and the design of the sliding mode controllers are presented in section 4. In section 5, the control of rotor flux and motor speed of IM are presented. Finally, we give some concluding remarks on the proposed controller and/observer of IM, and some simulation results are presented.

Mathematical model of induction motor

A three-phase induction motor with squirrel cage rotor is considered in the paper. Assuming the three-phase AC voltage is uniformly distributed over the stator windings are balanced and is based on the well-known in two phases. equivalent representation of the motor, the model of induction motor can be described in the fixed coordinate system (α,β) by a set of the first order nonlinear differential equations [1].

$$(1)\begin{cases} \dot{i}_{s\alpha} = -\gamma\, i_{s\alpha} + \dfrac{K}{T_r}\varphi_{r\alpha} + p\,\Omega\, K\,\varphi_{r\beta} + \dfrac{1}{\sigma L_s}U_{s\alpha} \\ \dot{i}_{s\beta} = -\gamma\, i_{s\beta} + \dfrac{K}{T_r}\varphi_{r\beta} - p\,\Omega\, K\,\varphi_{r\alpha} + \dfrac{1}{\sigma L_s}U_{s\beta} \\ \dot{\varphi}_{r\alpha} = \dfrac{M}{T_r}i_{s\alpha} - \dfrac{1}{T_r}\varphi_{r\alpha} - p\,\Omega\,\varphi_{r\beta} \\ \dot{\varphi}_{r\beta} = \dfrac{M}{T_r}i_{s\beta} - \dfrac{1}{T_r}\varphi_{r\beta} + p\,\Omega\,\varphi_{r\alpha} \\ \dot{\Omega} = \dfrac{C_e}{J_m} - \dfrac{f_m}{J_m}\Omega - \dfrac{\tau_L}{J_m} \end{cases}$$

$$C_e = \frac{pM}{L_r}\left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}\right)$$

with

$$\sigma = 1 - \frac{M^2}{L_s L_r};\; K = \frac{M}{\sigma L_s L_r};\; T_r = \frac{L_r}{R_r};$$

$$\gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r}$$

Here, $\varphi_{r\alpha}$, $\varphi_{r\beta}$ – the rotor flux components, $U_{s\alpha}$, $U_{s\beta}$ – the stator voltage components, $i_{s\alpha}$, $i_{s\beta}$ – are the stator current components, σ is the leakage factor and *p* – the number of pole pairs. R_s and R_r are stator and rotor resistances, L_s and L_r denote stator and rotor inductances, whereas *M* is the mutual inductance. C_e is the electromagnetic torque, τ_L is the load torque, J_m is the moment of inertia of the IM, Ω is the mechanical speed, f_m is the damping coefficient, T_r is the rotoric time-constant.

Nonlinear observer of induction motor

The proposed observer uses the measurements of the stator voltage and current, and the rotor speed. More precisely, the observer is designed up to an injection of the speed measurements so that only electrical equations are considered. Consequently, the gain can be updated directly, as described in the theorem, without making use of any kind of transformation.

The model is described by:

(2) $$\begin{cases}\dot{z} = F(\Omega)\, z + G(u,\Omega,z) \\ y = C\, z\end{cases}$$

where

$$z_1 = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}, z_2 = \begin{bmatrix} \varphi_{r\alpha} \\ \varphi_{r\beta} \end{bmatrix},$$

$$u = \begin{bmatrix} U_{s\alpha} \\ U_{s\beta} \end{bmatrix}, \quad y = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}, \quad s = \Omega$$

$$F_1(\Omega) = \begin{bmatrix} \dfrac{K}{T_r} & Kp\Omega \\ -Kp\Omega & \dfrac{K}{T_r} \end{bmatrix}, g_1(u,\Omega,z_1) = \begin{bmatrix} -\gamma i_{s\alpha} + \alpha U_{s\alpha} \\ -\gamma i_{s\beta} + \alpha U_{s\beta} \end{bmatrix}$$

and $g_2(u,\Omega,z) = \begin{bmatrix} \dfrac{M}{T_r} i_{s\alpha} - \dfrac{1}{T_r}\varphi_{r\alpha} - p\Omega\varphi_{r\beta} \\ \dfrac{M}{T_r} i_{s\beta} - \dfrac{1}{T_r}\varphi_{r\beta} + p\Omega\varphi_{r\alpha} \end{bmatrix}$

Assume that the system (2) satisfies Assumption's [8]

Then there exists $\theta > 0$ such that the System

(3) $$\dot{z} = F(\Omega)\hat{z} + +G(u,\Omega,\hat{z}) - \Lambda^{-1}(\Omega) S_\theta^{-1} C^T \left(C\hat{z} - y\right)$$

Where

$$\Lambda(\Omega) = \begin{bmatrix} I_2 & 0 \\ 0 & F_1(\Omega) \end{bmatrix}, \quad S_\theta^{-1} C^T = \begin{bmatrix} 2\theta\, I_2 \\ \theta^2 I_2 \end{bmatrix}$$

The choice of θ permits the pole placement of the motor and the observer according to the speed

Sliding mode control

The sliding mode technique is developed to solve the disadvantage of other design of non linear control systems. This technique adjust feedback by previously defining a surface, so that the system which is controlled will be forced to that surface, then the behaviour of the system slides to the desired equilibrium point.

The main feature of this control is that we only need to drive the error to a switching surface .When the system is in the sliding mode, the system behaviour is not affected by any modelling uncertainties and/or disturbances.

Calculation of control laws:

The control function will satisfy reaching conditions in the following form:

(4) $$u = u_e + u_i$$

here *u* is the control vector, u_e is the equivalent control vector, it can be interpreted as the average value swing , aside from, it is calculate *: S(X)=0 →S(X)=0* , u_i is the correction factor and must be calculated so that the stability conditions for the selected control are satisfied. $u_i = k\; sign\big(S(X)\big)$

Speed and flux sliding mode controller

The objective of SMC is designed for converge the modulus of the rotor flux vector (ø), and speed (Ω) to their reference value $ø_{ref}$ and Ω_{ref}, respectively. For that it is proposed that all states are measured, our objective is to build a control law $u=[u_a\ u_b]^T$ to force the states that are flux and speed to meet the slide surface $S=[S_1\ S_2]^T$ which is defined by:

(5) $$\begin{cases} S_1 = \dfrac{k_1}{\mu}\left(\Omega - \Omega_{ref}\right) + f_2 - \dfrac{\tau_L}{J_m \mu} - \dfrac{\dot{\Omega}_{ref}}{\mu} \\ S_2 = \dfrac{T_r}{2} k_2\left(\Phi - \Phi_{ref}\right) + M\, f_1 - \Phi - \dot{\Phi}_{ref}\dfrac{T_r}{2} \end{cases}$$

where

$$\mu = \frac{pM}{J_m L_r}; \quad f_1 = i_{s\alpha}\varphi_{r\alpha} + i_{s\beta}\varphi_{r\beta};$$

$$f_2 = i_{s\beta}\varphi_{r\alpha} - i_{s\alpha}\varphi_{r\beta}; \quad k_1, k_2 > 0$$

Φ_{ref} and Ω_{ref} are the time derivative of φ_{ref} and Ω_{ref}, respectively; along $S \equiv 0$ we have :

(6) $$\begin{cases} \mu f_2 - \dfrac{\tau_L}{J_m} = -k_1\left(\Omega - \Omega_{ref}\right) + \dot{\Omega}_{ref} \\ \dfrac{2}{T_r}\left(Mf_1 - \Phi\right) = -k_2\left(\Phi - \Phi_{ref}\right) + \dot{\Phi}_{ref} \end{cases}$$

Knowing that

(7) $$\begin{cases} \dot{\Omega} = \mu\left(i_{s\beta}\varphi_{r\alpha} - i_{s\alpha}\varphi_{r\beta}\right) - \dfrac{\tau_L}{J_m} \\ \dot{\Phi} = \dfrac{2}{T_r}\left(M\left(i_{s\alpha}\varphi_{r\alpha} + i_{s\beta}\varphi_{r\beta}\right) - \Phi\right) \end{cases}$$

We obtain

(8) $$\begin{cases} \dfrac{d}{dt}\left(\Omega - \Omega_{ref}\right) = -k_1\left(\Omega - \Omega_{ref}\right) \\ \dfrac{d}{dt}\left(\Phi - \Phi_{ref}\right) = -k_2\left(\Phi - \Phi_{ref}\right) \end{cases}$$

Consequently on S ≡ 0 the rotor speed and the square of rotor flux must converge to their references. However from following their reference, it is sufficient to make the sliding surface attractive and invariant. We consider the following proposition:

Proposition:

We consider the slid surface $S=[S_1\ \ S_2]^T$ defined in (4) and control law sliding mode $u = u_e + u_i$

(9) $$\begin{cases} u_i = -D^{-1}\begin{bmatrix} u_{01} & 0 \\ 0 & u_{02} \end{bmatrix}\begin{bmatrix} sign(S_1) \\ sign(S_2) \end{bmatrix} \\ u_e = -D^{-1}\begin{bmatrix} A \\ B \end{bmatrix} \end{cases}$$

(10) $$\begin{cases} u_{01} > |A| \\ u_{02} > |B| \end{cases}$$

where

$$D = \alpha\begin{bmatrix} -\varphi_{r\beta} & \varphi_{r\alpha} \\ M\varphi_{r\alpha} & M\varphi_{r\beta} \end{bmatrix} \textit{ with } \alpha = \frac{1}{\sigma L_s}$$

and

(11) $$\begin{cases} A = \left(k_1 - \dfrac{1}{T_r} - \gamma\right) f_2 - k_1\dfrac{\tau_L}{\mu J_m} - p\Omega\left(f_1 + K\Phi\right) \\ \quad - \dfrac{k_1}{\mu}\Omega_{ref} - \dfrac{1}{\mu}\ddot{\Omega}_{ref} \\ B = \left(\dfrac{T_r k_2}{2} - 1\right)\dot{\Phi} - \dfrac{T_r}{2}k_2\dot{\Phi}_{ref} - \dfrac{T_r}{2}\ddot{\Phi}_{ref} + \\ \quad M\left(\dfrac{M}{T_r}\left(i_{s\alpha}^2 + i_{s\beta}^2\right) - \left(\dfrac{1}{T_r} + \gamma\right) f_1 + \dfrac{K}{T_r}\Phi + p\Omega f_2\right) \end{cases}$$

Proof

Let consider the Lyapunov function $V = \dfrac{1}{2} S^T S$, so its time derivative $\dot{V} = S^T \dot{S}$

with $\dot{S} = \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} + D\, u_i$, we can be rewritten $\dot{S}$ as follows:

(12) $$\dot{S} = \begin{bmatrix} A \\ B \end{bmatrix} - \begin{bmatrix} u_{01} & 0 \\ 0 & u_{02} \end{bmatrix} sign(S)$$

The variety $\dot{S}$ is attractive if: $S^T \dot{S} < 0$
Then

(13) $$\begin{cases} u_{01} > |A| \\ u_{02} > |B| \end{cases}$$

We can choose u_{01} u_{02} such that

(14) $$\begin{cases} u_{01} = |A| + k_1 \dfrac{\tau_{L\max}}{\mu\, J_m} \\ u_{02} = |B| + k_2 \dfrac{\tau_{L\max}}{\mu\, J_m} \end{cases}$$

Where $\tau_{L\max} = \max(\tau_L)$

Then the condition of existence of sliding requires only to knowledge the maximum value of torque that the machine and support. However $S=0$ is invariant if $\dot{S} = 0$; that is to say:

(15) $$u_e = -D^{-1} \begin{bmatrix} A \\ B \end{bmatrix}$$

In the design of the control, we assume that all state was measured. Since is only measures the current and the speed are a variable, we will need to estimate a rotor flux for a real time application.

Results and simulations

Simulation blocks diagrams

As a first step, a Simulink/Matlab simulation was realized in order to simulate a motor model according with the proposed observer. The parameters of the motor model are given in the Table 1 [8]. The trajectories of the references speed, flux and load torque are given in Fig. 1. This benchmark shows that the load torque appears at the nominal speed. In spite of a varying speed, the resistive torque is zero. The desired flux remains constant in the asynchronous machine to satisfy the objectives of the field-oriented control.

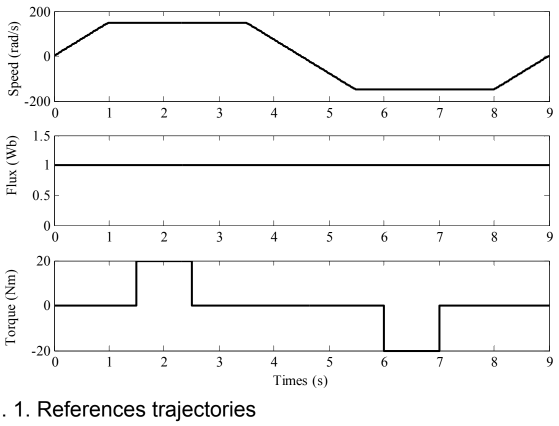

Fig. 1. References trajectories

Table 1: Inductor motor parameters

Parameter	Notation	Value
Rotor resistance	R_r	4.3047 Ω
Stator resistance	R_s	9.65 Ω
Mutual inductance	M	0.4475 H
Stator inductance	L_s	0.4718 H
Rotor inductance	L_r	0.4718 H
Rotor inertia	J_m	0.0293 kg/m²
Pole pair	p	2
Viscous friction coefficient	f_m	0.0038 Nm.sec/rad
Mechanical power	P_{mec}	1.1 KW
Nominal voltage	V_{sn}	220 V
Nominal current	T_{sn}	2.6 A
Nominal speed	Ω_{sn}	1410 rpm

Study of the nonlinear observer in an open loop

First we test the nonlinear observer are open loop (θ = 500) at low and high speed while varying the rotor resistance up to 180%. In the figure 2 and 3, we noted that the estimated flux and the real flux are fully in line after 0.02sec.

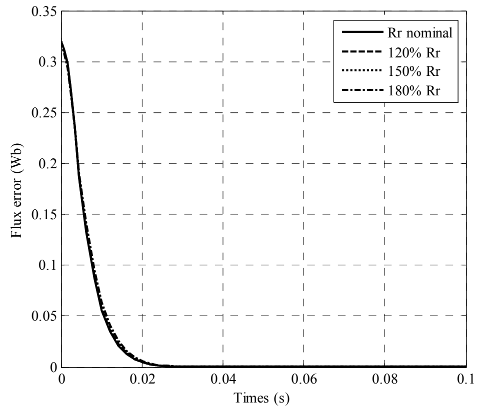

Fig. 2. Observation errors of the flux at a low speed of 230 rpm

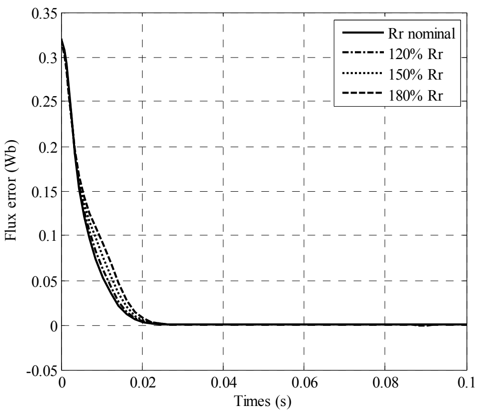

Fig. 3. Observation errors of the flux at a nominal speed of 1500 rpm

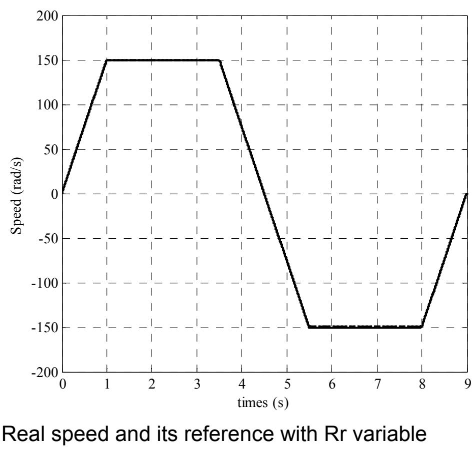

Fig. 4. Real speed and its reference with Rr variable

Sliding mode control SMC

In this section (Figure 4-7), the flux is considered measurable and the non-linear control SMC, when analyzing the variation of rotor resistance up to 180% and it also assumes that the torque is zero. we can clearly distinguish that this variation does not affect the controller because the error has not exceeded 0.07rad/s (Fig.5).

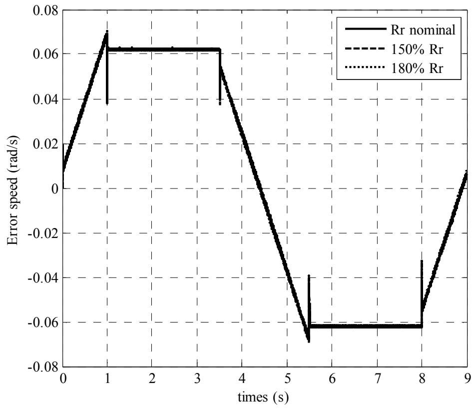

Fig. 5. Comparison of the error speed for 180% variation Rr

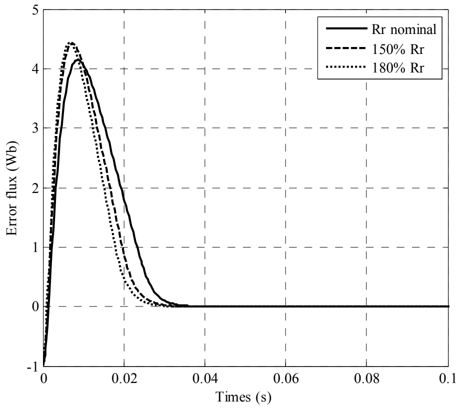

Fig. 6. Comparison of the error flux for 180% variation Rr

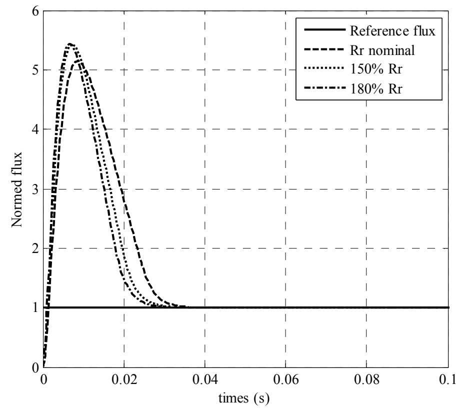

Fig. 7. Real flux and its reference with Rr variable

Performance SMC associated with the nonlinear observer

A periodic trapezoidal reference speed is used here to study the tracking performance of the drive system. It is shown in Fig.1, the speed is increased linearly from 0 at t = 1 sec to 150 rad/sec at t = 3.5sec, and decreased linearly to -150 rad/sec at t=5.5sec. Then, the speed is kept constant at -150 rad/sec till t = 8 sec and increased linearly to zero at t = 9 sec. The same trajectory is used to study the performance of sliding mode controller with variation of rotor resistance at 180%

Low speed

At a low speed of 230 rpm, there is an optimum speed error and flux not exceeding, respectively, 35 rad/s and 4.5 Wb and it vanishes very quickly to 0.03 sec (Fig 8 and 9).

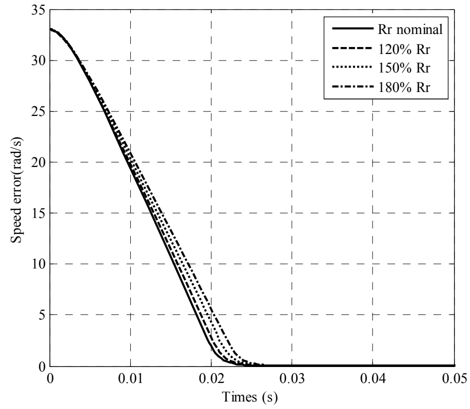

Fig. 8. Comparison of error speed for 180% variation in Rr

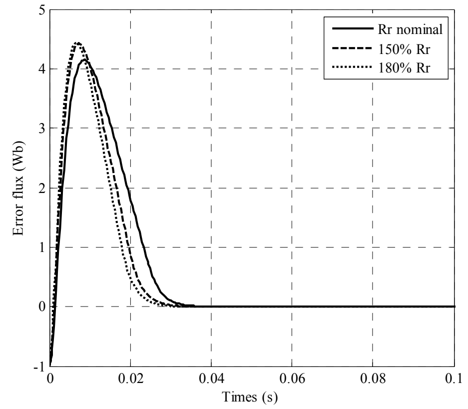

Fig. 9. Comparison of error flux for 180% variation in Rr 230 rpm

Nominal speed

By applying the reference trajectory (Fig. 1) with a load torque zero on interval 0 sec to 1.5 sec, which rises abruptly to 20 Nm stabilized until 2.5sec and then it drops to zero at -20 Nm no 6 s for 1 s, we note that the effect of the load torque is negligible on the speed and flow control. The influence of rotor resistance appears only when the torque is important. As a result the nonlinear controller SMC can be considered robust (Fig.10-13).

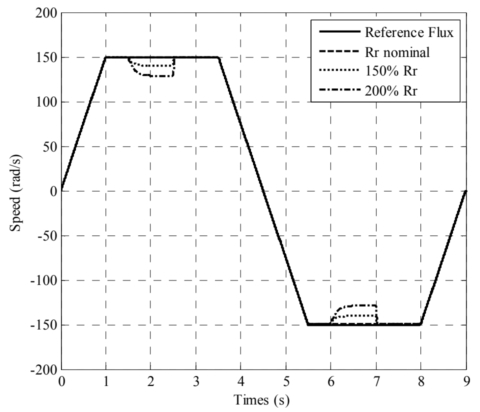

Fig. 10. Real speed and its reference

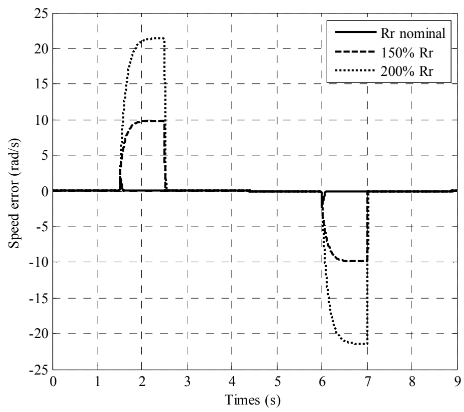

Fig. 11. Comparison of error speed for 200% variation in Rr

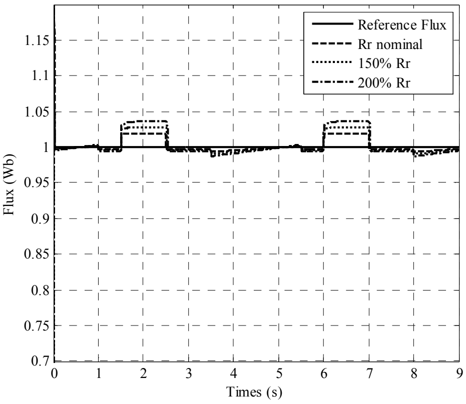

Fig. 12. Zoom on actual and reference flux

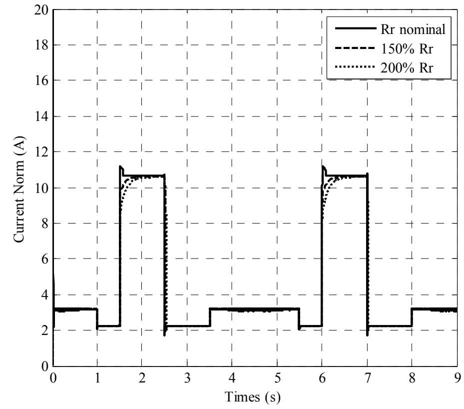

Fig. 13. Currents norm stator

Conclusion

The robust nonlinear observer of a special class associated with SMC for induction motor has been presented. The rotor flux observer accuracy is guaranteed through the stator currents observer, based on the Lyapunov stability theory. The results show that this nonlinear observer offers better performances while tracking the torque, speed and estimating the flux. A major advantage of the method is that very little tuning was required to obtain the convergence of the observation at low speeds.

The efficiency of the control-observer structure has been successfully verified by simulation. The proposed sliding mode control with nonlinear observer demonstrated very good performance, especially; it is robust under rotor resistance variation, external load disturbances and speed tracking. Future work is oriented at experimental validation, including stator time-constant estimation.

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