JI HAIYING, Luo YONGFEN, JI SHENGQIANG, LI JISHENG, LI YANMING

School of Electrical Engineering, Xi'an Jiaotong University

Simulation of Cross-shaped Ultrasonic Array Applied to Partial Discharge Location in Oil

Abstract A study on a combined sensor was made for partial discharge(PD) location in oil. The sensor includes a cross-shaped ultrasonic phased array of 13 elements and an ultra-high frequency (UHF) electromagnetic conformal array of 4 elements. The ultrasonic array can achieve the array performance of 61 elements with high-order cumulant processing technology. It can improve the array aperture and direction sharpness of the sensor and greatly reduce the following circuit and manufacturing costs. The method of PD location which use expanded ultrasonic array accompanied with UHF electromagnetic array has better location performance. The spatial spectrum comparing with MUSIC(multiple signal classification) method shows that virtual expanding array of the fewer number array elements with high-order cumulant processing technology has strong ability to suppress colored Gaussian noise interference. This provides the possibility for the practical application of array technology in power equipment.

Streszczenie. Zaprezentowano czujnik do badania wyładowania niezupełnego w oleju. Czujnik składał się z matrycy 13 czujników ultradźwiękowych l 4 wysokoczęstotliwościowych czujników elektromagnetycznych. Matryca czujników może spełniać rolę wirtualnej matrycy złożonej z 61 elementów. (Matryca krzyżowych czujników ultradźwiękowych do badania wyładowania niezupełnego w oleju)

Keywords: partial discharge; high-order cumulant; noise suppression; virtual array expansion. **Słowa kluczowe:** wyładowanie niezupełne, czujniki ultradźwiękowe.

Introduction

Partial discharge is the major reason resulting in insulation deterioration for the power equipment, especially for the power transformer. PD monitoring is also an important part of intelligent monitoring equipment online. The PD parameters are greatly affected by the location of the PD sources in power equipment. Fixing the PD location can reflects the insulation condition. It can make the maintenance strategy better and shorten the repair cycle. So PD location is significant.

With the electromagnetic wave, sound, light, heat and chemical changes that are produced in the PD process, the traditional PD location method includes the electric method, ultrasonic method and electric- ultrasonic method[1-3]. As the development of signal processing technology and the manufacturing technology, some new location methods had also been used. Like the UHF method and the method based on array technologies[4-7].

For the method of using array technologies to locate the PD source, the current research is based on rectangular plane array sensor, such as the 16×16 ultrasonic array sensor[5], the array has good performance and can locate multiple targets. However, it has too many elements and the following circuit and manufacturing cost is too much that affects its practicality. So to achieve the purpose of practical application of array technology, we should solve the problem of how to reduce the elements number and subsequent hardware on the premise of certain accuracy.

We had made a study of a combined sensor which includes a 13-element cross-shaped ultrasonic array and a 4-element conformal UHF array in this paper. The cross-shaped array is expanded with high order cumulants technology. The PD sources were located with expanded ultrasonic array and UHF array. And it is compared with the direction estimation effect of MUSIC method and high order cumulant method.

The structure of the combined sensor

The combined sensor in this paper is mainly used to locate PD source in power transformer. However, in practically application, it is limited by the installation position(such as oil valve, manhole), conformal with UHF sensor, following circuit and fabricating cost, etc. The sensor is the smaller size and fewer elements, the better.

The UHF sensor is usually larger that the UHF array can not have too many elements in limited space . It is often used

for providing the time delay of ultrasonic signal and estimating the DOA(direction of arrival) roughly. In this paper, a combined sensor which includes a 13-element cross-shaped ultrasonic array and 4-element conformal UHF array was designed. We arrange the ultrasonic array in the space between UHF elements as shown in Fig.1.

However, simply reducing array elements will make the location precision low. In order to improve the accuracy of the array sensor, it needs to use special signal technology to process the array data. The aim is that on the basis of existing array and following hardware it has the effect of more elements array with the technology of digital signal processing.



Fig.1. Structure diagram of combined sensor

The principle of PD location with virtual expanded array

High order cumulants analysis is an advanced subject in the signal processing domain in recent years. Its property of expansion can solve that problem above.

A. Array expansion based on fourth order cumulants The principle of array expansion could be explained by three elements. As shown in Fig.2, there are three elements in the space (denoted by solid circular) and the hollow circular denotes the virtual element obtained by fourth order cumulants of the other three elements. Assuming the reference element *r* receives a static signal r(t) in the space and r(t)=s(t). Then, the received data of elements *x*, *y* and *v* are:

1)
$$\begin{cases} x(t) = s(t) \exp(-j\vec{k} \cdot \vec{d}_x) \\ y(t) = s(t) \exp(-j\vec{k} \cdot \vec{d}_y) \\ v(t) = s(t) \exp(-j\vec{k} \cdot \vec{d}) \end{cases}$$

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where \vec{k} is propagation vector. \vec{d}_x , \vec{d}_y are the position vectors of the elements *x* and *y*. \vec{d} is the position vector of the virtual element *v*. Then the mutual correlation of *r*(*t*) and *v*(*t*) is:

(2)
$$\mu_{r,v} = E\{r(t)v^*(t)\} = \sigma_s^2 \exp(j\vec{k}\cdot\vec{d})$$

where σ_s^2 is the signal power, and is constant. And the fourth order cumulant is:

(3)

$$\mu_{r,x}^{r,y} = cum\{r(t), r(t), x^{*}(t), y^{*}(t)\}$$

$$= cum\{s(t), s(t), s^{*}(t) \exp(j\vec{k} \cdot \vec{d}_{x}), s^{*}(t) \exp(j\vec{k} \cdot \vec{d}_{y})\}$$

$$= cum\{s(t), s(t), s^{*}(t), s^{*}(t)\} \exp(j\vec{k} \cdot \vec{d}_{x}) \exp(j\vec{k} \cdot \vec{d}_{y})$$

$$= \gamma_{t,c} \exp(i\vec{k} \cdot \vec{d})$$

where $\gamma_{4,s}$ is constant, * represents the conjugate operation.



Fig.2. Schematic diagram of array virtual expansion based on fourth order cumulant

The relation between second order and fourth order statistic can be seen with the equations (2) and (3). The mutual correlation between real elements and virtual elements can be expressed by fourth order cumulants of real elements. Therefore, while estimating the DOA of PD, the covariance matrix of array signal $\{x(t), y(t), r(t), v(t)\}$ can be represented by the real array signal $\{x(t), y(t), r(t)\}$. That is to say, we can use three elements to achieve the effect of four elements. This is the expansion property of fourth order cumulants.



Fig.3 Expansion diagram of cross-shaped array

B. Virtual expansion of cross-shaped array

The mathematical model of ultrasonic array is built as Fig.3. The array elements lie on the *X* and Y axis and equally spaced. *P* point is the position of PD. φ and θ is the azimuth and elevation angle of the signal direction. By coordinate subtracting each orther, the coordinate of the

elements after expanded can be obtained. The diagram of array element location pre- & post- expanded is shown in Fig. 3. The solid circular and hollow circular represent the array pre- & post- expanded. The expanded array elements lie on the rectangular surface that is occupied by the cross-shaped array and the extended line of the original array. The number of cross-shaped elements is 13, after expanded is 61.

C. Beampattern comparison of pre- & post- expanded array

The beam pattern can intuitively reflect the directivity and resolution of the array. Fig. 4 is the comparison of beam pattern slice. u_r is the direction $cosine(u_r = sin \varphi)$. Curve 1, 2 and 3 represent the slices of the 13-element cross-shaped array, the expanded array and the 7×7 rectangular surface that occupied by the cross-shaped array.



Fig.4 Beampattern comparison of the pre- & post- expanded array(X- or Y- Cross plane)

The sidelobe level and half power beam width(HPBW) of the expanded array is better than the original array. And its directivity is better than the rectangular array on the whole. This is because the aperture and the elements number have improved dramatically.

Simulation of PD location with expanded L-shaped array

The ultimate aim of array expansion study is that this technology can used in PD location. Here the ultrasonic and UHF signals of PD are considered to be radio-frequency (RF) signals.

A. PD location process based on fourth order cumulants

The PD location process based on fourth order cumulants is follow-up:

1) Expanding to the cross-shaped array, then calculating the array flow vector $B(\theta)$ of the virtual array;

2) Calculating the fourth order matrix R_{cum} of ultrasonic signal;

3) Doing eigenvalue decomposition to the R_{cum} , then the eigen-subspace E_n will be obtained. By the spectrum estimation formula as the equation (4), doing spectrum peak search. The value (θ, φ) corresponding to the maximum of spatial spectrum is the direction of the signal.

(4)
$$P(\theta) = \frac{1}{\left\| B(\theta) E_n \right\|^2}$$

4) Synthetizing the array signal in the direction (θ, φ) that is estimated in the last step (namely doing weighted summation of the array received data in this direction), then the output of ultrasonic and UHF array y(t) and $y_1(t)$ will be obtained. The time difference of the two envelope peaks is the propagation time delay au of ultrasonic.

5) According to speed of ultrasonic *c* in oil, the distance between the PD source and sensor is $l=c \times \tau$. Combined with θ , φ and l, the cylindrical coordinate can be translated into cartesian coordinate.

B. PD location of single source

Assuming that there is one PD source in the space, its direction is $(\varphi, \theta) = (45^{\circ}, 45^{\circ})$ and distance *l*=3m. Corresponding to the cartesian coordinate is (1.50m, 1.50m, 2.12m). The background noise is colored Gaussian noise, and the SNR is 10dB.



(a) Spatial spectrum based on ultrasonic signal



(b) Beamforming envelope of ultrasonic and UHF signals

Fig.5 Simulation results of single PD source location

Fig. 5 shows the simulation results of PD location of one source. Subfigure(a) shows the DOA spatial spectrum based on fourth order cumulants of ultrasonic signal. By the coordinate corresponding to the spectrum peak, the direction of the PD source could be obtained that $(\varphi', \theta') = (45^{\circ}, 45.5^{\circ})$.

Synthetizing the ultrasonic and UHF signals in the direction (φ', θ') , the envelope of beams y(t) and $y_1(t)$ are shown in subfigure(b). It can be seen that, there are two peaks at the time 1ms and 2.98ms, so the distance *l* between the PD source and sensor is 2.97m(c=1500m/s) Translated into cartesian coordinate, the simulated location is (1.47m,1.47m,2.12m). By the hypothetic location and the simulated location, the relative error is 1.4%.

C. PD location of two sources

It is assuming that there are two PD sources in the space. The direction of the first one is $(\varphi_1, \theta_1) = (45^\circ, 45^\circ)$ and its distance is 3m, corresponding to the Cartesian coordinate is (1.50m, 1.50m, 2.12m); The direction of the second one is $(\varphi_2, \theta_2) = (150^\circ, 60^\circ)$ and its distance is 3.9m, corresponding to the Cartesian coordinate is (-2.93m, 1.69m, 1.95m).

The background noise is colored Gaussian noise, and the SNR is 10dB.

Fig.6 shows the simulation results of PD location of two sources. Subfigure(a) shows the DOA spatial spectrum based on fourth order cumulants of ultrasonic array signal. By the coordinate corresponding to spectrum peaks, the direction of the two PD sources are $(\varphi_1', \theta_1') = (45^\circ, 45.5^\circ)$ and $(\varphi_2', \theta_2') = (149.5^\circ, 59^\circ)$.



(a) Spatial spectrum based on ultrasonic signal





Synthetizing the array signal in the two directions (φ_1', θ_1') and (φ_2', θ_2') , the envelopes of the array output are shown in subfigure(b). Curve 1 and 3 are the ultrasonic and UHF signal envelopes of the first PD source, the time difference between the two peaks is 1.97ms. So the distance between the first PD source and sensor l_1 is 2.96m. Curve 2 and 4 are the ultrasonic and UHF signal envelopes of the second PD source, the time difference between the two peaks is 2.62ms. So the distance between the second PD source and sensor l2 is 3.93m. Translated into cartesian coordinate, the simulated locations are (1.49m, 1.49m, 2.07m) and (-2.90m, 1.71m, 2.00m). By the hypothetic locations and the simulated locations, the relative errors are 1.7% and 1.9%

In addition, compared with the two curves in Fig.5(b), at the time 4ms the amplitude of the envelope (solid curve) of the first PD source is lower. Also, at the time 2ms the amplitude of the envelope (dotted curve) of the second PD source is lower too. This is to say, the synthesizing beam has better directionality and can well suppress the signal coupling. And this provides possibility to analyze PD source alone in actual applications.

Noise suppression of fourth order cumulants

While detecting the PD signals, the actual interference noise is usually the colored Gaussian noise.It should be suppressed to improve the precision.

The existing direction methods usually include the

MUSIC method and fourth order cumulants method. Here, we compared the suppression effect of the two methods.

A. Cumulant characteristics of Gaussian signal

For the random vector of Gaussian signal, it can be represented as $\mathbf{X} = (X_1, ..., X_k)^T$. And it obeys the normal distribution $N(\mu, \Sigma)$. The eigenfunction of X is:

(5)
$$\Phi(\omega_1, \cdots, \omega_k) = \exp\{j\mu^T \omega - 0.5\omega^T \Sigma \omega\}$$

Then the cumulant generating function is:

$$\Psi(\omega_1,\cdots,\omega_k) = \ln \Phi(\omega_1,\cdots,\omega_k)$$

(6)

$$= j \sum_{i=1}^{k} \mu_i \omega_i - 0.5 \sum_{i=1}^{k} \sum_{l=1}^{k} \sigma_{il} \omega_i \omega_l$$

The γ th($\gamma = \gamma_1 + \cdots + \gamma_k$) order cumulant of X_1, \cdots, X_k is $C_{\gamma_1 \cdots + \gamma_k}$.

 $= i\mu^T \omega - 0.5 \omega^T \Sigma \omega$

When $\gamma = 2$, there are two situations,

1) One element γ_i =2, the others are zeros. $C_{\gamma_1 \cdots \gamma_k}$ is

(7)
$$c_{0\cdots 020\cdots 0} = (-j)^2 \frac{\partial^2 \Psi(0,\cdots,0)}{\partial \omega_i^2} = \sigma_{ii}$$

2) Two elements $\gamma_i = \gamma_l = 1, i \neq l$, the others are zeros. $c_{\gamma_1 \cdots \gamma_k}$ is

(8)
$$c_{0\cdots 010\cdots 010\cdots 0} = (-j)^2 \frac{\partial^2 \Psi(0,\cdots,0)}{\partial \omega_i \omega_l} = \sigma_{il}$$

From the equations (7) and (8), we can see the second order cumulant of Gaussian signal has a relation to σ_{ii} and σ_{il} . If the σ_{ii} and σ_{il} are unknown, namely the statistical property is indeterminacy, and then the second order cumulants can not describe it. So the MUSIC method based on correlative has worse effect in this case.

When $\gamma \geq 3$, because $\Psi(\omega_1, \cdots, \omega_k)$ is quadratic polynomial of $(\omega_1, \cdots, \omega_k)$, so no matter what $\gamma_1, \cdots, \gamma_k$ are, $c_{\gamma_1, \cdots, \gamma_k}$ always is:

(9)
$$c_{\gamma_1\cdots\gamma_k} = (-j)^{\gamma} \frac{\partial^{\gamma} \Psi(0,\cdots,0)}{\partial \omega_1^{\gamma_1}\cdots \partial \omega_k^{\gamma_k}} = 0$$

As shown in equation (9), the γ th($\gamma \ge 3$) order cumulants of Gaussian process is zero, which means high order cumulants has the blind Gaussian signal speciality. Theoretically it can suppress all kinds of Gaussian signal, including the colored Gaussian noise.

Because the third order cumulants of the symmetric distribution signal is zero, so fourth order cumulant is often used in practical application.

B. Comparison of direction estimation with the MUSIC and fourth order cumulants method

With the same signal and SNR, simulation of direction estimation under white Gaussian noise and colored Gaussian noise background with MUSIC method is shown in Fig.7.

Comparison with Fig.5(a) and Fig.6(a), it can be seen that the sharpness of spatial spectrum with MUSIC method is

worse than the high order cumulant method. It will be difficult to distinguish the adjacent spectrum peak. Especially for the colored noise, the estimation effect is more worse. When the PD source number is greater than two, it can not estimate the correct direction then. This shows that the MUSIC method works worse in the colored noise background. In this case, the location method based on fourth order cumulants is better.



(a) White noise, single source (b)Colored noise, single source



(c) White noise, two sources (d)Colored noise, two sources Fig.7 Spatial spectrum diagram of MUSIC under different noise background

Conclusions

A study of a combined sensor which includes a 13-element cross-shaped ultrasonic array and 4-element conformal UHF array was made in this paper. The cross-shaped array is expanded with high order cumulants technology. The PD sources were located with expanded ultrasonic array and UHF array.

The method of PD location which use expanded ultrasonic array accompanied with UHF electro- magnetic array has better location effect. The spatial spectrum comparison with MUSIC method shows that virtual expanding array of the fewer number array elements with high-order cumulant processing technology has strong ability to suppress colored Gaussian noise interference. This provides the possibility for the practical application of array technology in power equipment.

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Authors: Master student Ji Haiying High Voltage Division, School of Electrical Engineering, Xi'an Jiaotong University, China, 710049, E-mail:<u>haiy.ji@stu.xitu.edu.cn;</u> Doctor Luo Yongfen, High Voltage Division, School of Electrical Engineering, Xi'an Jiaotong University, China, 710049, E-mail:<u>yfluo@mail.xitu.edu.cn;</u> Prof. Li Yanming, High Voltage Division, School of Electrical Engineering, Xi'an Jiaotong University, China, 710049, E-mail: <u>ymli@mail.xitu.edu.cn</u>