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Energy function analysis of a two-machine infinite-bus power system by Lyapunov's second method

***Abstract**. This paper is devoted to the study and analysis of a two-machine infinite-bus (TMIB) power system by applying Lyapunov (energy) function based on the variable gradient method. Firstly, the TMIB power system's energy function is constructed. More, numerical simulation is studied for Lyapunov (energy) function of the TMIB power system. Finally, the simulation results are obtained by using MACSYMA and some useful conclusions are obtained.*

***Streszczenie.** Przedstawiono analizę system z dwoma maszynami i nieskończponą szyną przy zastosowaniu funkcji Lapunova w metodzie zmiennego gradientu. Skonstruowany system poddano badaniom symulacyjnym przy wykorzystaniu MACSYMA. (**Analiza funkcji energii w systemie z nieskończoną szyną i dwoma maszynami przy wykorzystaniu metody Lapunova**)*

Keywords: Lyapunov (energy) function, variable gradient method, TMIB power system, transient stability.
Słowa kluczowe: funkcja energii Lapunova, metoda zmiennego gradientu

Introduction

Electric power system stability was first recognized as an important problem in the 1920s [1, 2]. It has a long history of research [3-5]. The question of transient stability has been studied primarily by examining solutions of systems of differential equations based on the swing equation or driven pendulum. With the Lyapunov methods, power system stability can be estimated using the valuable gradient method.

One of the most important developments in power system stability studies is the Lyapunov (energy) function method. It is based on the Lyapunov's second method (also called Lyapunov's direct method). It is well-known that the Lyapunov's second method is one of the most useful and effective way to analyze the stability of a power system without solving the differential equations [6-8]. It has been applied over the last few decades for power systems [9-12].

Lyapunov has defined a function which will install relationship between accumulated energy in the system and the dynamics of the system. This function is given by considering the concept of energy. If a system's energy is continuously decreasing until an equilibrium state is reached, the system is stable. If the total energy of a system is continuously decreased, the time derivative of energy function is negative definite [13]. In using the Lyapunov's second method to analyze the stability of a nonlinear system, the goal is to construct a scalar energy function (a Lyapunov function) for the system [14].

There is currently no generally applicable way to construct or find Lyapunov functions. Some methods which arise from Lyapunov's second method are proper to examine the stability of nonlinear systems. One of them is the variable gradient method which is used for the generalization of Lyapunov functions. The variable gradient method provides a systematic approach to determining a suitable Lyapunov function. It assumes a certain form for the gradient of an unknown Lyapunov function, and then finding the Lyapunov function itself by integrating the assumed gradient [14].

The variable gradient method

Consider a nonlinear dynamical system described by

$$\dot{x} = f(x,t) \tag{1}$$

Accept an equilibrium point at the origin of the space. Denote a test Lyapunov function by using V and its time derivative $\dot{V}$. Assume that in (1), V is x's open function but not t's. Then,

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + ... + \frac{\partial V}{\partial x_n}\dot{x}_n \tag{2}$$

can be written. Hence,

$$\dot{V} = (\nabla V)^* \dot{x} \tag{3}$$

In (3), $(\nabla V)^*$ is ∇V's transpose. The gradient of V, denoted by ∇V as follows:

$$\nabla V = \begin{bmatrix} \frac{\partial V}{\partial x_1} \\ . \\ . \\ . \\ \frac{\partial V}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ . \\ . \\ . \\ \nabla V_n \end{bmatrix} \tag{4}$$

∇V's line integral can be expressed by

$$V = \int_0^x (\nabla V)^* dx \tag{5}$$

In (5), integral's upper limit does not point that V is a vector magnitude, but integral is preferred to line integral of a random point (x_1,x_2,…,x_n) at the space. This integral can be done separately from integration method.

Investigation of Lyapunov function using gradient system

A special class of dynamical system is particularly well suited to the Lyapunov method. This system arises from the gradient of a function [15]. A gradient dynamical system is given as

$$\dot{x} = -A\nabla v(x,x_0) \tag{6}$$

In (6), $v: \Re^n x \Re^n \to \Re$ can be a continuously differentiable. $A \in \Re^{nxn}$ is defined as det(A)≠0 and $v(x,x_0)=0$ for $x=x_0$. If $v(x,x_0)$'s Hessian is completely positive definite at x_0, equilibrium point is asymptotically stable at x_0. Lyapunov function is given as

(7) $$V(x) = \int_{x_0}^{x} \left[f(\xi)\right]^T d\xi$$

Lyapunov function which is given above will be used in order to find the TMIB power system's energy function.

Energy function of the TMIB power system

We consider the power system model shown in Fig. 1, which is taken from [16].

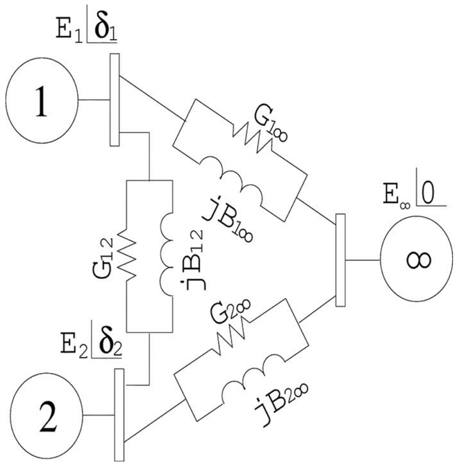

Fig.1. Two-machine infinite-bus power system model [16]

The TMIB power system parameters have been presented in the Appendix. First-order differential equations are expressed which show power system model's equations of state as follow [16].

(8) $$\dot{\delta}_1 = w_1$$

(9) $$M_1\dot{w}_1 = -T_1 w_1 + P_1 - C_1 \sin\delta_1 - D_1 \cos\delta_1 - C_{12}\sin(\delta_1-\delta_2) - D_{12}\cos(\delta_1-\delta_2)$$

(10) $$\dot{\delta}_2 = w_2$$

(11) $$M_2\dot{w}_2 = -T_2 w_2 + P_2 - C_2 \sin\delta_2 - D_2 \cos\delta_2 - C_{12}\sin(\delta_2-\delta_1) - D_{12}\cos(\delta_2-\delta_1)$$

The system differential equations above can be written again under the condition that generator mechanical power is equivalent to active load requirement ($P_m=P_l$).

(12) $$\dot{\delta}_1 = \frac{1}{M_1} M_1 w_1$$

(13) $$\dot{w}_1 = -\frac{T_1}{M_1^2} M_1.w_1 - \frac{1}{M_1} f(\delta_1,\delta_2)$$

(14) $$\dot{\delta}_2 = \frac{1}{M_2} M_2 w_2$$

(15) $$\dot{w}_2 = -\frac{T_2}{M_2^2} M_2.w_2 - \frac{1}{M_2} g(\delta_1,\delta_2)$$

Here,

(16) $$f(\delta_1,\delta_2) = -(P_1 - C_1\sin\delta_1 - D_1\cos\delta_1 - C_{12}\sin(\delta_1-\delta_2) - D_{12}\cos(\delta_1-\delta_2))$$

(17) $$g(\delta_1,\delta_2) = -(P_2 - C_2\sin\delta_2 - D_2\cos\delta_2 - C_{12}\sin(\delta_2-\delta_1) - D_{12}\cos(\delta_2-\delta_1))$$

The derivation of Lyapunov Function for the TMIB power system in Fig. 1, equations (12), (13), (14) and (15) could be determined as

(18) $$\begin{bmatrix}\dot{\delta}_1\\ \dot{w}_1\\ \dot{\delta}_2\\ \dot{w}_2\end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{M_1} & 0 & 0\\ \frac{1}{M_1} & \frac{T_1}{M_1^2} & 0 & 0\\ 0 & 0 & \frac{1}{M_2} & \frac{T_2}{M_2^2}\\ 0 & 0 & 0 & -\frac{1}{M_2}\end{bmatrix} \begin{bmatrix} f(\delta_1,\delta_2)\\ M_1 w_1\\ g(\delta_1,\delta_2)\\ M_2 w_2\end{bmatrix}$$

The equation (18) for the TMIB power system defined in equations (8), (9), (10) and (11), is an alternative definition for this system's dynamics.

For the (δ_1^0, w_1^0, δ_2^0, w_2^0)'s equilibrium point, a candidate energy function which is seen on the right of the equation (18), ((4x1) gradient matrix seen on the right of the equation (18)) is obtained and therefore it can be used in equation (7). The candidate energy function can be written in equation (7) as

(19) $$v(\delta_1,w_1,\delta_2,w_2) = \int_{(0,0,0,0)}^{(\delta_1,w_1,\delta_2,w_2)} \begin{bmatrix} f(\delta_1,\delta_2)\\ M_1 w_1\\ g(\delta_1,\delta_2)\\ M_2 w_2\end{bmatrix}^T \begin{bmatrix} d\delta_1\\ dw_1\\ d\delta_2\\ dw_2\end{bmatrix}$$

If $f(\delta_1,\delta_2)$ and $g(\delta_1,\delta_2)$ are replaced on equation (19), the TMIB power system's energy function is obtained as

(20)
$$v(\delta_1,w_1,\delta_2,w_2) = D_1\sin\delta_1 - C_{12}(\cos(\delta_1-\delta_2)-\cos\delta_2) - C_1(\cos\delta_1 - 1) - P_1\delta_1$$
$$+ D_{12}(\sin(\delta_1-\delta_2)+\sin\delta_2) - \frac{1}{2}M_1 w_1^2 + D_2\sin\delta_2 - C_{12}(\cos(\delta_1-\delta_2)-\cos\delta_1)$$
$$- D_{12}(\sin(\delta_1-\delta_2)-\sin\delta_1) - C_2(\cos\delta_2 - 1) - P_2\delta_2 - \frac{1}{2}M_2 w_2^2$$

Simulation Results of the TMIB Power System

The simulation results are obtained by using MACSYMA for Lyapunov (energy) function of the TMIB power system. Fig. 2, Fig. 3, and Fig. 4 present stored energy of the TMIB power system as two-dimensional representation and three-dimensional representation.

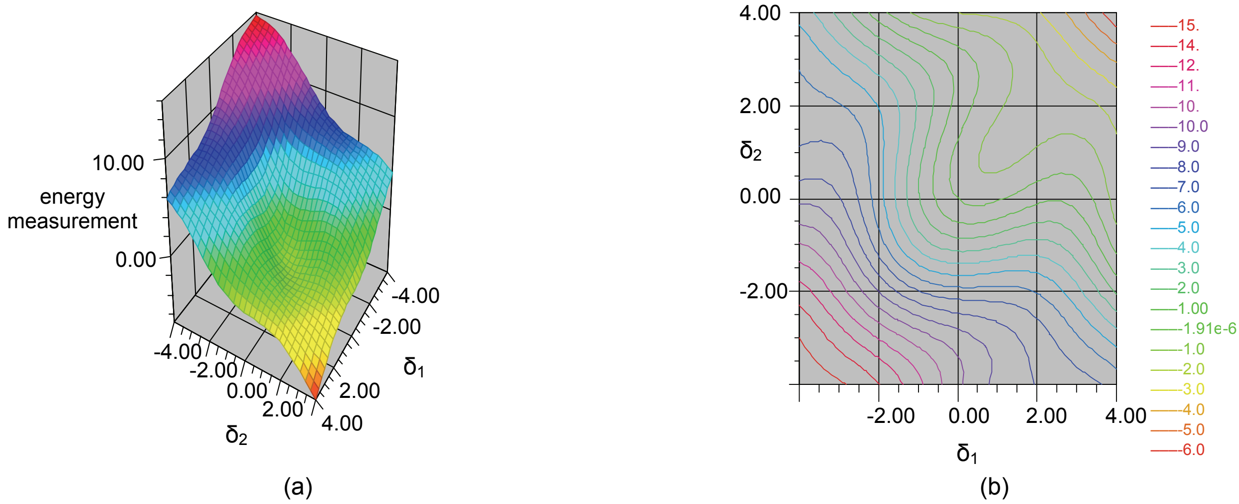

Fig.2. System's stored energy for δ_1 and δ_2 (a) Three-dimensional representation (b) Two- dimensional representation

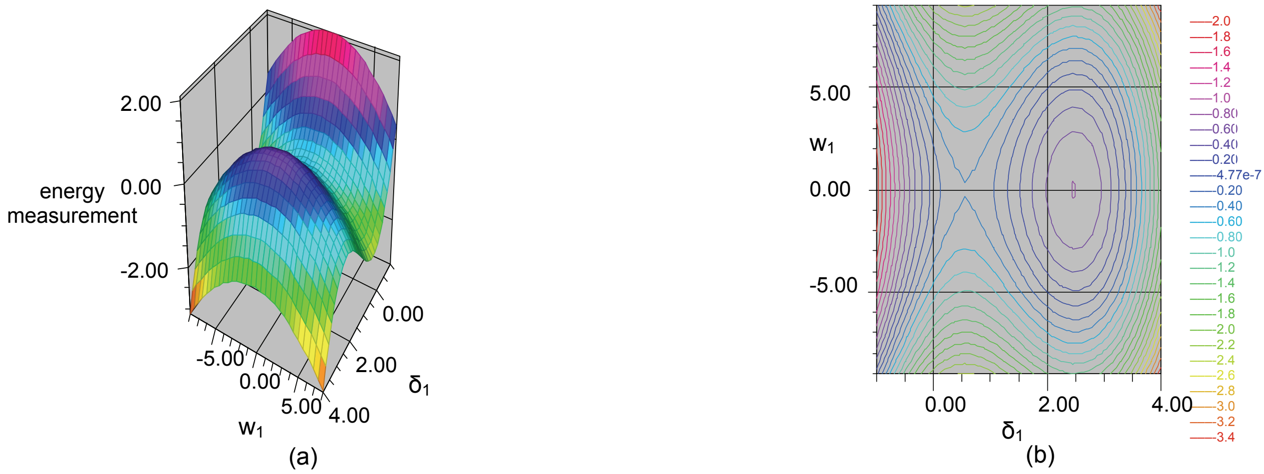

Fig.3. System's stored energy for δ_1 and w_1 (a) Three-dimensional representation (b) Two- dimensional representation

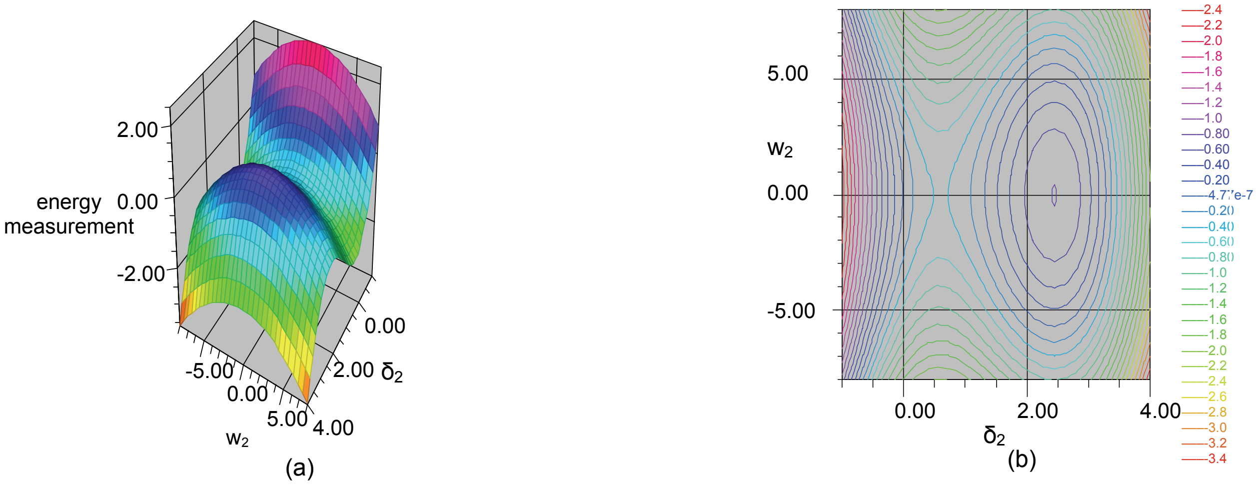

Fig.4. System's stored energy for δ_2 and w_2 (a) Three-dimensional representation (b) Two- dimensional representation

Conclusion

In this paper, the Lyapunov's second method based on the valuable gradient method is applied to the TMIB power system. The TMIB power system's energy function is constructed. This function is shown to be suitable for transient stability studies. The simulation results show the benefits of the Lyapunov's second method in the stability analysis of power systems. An important conclusion is reached that a more practical energy function can be obtained for power systems.

Appendix

The network, load and generator parameters belonging to the TMIB power system [16].

Parameter	Value	Parameter	Value
P_1	1.25	D_{12}	0.05
P_2	1.5	T_1	0.1
C_1	1.7	T_2	0.1
C_2	2.0	M_1	0.05
D_1	0.1	M_2	0.05
D_2	0.1	w_1	-0.4
C_{12}	0.5	w_2	-0.4

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