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Velocity and acceleration computations by single-dimensional Kalman filter with adaptive noise variance

***Abstract**. The velocity and acceleration estimations by first-order and second-order differentiation of the position data from optical incremental encoder for servo motors have serious errors. Single-dimensional Kalman filter (SDKF) is proposed to estimate velocity with less complexity. Based on velocity and its variation, a polynomial expression multiplied by exponential is established to compute in real time the adaptive noise variance approximately, which can be applied to the Kalman filter. Regarding SDKF-filtered velocity as reference input, phase-locked loop (PLL) structure is employed to estimate acceleration, which adopts integral operation instead of differentiation. The simulation and experimental results demonstrate that the estimated velocity and acceleration by the proposed methods have more precision and less noise than that by differentiation method.*

***Streszczenie.** Analizowano błąd określania szybkości i przyśpieszenia na podstawie danych o pozycji otrzymanych z optycznego enkodera. Zaproponowano wykorzystanie filtru Kalmana do określania prędkości. Do określania przyśpieszenia wykorzystano układ PLL. Uzyskano poprawę dokładnoćci i zmniejszenie szumów. (**Wyznaczanie szybkości i przyśpieszenia z wykorzystaniem filtru Kalman z adaptacyjną wariancją szumów**)*

Keywords: optical incremental encoder, single-dimensional Kalman filter, PLL, velocity, acceleration
Słowa kluczowe: encoder optyczny, filtr Kalmana, pomiar prędkości..

Introduction

Optical incremental encoders are widely equipped in rotary motors to obtain position signals for servo control due to their high resolution, significant precision, excellent robustness, and moderate cost, even though the research of sensorless control has attracted widespread attention [1]. Once the position signals are acquired, the velocity and acceleration can be obtained by first-order and second-order differentiation calculations [2]. Accurate velocity benefits closed-loop dynamic response and stable tracing performance, and precise acceleration plays an important role in identification of moment of inertia and load torque, which determine PI parameters and current compensation for robustness control [3].

However, the position signals appearing in pulses form from the encoders have quantization errors. Furthermore, the direct differentiation of the position signals tends to magnify the errors, particularly in low-velocity and low-acceleration regions, as will degrade the servo performance. So far, several methods have been proposed to reduce the errors and improve the accuracy of velocity and acceleration computation. Firstly, conventional approaches adopt M-method, T-method, or M/T-method[4], then utilize FIR or IIR filters to obtain velocity and acceleration. One obvious drawback is that lag effect inherently exists. In addition, the filters with constant gains can not satisfy a wide range of velocities[5]. Secondly, Lee S. H. and Song J. B. abandoned the differential method and had proposed a special structure combining single integral and double integral to estimate the velocity and acceleration under the premise of providing accurate position information [6], but it is difficult to obtain very accurate position information from the encoders, especially from low-resolution ones. Thirdly, as optimal filtering methods, Wiener filter, and Kalman filter with variable gains can improve significantly accuracy, but the former is suitable for stable signals, and in principle needs unlimited past data; Kalman filter will be time-consuming if it has multi-dimensional state variable vector including current, voltage, velocity, etc.[7].

In this paper, a new algorithm for estimating velocity and acceleration is proposed. The algorithm is based on M-method, and consists of two kernel parts: single-dimensional Kalman filter(SDKF) with adaptive noise variance for estimating velocity, and a phase-locked loop (PLL) structure for estimating acceleration. The former can reduce complexity, and the latter can avoid differentiation. Simulations and experiments will be performed to verify the algorithm.

Analysis for optical incremental encoder data

By M-method, which calculates the motor angular velocity based on the number of pulses in a constant time slice, the computing expression is written as follows:

$$n'_k = \frac{60m_k}{LT} \tag{1}$$

where: T – sampling period, m_k – the number of pulses in the sampling period k, L – the encoder resolution, referring to the generated pulses per revolution, n'_k – the motor velocity(r/min).

Taking an incremental encoder of 2500 lines for example, 10,000 pulses per revolution can be gotten from the QEP circuit in DSP , and one pulse per sampling period corresponds to 60r/min when T = 100us, here called unit velocity. If the motor is running at rated velocity n_N = 3000r/min, 50 pulses are produced in one sampling period, meaning 50 unit velocities.

A further example here affords more data about the encoder for analysis. If the motor runs at the velocity of 96r/min, the nominal number of pulses within each sample period should be 1.6. However, the practical numbers in real applications are only integers without fractional components, and they are obtained as follows：

$$m_k = \left\lfloor \sum_{i=0}^{k} V_k \right\rfloor - \sum_{i=0}^{k-1} m_i = P_k - \sum_{i=0}^{k-1} m_i = P_k - P_{k-1} \tag{2}$$

where: V_k – the motor's actual velocity in the sampling period k, $\lfloor\ \rfloor$ – rounding operation, P_k – the practical number of accumulated pulses.

V_k is described by the nominal number of pulses, e.g, V_k = 1.6 pulses represents 96r/min. The detailed data at the velocity of 96r/min are listed in Table 1, where E_k is the error expressed by the nominal number, n_k the actual velocity. Fig. 1 a) shows the encoder's output signals, which consists of A and B routes of pulse waveforms. Fig. 1 b) shows the pulse-counting process of the QEP circuit in DSP at the rising and falling edges, and the values of P_k are placed on top. Fig. 1 c) shows the velocity waveform of M-method, and Fig. 1 d) shows the acceleration waveform of differential method.

Table 1. Related data at velocity of 96r/min

Samples	n_k[r/min]	V_k	P_k	m_k	E_k	n'_k [r/min]
1	96	1.6	1	1	-0.6	60
2	96	1.6	3	2	0.4	120
3	96	1.6	4	1	-0.6	60
4	96	1.6	6	2	0.4	120
5	96	1.6	8	2	0	120
6	96	1.6	9	1	-0.6	60
7	96	1.6	11	2	0.4	120
8	96	1.6	12	1	-0.6	60
9	96	1.6	14	2	0.4	120
10	96	1.6	16	2	0	120

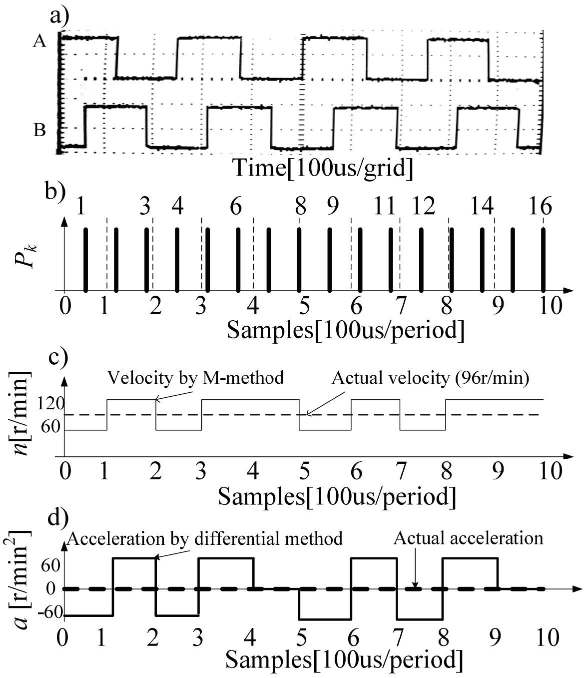

Fig. 1. Pulses, velocity by M-method and acceleration by differential method at 96r/min. a) Actual pulse waveforms of A and B routes from the encoder. b) QEP pulse numbers in DSP. c) Velocity by M-method and actual velocity. d) Acceleration by differential method and actual acceleration.

Theoretically, the waveforms of the velocity should be straight, but in the real velocity waveform of M-method, some jumps appear due to the encoder's quantization process, as shown in Fig. 1 c), so the measurement noise exists. From equation (2), it can be inferred that the noise mean will be zero exactly if the data inside the rounding part happens to be an integer, and even if not, the error will not add up to more than 1 pulse, much less than the total location trip, so its noise mean can also be approximately zero. Therefore, position error and velocity error can be regarded as Gaussian white noise. At low velocity, any increase or decrease of the pulse number will produce a mutation to the velocity, especially to the acceleration, as shown in Fig. 1 d).

The measurement noise variance can be considered as a constant as long as the encoder resolution and the sampling period remain unchanged. As for its value, no rigorous theory of the exact model exists, and here it can be expressed approximately as follows:

$$R = (1+\delta)N_{\mathrm{I}}\sum_{i=1}^{m_{\mathrm{N}}}\rho_i\frac{1}{N_i^2} \tag{3}$$

where: R – the measurement noise variance, N_{I} – unit velocity, N_i – quantized velocity based on the encoder pulses, ρ_i – the probability for each quantized velocity, m_{N} – the nominal number at rated velocity, δ– the correction factor, usually between ± 0.2.

Generally, i =1,2, ..., m_{N}, and ρ_i = 1/m_{N} when the probability is generally equal. They can be described as follows:

$$m_{\mathrm{N}} = \frac{n_{\mathrm{N}}}{60}LT, N_{\mathrm{I}} = \frac{n_{\mathrm{N}}}{m_{\mathrm{N}}} = \frac{60}{LT}, N_i = i \times N_{\mathrm{I}} \tag{4}$$

Velocity estimation by SDKF

Kalman filter, as an optimal filter with variable gain, has excellent filtering effect in dealing with the signals that contains Gaussian noises. Based on covariance matrixes, which are determined by systematic noise and measurement noise, Kalman filter can adjust its gains according to innovation information[19]. However, it has in general large computation complexity due to its multiple state variables and has difficulty in calculating the covariance matrixes. In this paper, SDKF is proposed to estimate velocity on the aforementioned condition that the position and velocity signals from the decoder have Gaussian noises.

The mechanical motion model for the servo motor can be expressed as follows [8]:

$$\begin{bmatrix}\theta_{k+1}\\ \omega_{k+1}\\ \alpha_{k+1}\end{bmatrix} = \begin{bmatrix}1 & T & T^2/2\\ 0 & 1 & T\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta_k\\ \omega_k\\ \alpha_k\end{bmatrix} + \begin{bmatrix}T^3/6\\ T^2/2\\ T\end{bmatrix}w_k \tag{5}$$

$$y_{k+1} = \theta_{k+1} + v_k$$

where: θ, ω, α – position, velocity, acceleration, respectively, y – the exact position signal, w, v – systematic noise and measurement noise, respectively.

The noises w and v, as Gaussian white noises with zero mean, can be characterized by their noise variances, denoted by Q and R, respectively. Equation (5) illustrates that this model is only related to the encoder signal, not related to current, voltage, etc., thus fewer noise sources exist. Unfortunately, this motion model is a third-order model, and the use of five formulas for Kalman filter [7] may cause more complex calculations, not suitable for digital application, so it is necessary to reduce its order. The position has high accuracy since the commonly used 13-bit encoder can offers enough high resolution that can satisfy general requirements, and the acceleration can be estimated from the estimated velocity, so the position and acceleration in this model need not consideration, and the key problem is to estimate the velocity, that is to say, single variable is needed to be estimated. Therefore, SDKF is available to deal with the velocity data as a SISO system, since the velocity obtained from the M-method has Gaussian white noise.

In servo system, if the velocity changes little or the sampling period is short, the velocity in the next sampling period can be predicted to be the same as that in the current period. Of course, it is impossible in the actual situation, but the small prediction error can be regarded as systematic noise. By Kalman filtering theory, the velocity signal is filtered as follows:

(1) Predict the velocity in this current period to be the same as that in its previous one. This prediction is reasonable because little velocity change occurs in the two adjacent periods if the interval is short. The unconformity, if exists, can contribute to the systematic noise.

(2) Calculate the variance corresponding to the predicted velocity. The variance is composed of two parts:

one is the variance of the previous velocity from SDKF, the other is the systematic noise variance Q caused by the prediction.

(3) Calculate the adaptive gain G_k according to the variance of the predicted velocity and the measurement noise variance R.

(4) Calculate the optimal filtered velocity in term of the adaptive gain, the predicted velocity, and the M-method velocity.

(5) Calculate the variance corresponding to the above filtered velocity, preparing for the next filtering process.

By digital implementation, five formulas are utilized to describe the above process:

$$(6)\qquad \begin{cases} \omega(k|k-1) = \omega(k-1|k-1) \\ P(k|k-1) = P(k-1|k-1) + Q \\ G(k) = P(k|k-1) \;/\; \left(P(k|k-1) + R\right) \\ \omega(k|k) = \omega(k|k-1) + G(k)\left(\omega(k) - \omega(k|k-1)\right) \\ P(k|k) = (1-G(k))\, P(k|k-1) \end{cases}$$

where: $\omega(k\text{-}1|k\text{-}1)$, $\omega(k|k)$ – the filtered velocity from the Kalman filter in the period k-1, k, respectively, $\omega(k|k\text{-}1)$ – the predicted velocity in the period k, $\omega(k)$ – the M-method velocity in the period k, $P(k\text{-}1|k\text{-}1)$, $P(k|k\text{-}1)$, $P(k|k)$ – the variances corresponding to $\omega(k\text{-}1|k\text{-}1)$, $\omega(k|k\text{-}1)$, $\omega(k|k)$, respectively, $G(k)$ – the adaptive gain in the period k, Q – the systematic noise variance, R – the measurement noise variance.

Q and R can be regarded as constant greater than zero, when the sampling period and the encoder resolution maintain unchanged.

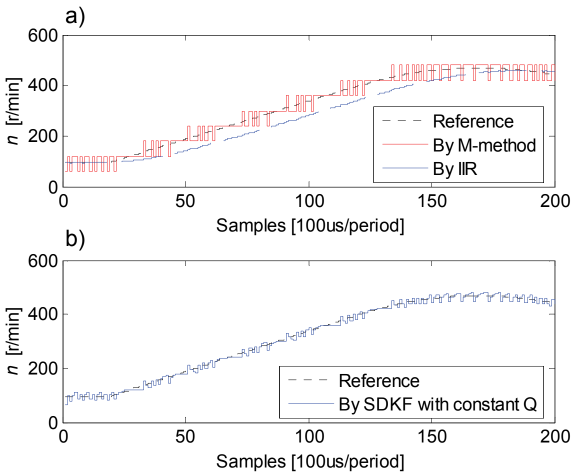

Fig. 2. Velocity estimations by conventional methods. a) By M-method and IIR filter. b) By SDKF with constant Q

In Fig. 2, the estimated velocities by various methods are covered by simulation in the situation that the motor runs from the velocity of 96r/min to that of 420r/min. Fig. 2 a) shows that the M-method velocity has clear jumps and that although the IIR-filtered velocity has not jumps, its delay is significant. Fig. 2 b) shows that the SDKF-filtered velocity well conforms to the reference when R = 5, Q = 10, but there are still some obvious ripples.

SDKF with adaptive noise variance

Usually, the noise variance Q in Kalman filter model is set as constant for reducing complexity, but it will result in error accumulation in filtering process, even lead to filtering divergence. Indeed, Q is not a constant since the premise that the velocity maintains unchanged in two adjacent periods doesn't exist actually. The ripples in Fig.2 b) verify this fact. Therefore, Q should be adaptive based on the velocity innovation information to abolish the systematic error because of the premise, for robustness.

Jazwinski and other scholars[9] adopted Kalman filter for the research of orbit determination, and they proposed a virtual error model to calculate the variance matrix in order to relieve the negative effect. In fact, it was to produce a variable coefficient matrix. This idea may inspire us to determine Q dynamically here. In order to obtain the dynamic value of Q, two impacts of M-method on Q are considered:

(1) When the velocity is relatively low, even if the absolute velocity fluctuation between two adjacent periods is very small, the systematic error caused by the premise of constant velocity prediction is large; when the velocity is relatively high, even though the absolute velocity fluctuation between two adjacent periods is very obvious, the systematic error is small. Therefore, Q is monotonically decreasing in relationship with the velocity.

(2) On the other hand, when the velocity changes much in two adjacent periods, the systematic noise caused by the premise is relatively large, or small in adverse, so Q is monotonically increasing in relationship with the velocity variation. Hence, in accordance with the absolute velocity and the velocity variation, Q can be adaptive through the following virtual model:

$$(7)\qquad Q_k = \lambda^2 T^2 \Gamma_k^{\,2} \mathrm{e}^{-\gamma\omega^2(k)}$$

where: $\Gamma_k = \omega(k) - \omega(k\text{-}1|k\text{-}1)$, λ, γ – the constant coefficients that can be determined by experience or experiment.

Equation (7) complies with the above analysis. In practice, the exponential computation is time-consuming in DSP programming. Replacing the exponent term with $\dfrac{1}{1+\gamma\omega^2(k)}$, the systematic noise variance Q can be approximately calculated as follows:

$$(8)\qquad Q_k = \frac{\lambda^2 T^2 \Gamma_k^{\,2}}{1+\gamma\omega^2(k)}$$

Fig. 3 shows the filtered velocity by SDKF with adaptive Q in simulation, where λ=10, γ=1.

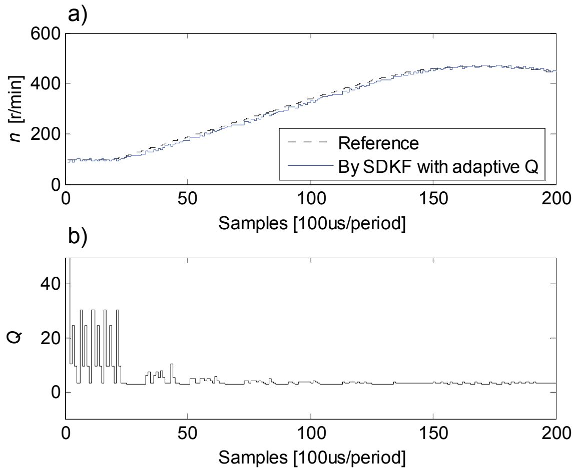

Fig. 3. Velocity by SDKF with adaptive Q. a) Velocity waveform. b) Adaptive Q

Fig. 3 indicates that Q can be adaptive, and the filtered velocity is smoother than that from SDKF with constant Q, and almost no delay exists. Moreover, the jumping phenomenon in velocity brought up by M-method is eliminated, and the ripple phenomenon vanishes clearly.

Acceleration estimation based on PLL structure

In servo system, the angular acceleration can be calculated by differentiation of the SDKF-filtered velocity. Unfortunately, the acceleration error is very large when the motor runs in low-velocity regions or low-acceleration regions. Thus it is better if acceleration estimation can avoid differentiation operation. To obtain more precise acceleration, an algorithm based on phase-locked loop (PLL) structure[10] is proposed to calculate acceleration by integral calculation, instead of differentiation. Fig. 4 synthetically depicts the velocity and acceleration estimating process by SDKF and PLL structure.

For contributing to understanding, Fig. 4 describes the SDKF filtering process in discrete-domain in the left side, and describes the acceleration estimation by PLL structure in continuous-domain denoted in the dashed box, where the reference input ω is the SDKF-filtered velocity, ω' the velocity estimated through integral computation of the acceleration through the PLL method. According to the error between ω and ω', the acceleration by the PI regulator in the PLL structure is computed as follows:

(9) $$\alpha' = k_{\mathrm{p}}\Delta\omega + k_{\mathrm{i}}\int \Delta\omega \mathrm{d}t$$

where: k_{p} – proportional gain, k_{i} – integral gain.

The open-loop transfer function from ω to ω' is a Ⅱ-type model:

(10) $$G(s) = \frac{\omega}{\omega'} = \frac{k_{\mathrm{p}}s + k_{\mathrm{i}}}{s^2(1+\tau s)}$$

where: τ – the time constant of first-order low-pass filter.

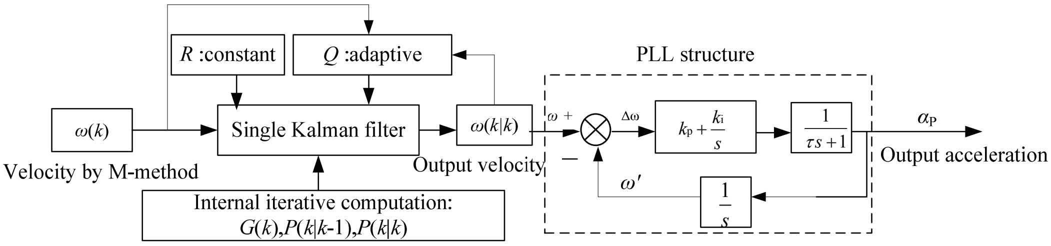

Fig. 4. Velocity estimation by SDKF with adaptive *Q* and acceleration estimation by PLL structure

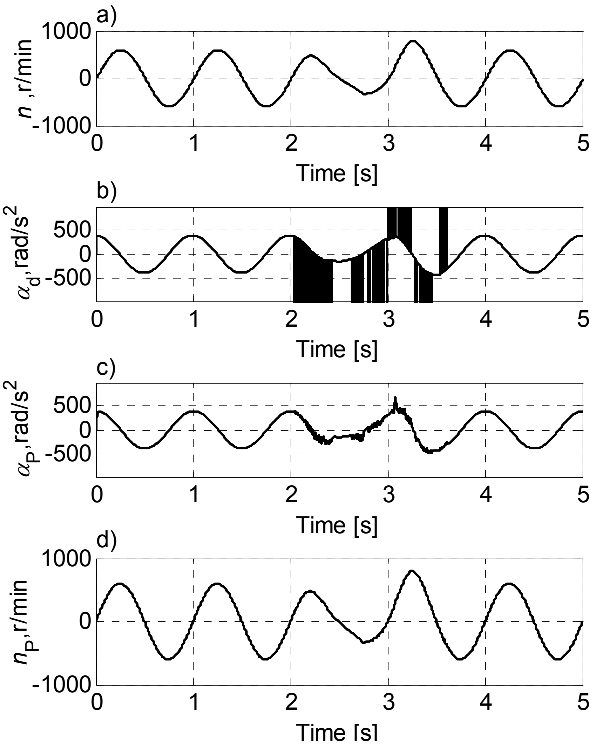

Fig. 5. Acceleration estimation by PLL method. a) Reference velocity. b) Acceleration by differential method. c) Acceleration by PLL. (d) Velocity by integral of the acceleration in c)

On the basis of the principles for designing Ⅱ-type system, with regard to the bandwidth of velocity loop and the sampling period, τ, k_{p}, k_{i} can be chosen to satisfy the tracking performance of the velocity loop[15]. At this moment, the acceleration obtained by PLL method becomes more accurate. It is implemented in incremental digital form as follows:

(11) $$\alpha_k = \alpha_{k-1} + (k_{\mathrm{p}} + k_{\mathrm{i}}T)\Delta\omega_k - k_{\mathrm{p}}\Delta\omega_{k-1}$$

Fig. 5 shows its simulation results where k_{p} = 0.9, k_{i} = 0.5, τ= 100us. At first, a sinusoidal velocity profile of $600\sin(2\pi t)$ is given. The distinction between the estimated acceleration by differential method, as shown in b), and that by PLL method, as shown in c), is almost negligible, and the two estimated accelerations are consistent with the theoretical ones. However, when the velocity profile happens to be irregularly sinusoidal, the acceleration computed by differential method has obvious noise and glitches, while the estimated acceleration by PLL method has less noise.

Experiments

The experimental setup is composed of a surface-mount servo PMSM and an optical incremental encoder of 2500 lines. Related parameters: rated power is 1.8 kW, rated velocity 3000 r/min, the number of pole pairs 4, moment of inertia 0.001 kg·m^2. The drive circuit adopts a three-phase voltage source inverter, employing IPM module to drive motor. The control circuit mainly consists of TMS320F2810DSP chip and EPM3128CPLD chip. The above components constitute all-digital control system including the current, velocity, and position loops. The motor runs without load under the vector-control scheme of i_{d}=0, and the velocity and acceleration signals are outputted by a simple analog D/A circuit including a low-pass second-order Butterworth filter connected to the DSP's T1PWM pin.

Fig. 6 shows the estimated velocity and acceleration responses by different methods when the motor runs with a step velocity profile from 0 to 500r/min. The comparison between Fig. 6 a) and Fig. 6 b) shows that the filtered velocity by SDKF with adaptive noise variance is smoother than that from SDKF with constant noise variance, and the glitches in the waveform drop a lot, as implies that fewer jumps and ripples appear by using SDKF with adaptive noise variance. In the experiment, R=5, Q is in ranges of [1,20]. Fig. 6 c) shows that the acceleration computed by differential method is prone to have mutations, and its error is relatively large, while Fig. 6 d) shows that the acceleration waveform by PLL method is very smooth. In the PLL structure, k_p = 90, and the actual coefficient is 0.9 after divided by the calibration 100; k_i = 50, and the actual is 0.5 after divided by the calibration 100. In the PI-controller for velocity, the proportional gain is 10, and that is 0.01 after divided by the calibration 1000, and the integral time constant is 10ms.

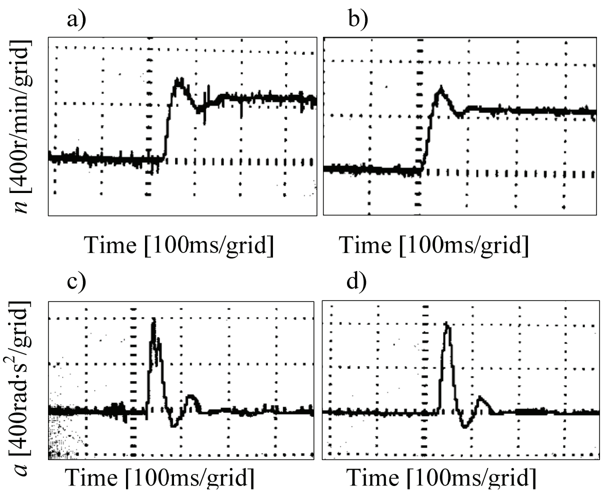

Fig.6. Velocity and acceleration responses of 500r/min step profile. a) Velocity by SDKF with constant Q. b) Velocity by SDKF with adaptive Q. c) Acceleration by differential method. d) Acceleration by PLL method.

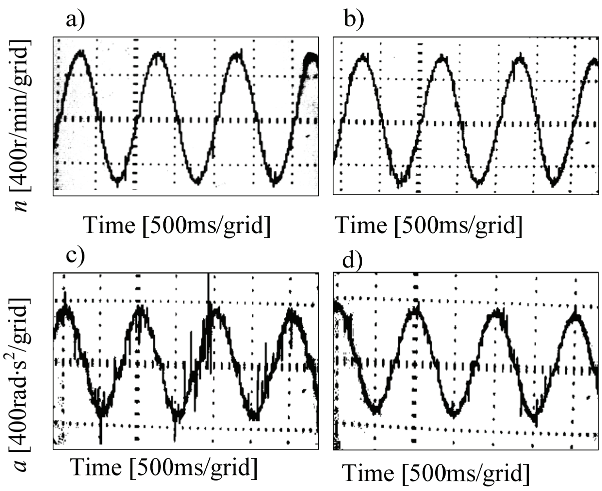

Fig. 7. Velocity and acceleration responses of 600$\sin(2\pi t)$r/min profile. a) Velocity by SDKF with constant Q. b) Velocity by SDKF with adaptive Q. c) Acceleration by differential method. d) Acceleration by PLL method..

Fig. 7 employs 600$\sin(2\pi t)$ r/min velocity profile to verify the proposed methods. Also, it shows the advantages of SDKF with adaptive noise variance for computing velocity and the PLL structure for estimating acceleration. Since the analog output of the controller is by Butterworth second-order low-pass filter, the difference between Fig. 7 a) and Fig. 7 b) is not obvious, but Fig. 7 c) shows clear ripples in the acceleration waveform by differential method , while Fig. 7 d) shows a smooth acceleration waveform by PLL method.

Conclusions

According to the data from optical incremental encoder, a simplified SDKF with adaptive noise variance is proposed to calculate the angular velocity for motor. When the systematic noise variance is adjusted with regard to the velocity and its variation in real time, the velocity estimated by the filter is very close to the actual one with fewer jumps and ripples. The higher the encoder's resolution is, the more excellent this method is. Using PLL structure, which contains PI algorithm, the estimated acceleration by integral operation waveform is more smooth, having fewer mutations and glitches than that by differentiation. Of course, appropriate compensation is needed for pursuing more accurate acceleration since the PLL design refers to the bandwidth of the velocity loop. The algorithm is simple and rapid. The obtained velocity and acceleration can contribute to the feedback control of velocity, the inertia identification, and the compensation for disturbance, which can benefit the high performance of the servo system.

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