Frequency symbolic models of linear parametric circuits

Abstract. The methods of constructing symbolic models of linear parametric circuits in frequency domain are considered. The models introduce the systems of linear algebraic equations. The circuit is considered to contain only one parametric element and in the model it is replaced with additional source of signal or frequency model of parametric element or controlled source. Frequency symbolic models of single-circuit parametric amplifier are formed according to the given methods.

Streszczenie. Podano metodę tworzenia symbolicznego opisu obwodu liniowego, zmiennego w czasie. Rozpatrano obwód zawierający jeden kondensator zmienny w czasie. Kondensator jest zastępowany przez dodatkowe źródło stawane zależne od czasu i częstotliwości. Korzystając z opracowanej metody pokazano przykład modelowania zawierającego kondensator zmienny w czasie. (Symboliczne modele częstotliwościowe obwodów zmiennych w czasie)

Keywords: Linear parametric circuits, symbolic analysis, frequency symbolic models.

Introduction
Frequency symbolic method of analysis of linear parametric circuits with periodically variable parameters which allows for a known input signal $X(s)$ using the parametric transfer function $W(s,t)$ to determine the output signal $Y(s,t)$ in frequency domain is considered in [1]:

$$Y(s,t)= W(s,t) \cdot X(s),$$

where $s,t$ - complex variable and time variable, respectively. The approximation $\hat{W}(s,t)$ of parametric transfer function $W(s,t)$ is determined in the form of truncated Fourier series which contains $k$ harmonic components.

In this paper on the basis of frequency symbolic method the frequency symbolic models of parametric circuit are constructed and they represent the system of linear algebraic equations (SLAE) and they allow to conduct the further circuit analysis only in frequency domain. Three methods of construction of such models are considered in the paper.

Method 1. The method of additional independent source.

According to expression (1) we determine the current that goes through parametric element of the circuit $I_p(s,t)$:

$$I_p(s,t) = \Lambda(s,t) \cdot I(s),$$

where $I(s)$ - input current, $\Lambda(s,t)$ - parametric transfer function of signal from input source of current to the current of parametric element. Then it can be seen from the theorem of substitution [2] that voltages and currents in parametric circuit will not change if its branch with parametric element is replaced with branch with source of current $I_p(s,t)$ that is determined by expression (2). After such replacement given parametric circuit becomes a circuit with constant parameters and two sources of current – input source $I(s)$ and source of current $I_p(s,t)$ respectively. Such treatment of parametric circuit gives the possibility to construct its frequency model in the form of SLAE constructed according to the rules of nodal solution in the same way as for the circuit with constant parameters:

$$Y(s) \cdot U(s,t) = I(s,t),$$

where conductivity matrix $Y(s)$ contains the parameters of elements of given parametric circuit except for parametric element; the vector of sources of current $I(s,t)$ has non-zero values only in the elements that correspond to nodes of connecting sources $I(s)$ and $I_p(s,t)$; $U(s,t)$ - vector of unknown nodal voltage; $s$ - complex variable of input signal and parametric transfer function from vector $I(s,t)$; $s_i$ - complex variable which proves that signal $I_p(s,t)$ contains harmonic components with frequencies $(\omega \pm k\Omega)$ and $i=0,1,2,...,k$, that is why according to the principle of superposition the SLAE (3) should be done $(2k+1)$ times, substituting different values of complex frequency $j(\omega \pm k\Omega)$ every time; and the results should be added. But frequency symbolic method is symbolic and it is enough to solve SLAE (3) one time with symbolic frequency $s_i$, according to some rules in the given solution the symbol $s_i$ should be replaced with symbols of complex frequencies $s_{-1},...,s_{i-1},s_{i+1},...,s_{k}$ and nodal voltages should be formed like the sums of nodal voltages determined for every harmonic of the signals. So, according to model (3) in complex expressions of circuit functions (input resistance of the circuit, transfer of current from input to output) and nodal voltages there are two symbol complex variables $s$ and $s_i$ but only during the last section of calculations before determining time dependences in expressions for nodal voltages, $s_i$ will be replaced with symbolic complex variables from series $s_{-1},...,s_{i-1},s_{i+1},...,s_{k}$.

As model (3) of parametric circuit is algebraic, it can be analysed with the help of programs of analysis of linear circuits with constant parameters. It is also clear that the change of parameters of input signal $I(s)$ must be accounted for by corresponding change of current $I_p(s,t)$.

Example 1. According to the method of additional independent source it is necessary to construct frequency symbolic model of parametric amplifier (Fig. 1.a) with additional source of current $I_p(s,t)$, and to determine instantaneous value of voltage $U_{i1}$ in separate points of time and compare them with instantaneous values of voltages determined with the help of MicroCap 7 program in the same points of time.

Equation that connects input current and current of parametric capacity in time domain is the following:
Despite having the parameter of independent source \( I_c(s,t) \) that changes with time model (8) describes parametric amplifier (Fig. 1.a) in frequency domain and can be the basis for further analysis, statistical investigation and optimization of given circuit only in this frequency domain without the help of differential equation. So, for example, symbolic nodal voltage \( U_1 \) is determined from model (8) in the following form:

\[
(9) \quad U_1 = \left[ \left( Y_1 + Y + Y_1 L_s \right) - Y_1 \Lambda(s,t) \right]/\left( Y_1 + Y_1 L_s \right) \cdot I(s). 
\]

Substituting (9) by expression for \( \Lambda(s,t) \) from (6) we receive complex voltage \( U_1 \) of amplifier during approximation of function \( \Lambda(s,t) \) by one harmonic:

\[
(10) \quad U_1 = \left[ \left( Y_1 + Y + Y_1 L_s \right) - Y_1 \Lambda(s,t) + \Lambda(s) + \Lambda(s,s) \right]/\left( Y_1 + Y_1 L_s \right) \cdot I(s). 
\]

Further calculation of expression (10) should be done considering some peculiarities that result from principle of superposition of signals and concern algebraic operations of multiplication and summation of expressions in one of which there is variable \( s \) and exponential multiplier that is determined by inferior index and in the second – variable \( s \).

The main rules of such operations are the following.

The rule of multiplication:

\[
A_1(s,t) \cdot B(s,t) = A_1(s,t) \cdot B(s,t), \quad r = -k, -k-1, ..., -1, 0, 1, ..., (k-1), k, s_1 = s. 
\]

Rule of summation:

\[
A_2(s,t) + A_2(s,t) + A_2(s,t) + A_2(s,t) + ... + A_2(s,t) = A_2(s,t) + A_2(s,t) + A_2(s,t) + ... + A_2(s,t). 
\]

So, considering (11)-(12), we receive the following in (10):

\[
U_1 = \left[ \left( Y_1 + Y + Y_1 L_s \right) - Y_1 \Lambda(s,t) + \Lambda(s) + \Lambda(s,s) \right]/\left( Y_1 + Y_1 L_s \right) \cdot I(s). 
\]

The results of calculation of expression (13) and similar to it but with more harmonic components \( k \) in parametric function \( \Lambda(s,t) \) for different meanings of time \( t \) are presented in Table 1.

**Method 2. The method of frequency symbolic model of parametric element.**

We determine the current that goes through parametric element and voltage on it according to expression (1). The ratio of these values

\[
I(s,t) = \Lambda(s,t) \cdot I(s), \quad U_1(s,t) = \Lambda(s,t) \cdot Z(s,t), \quad S(s,t) = S(s, t) 
\]

due to circuit linearity does not depend on its voltages and currents and that is why it determines some parameter (conductivity) \( S(s,t) \) that will be called frequency symbolic model in particular conductivity of parametric capacity.
Example 2. Using expression (14) we construct the frequency model of amplifier:

\[ Y(s, s) \cdot U(s, t) = I(s) \]  

where variables \( s, s \) have the same content as for model (3).

\[ \begin{align*}
Y_1 & \quad I(s) \\
U_1 & \quad U_1 \\
Y_2 & \quad L_2 \\
U_2 & \quad I(s) \\
S(s, t) & \quad = \text{controlled source}
\end{align*} \]

(a)

\[ \begin{align*}
U_1 & \quad I(s) \\
Y_1 & \quad Y_1 \\
Y_2 & \quad L_2 \\
I(s) & \quad = S(s, t)U_{Y_1}
\end{align*} \]

(b)

Fig.2. Equivalent circuit of parametric amplifier from Fig.1.a made according to the method of frequency symbolic model of parametric element – (a); controlled source – (b).

Example 3. Using expression (17) as \( S_i(s, t) = I_p(s, t)/U_i(s, t) = \Lambda(s, t)/Z_i(s, t) \)

\[ \begin{pmatrix}
Y_i \\
-Y_i + Y_i + S_i(s, t) \\
-Y_i + S_i(s, t) \\
Y_i + Y_i + Y_i + U_i - S_i(s, t)
\end{pmatrix}
\begin{pmatrix}
U_i \\
U_i \\
U_i \\
U_i - U_i
\end{pmatrix}
= \begin{pmatrix}
I(s) \\
0
\end{pmatrix} \]

Obtained frequency symbolic model (18), as frequency symbolic model (16), contains all the information about the parametric amplifier in frequency domain.

Conclusions

1. The analyzed frequency models fully describe parametric circuit in frequency domain and as shown by experiments are adequate.
2. The accuracy of results received by frequency symbolic models of the circuit increases when the number of harmonic components in approximations of parametric transfer functions increases.
3. Frequency models of parametric elements are determined by the circuit in which these parameters are present. It is most probably that frequency models of parametric elements do not exist beyond the circuit.
4. The parameters of elements that are going to be changed in the process of circuit investigation must be left in symbol both in frequency model and in parametric transfer functions that are present in these frequency models. In this case the necessary change of such parameters is quite quick because of substitution of new values in frequency model of the circuit and parametric functions without any additional calculations.
5. The developed frequency models of the circuits contain two complex variables as they describe the circuit on a frequency of input signal and frequencies that appear in the circuit of harmonic components of signals.
6. The developed frequency models allow to carry out the analysis, statistical investigation, optimization and designing of parametric circuits in frequency domain by analogy with circuits with constant parameters.
7. The developed frequency models are the systems of linear algebraic equations. That is why during their analysis the software for symbolic analysis of linear circuits with constant parameters can be widely used. We consider this fact to be the most practical result of the given paper.

REFERENCES


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