Outage probability of the SSC/SC combiner at two time instants in the presence of lognormal fading

Abstract. The outage probability of the Switch and Stay Combining/Selection Combining (SSC/SC) combiner output signal at two time instants, in the presence of log-normal fading, is determined in this paper. The probability density function (PDF) of the combiner output signal is derived. Then, the outage probability is numerically calculated using this PDF. The results are shown graphically to compare performances of the SSC/SC combiner with regard to classical SSC and SC combiners at one time instant.

Streszczenie. W artykule analizuje się prawdopodobieństwo przerw połączenia SSC/SC w obecności zaników o rozkładzie logarytmiczno-normalnym. Prawdopodobieństwo to jest liczone na podstawie krzywej gęstości prawdopodobieństwa PDF. Porównano połączenia SSC/SC z klasycznymi SSC i SC. (Prawdopodobieństwo przerw połączenia SSC/SC w obecności zaników o rozkładzie logarytmiczno-normalnym)

Keywords: lognormal fading, SSC Combining, SC combining, two time Instants, outage probability

Słowa kluczowe: zaniki połączenia, prawdopodobieństwo

Introduction

In wireless communications, a variation of an instantaneous value of the received signal, i.e. fading of signal envelope is very common effect, due to the multipath propagation. Fading is one of the main causes of performance degradation of the receiver. Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading. Particular diversity methods and combining techniques are presented in [1].

Since the selection combining (SC) and switch and stay combining (SSC) do not require signal cophasing and fading envelope estimation, they are often implemented in practice. The SC is combining technique where the strongest signal is chosen among L branches of diversity system [1]. In the case of dual branch SSC, the first branch stay selected as long as its instantaneous signal-to-noise ratio (SNR) is greater than predetermined switching threshold, even if the instantaneous SNR in the second branch maybe has a larger value at that time [1, 2].

The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two [3-5]. In the paper [5] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC).

The probability density function (PDF) of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh, Nakagami-m, Weibull and lognormal fading are determined in [7-10], respectively. The probability density function and the outage probability of the SSC/SC combiner output signal at two time instants in the presence of Rayleigh fading is determined in [11].

In this paper the probability density function and the outage probability of the SSC/SC combiner output signal at two time instants in the presence of lognormal fading will be determined.

The remainder of this paper is organized as follows. The next section presents the system model and determines the probability density function, the joint PDF and the outage probability for the SSC/SC combiner output signal at two time instants. Sections III presents the numerical results obtained for performances introduced in section II. Finally, the main results of the paper are given as conclusions.

System model

The model of the SSC/SC combiner with two inputs at two time instants, considering in this paper, is shown in Figure 1. The SSC combiner input signals are \(r_{1i}\) and \(r_{2i}\) at first time moment and they are \(r_{1i}\) and \(r_{2i}\) at second time moment. The output signals from SSC part are \(r_{1}\) and \(r_{2}\). The indexes for the input signals are: first index is the ordinal branch number and the other signs time instant observed. For the output signals, the index represents the time instant observed. So found SSC combiner output signals \(r_{1}\) and \(r_{2}\) become the inputs in SC combiner. The overall output signal from the entire system is \(r\).

![Fig.1. Model of the SSC/SC combiner with two inputs at two time instants](image)

The probability densities of the combiner input signals, \(r_{1}\) and \(r_{2}\), in the presence of lognormal fading, are [1]:

1. \(p_{n}(r_{1i}) = \frac{1}{\sqrt{2\pi \sigma_{1i}^{2}}} e^{-\frac{(\ln r_{1i} - \mu_{i})^{2}}{2\sigma_{1i}^{2}}}, \quad r_{1i} \geq 0\)

2. \(p_{n}(r_{2i}) = \frac{1}{\sqrt{2\pi \sigma_{2i}^{2}}} e^{-\frac{(\ln r_{2i} - \mu_{i})^{2}}{2\sigma_{2i}^{2}}}, \quad r_{2i} \geq 0\)

where \(i=1,2\), \(\mu_{i}\) is mean value and \(\sigma_{i}\) is standard deviation of log-normal fading.

The probability of the event that combiner first examines the signal at the first input is \(P_{1}\), and for the second input is \(P_{2}\). The first case is: \(r_{1} < r_{2} < r_{T}\). In this case all signals at the input are below \(r_{T}\), i.e.: \(r_{11} < r_{T}, r_{12} < r_{T}, r_{21} < r_{T}, r_{22} < r_{T}\) and \(r_{22} < r_{T}\). Let the SSC combiner first examines the signal \(r_{11}\). Because \(r_{11} < r_{T}\), it follows that \(r_{1} = r_{11}\), and since \(r_{22} < r_{T}\) it is \(r_{2} = r_{22}\). The probability of this event is \(P_{1}\). When SSC combiner first examines the signal \(r_{21}\), then \(r_{1} = r_{21}\) because \(r_{21} < r_{T}\). Since \(r_{21} < r_{T}\), then it is \(r_{2} = r_{21}\). The probability of this event is \(P_{2}\). After the previous, the joint probability density of the SSC combiner output signals at two time instants, \(r_{1}\) and \(r_{2}\), is [10]:

\(P_{r}(r_{1}, r_{2}) = P_{r_{1}}(r_{1}) \cdot P_{r_{2}}(r_{2})\)
For \( r_1 < r_T \) and \( r_2 < r_T \)

\[
p_{r_1,r_2}(r_1, r_2) = P_1 \int_{0}^{r_1} \int_{0}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_2 \int_{0}^{r_1} \int_{0}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2)
\]

(3)

For \( r_1 < r_T \) and \( r_2 \geq r_T \)

\[
p_{r_1,r_2}(r_1, r_2) = P_1 \int_{0}^{r_1} \int_{r_T}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_1 \int_{0}^{r_1} \int_{0}^{r_T} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_2 \int_{0}^{r_1} \int_{0}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2)
\]

(4)

For \( r_1 \geq r_T \) and \( r_2 < r_T \)

\[
p_{r_1,r_2}(r_1, r_2) = P_1 \int_{r_T}^{r_1} \int_{0}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_1 \int_{r_T}^{r_1} \int_{r_T}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_2 \int_{r_T}^{r_1} \int_{0}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2)
\]

(5)

For \( r_1 \geq r_T \) and \( r_2 \geq r_T \)

\[
p_{r_1,r_2}(r_1, r_2) = P_1 \int_{r_T}^{r_1} \int_{r_T}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_1 \int_{r_T}^{r_1} \int_{r_T}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2) + \\
+ P_2 \int_{r_T}^{r_1} \int_{r_T}^{r_2} d\gamma_1 d\gamma_2 \rho_{\gamma_1\gamma_2}(\gamma_1, \gamma_2, r_1, r_2)
\]

(6)

The probabilities \( P_1 \) and \( P_2 \) are [10]:

\[
P_i = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\ln r_i - \mu_i}{\sigma_i \sqrt{2}} \right) \right] + \text{erf} \left( \frac{\ln r_i - \mu_i}{\sigma_i \sqrt{2}} \right)
\]

(7)

where \( i = 1, 2; \ i \neq j \).

The joint probability density function of correlated signals \( r_1 \) and \( r_2 \) with lognormal distribution and the same \( \sigma \) is [12]:

\[
p_{r_1,r_2}(r_1, r_2) = \frac{1}{\sqrt{2\pi} \sigma r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2 \sigma_1^2}} \cdot e^{\frac{1}{\sqrt{2\pi} \sigma_1 \sqrt{1 - \rho^2} r_2^2} \cdot \left[ \ln r_2 - \left( \rho \cdot \frac{\sigma_2}{\sigma_1} \cdot (\ln r_1 - \mu_1) \right) \right]^2}
\]

(8)

where \( \rho \) is correlation coefficient.

The PDF of \( r \) at the output of SC combiner can be expressed in terms of joint PDF of \( r_1 \) and \( r_2 \) as [1]:

\[
p_r(r) = \int_{0}^{r} p_{r_1,r_2}(r, r') dr' + \int_{0}^{r} p_{r_1,r_2}(r, r') dr
\]

(9)

If it is assumed that channels are identical, the distribution parameters for the both branches are the same.

Replacing (3-6) in (9), and after some manipulations we obtain PDF of \( r \) as:

For \( r < r_T \) \( (r_1 < r_T, r_2 < r_T) \)

\[
p_r(r) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\ln r - \mu + \rho (\ln r - \mu)}{\sigma \sqrt{1 - \rho^2} \sqrt{2}} \right) \right] + \text{erf} \left( \frac{\ln r - \mu + \rho (\ln r - \mu)}{\sigma \sqrt{1 - \rho^2} \sqrt{2}} \right)
\]

(10)

where \( \text{erf} \) is error function defined by [13]

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt
\]

(11)

For \( r_1 \geq r_T \) \( (r_1 > r_T, r_2 < r_T) \)
\( p_{c} (r) = \int_{0}^{r} p_{c1, c2} (r, r_2) dr_2 + \int_{r}^{\infty} p_{c1, c2} (r_1, r) dr_1 \)  
(12)  

\[ p_{c} (r) = \frac{1}{2\sqrt{2\pi} \sigma^2} e^{-\frac{(ln \mu - \sigma^2)^2}{2\sigma^2}} \left( \frac{1}{2} + \text{erf} \left( \frac{ln r - \mu}{\sqrt{2} \sigma^2} \right) \right), \]  
(13)  

For \( r_1 \geq r_2 \) \((r_1 < r_2, r_2 \geq r_1)\)  
\[ p_{c} (r_1) = \int_{0}^{r_1} p_{c1, c2} (r_2) dr_2 + \int_{r_1}^{\infty} p_{c1, c2} (r_1, r) dr_1 \]  
(14)  

\[ p_{c} (r_2) = \frac{1}{2\sqrt{2\pi} \sigma^2} e^{-\frac{(ln \mu - \sigma^2)^2}{2\sigma^2}} \left( \frac{1}{2} + \text{erf} \left( \frac{ln r_1 - \mu}{\sqrt{2} \sigma^2} \right) \right), \]  
(15)  

For \( r_1 \geq r_2 \) \((r_1 < r_2, r_2 < r_1)\)  
\[ p_{c} (r_1) = \int_{0}^{r_1} p_{c1, c2} (r_2) dr_2 + \int_{r_1}^{\infty} p_{c1, c2} (r_1, r) dr_1 \]  
(16)  

\[ p_{c} (r_2) = \frac{2}{2\pi\sigma^2 \sqrt{1-\rho^2} r} \cdot \left[ \frac{1}{2} + \text{erf} \left( \frac{ln r_1 - \mu}{\sqrt{2} \sigma^2} \right) \right], \]  
(17)  

Overall pdf for \( r_1 \geq r_2 \) \((r_1 < r_2, r_2 < r_1)\)  
\[ p_{c} (r) = p_{c} (r_1) + p_{c} (r_2) + p_{c} (r) \]  
(18)  

The PDF is obtained in the form of one dimension integral with finite bounds.  
The outage probability is very useful performance measure for diversity systems operating in fading environments defined as the probability that output SIR of the SC falls below a given threshold \( r \) also known as a protection ratio. The outage probability \( P_{out}(r) \) is defined as:  
\[ P_{out}(r) = P(r_1 \leq r, r_2 \leq r) \]  
(19)  

For \( r < r_1 \) \((r_1 < r, r_2 < r)\)  
\[ P_{out}(r) = \int_{0}^{r} p_{c} (r) dr \]  
(20)  

For \( r_1 \geq r_2 \) \((r_1 < r_2, r_2 < r_1)\)  
\[ P_{out}(r) = \int_{0}^{r_1} p_{c} (r) dr + \int_{r_1}^{\infty} p_{c} (r_1, r) dr \]  
(21)  

The Numerical Results  
We present some values of the outage probabilities for different types of combiners and correlation parameters in Figures 2 and 3. The optimum thresholds \( r_1 = \exp(\mu/2) \) are adopted for SSC combiners [1].  
In Fig. 2. we show the family of curves for the outage probabilities for one channel receiver and for SSC and SC combiners at one time instant and SSC/SC combiner at two time instants for uncorrelated case and for very strong correlation. We can see that SSC/SC combiner has significant better performances for uncorrelated case and for \( \rho = 1 \) the CDF of SSC/SC combiner for \( r < r_1 \) follows the results for SC combiner at one time instant and for SSC combiner otherwise.
better system performances. If there is a correlation between input signals, the use of this kind of combiner increases with decreasing two time instants from Figure 3. It is obvious that benefits of parameters µ=1, σ=0.5, ρ=0;1 for different values of µ≠1, σ≠0.5, and ρ≠0; for the very strong correlation of complex combiner between inputs, where there is not a gain of using this type that this is not valid only for the very strong correlation of correlation between input signals, and therefore results in better system performances.

Conclusion
The probability density function of dual SSC/SC combiner output signal at two time instants in the presence of lognormal fading is determined. The system performances deciding by two samples can be determine by the joint probability density function of the SSC combiner output signal at two time instants and putting them as inputs of SC combiner. Then, the outage probability is numerically calculated using this PDF. The obtained results are shown graphically to compare performances of the SSC/SC combiner with regard to classical SSC and SC combiners at one time instant. We emphasize improving of the SSC/SC combiner characteristics at two time instants comparing with classical SSC and SC combiners at one time instant. It can be seen that this is not valid only for the very strong correlation between inputs, where there is not a gain of using this type of complex combiner.

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