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# Research on Angle Measurement Model of Anti-radiation Missiles PRS under Coherent Decoys

Abstract. How the monopulse passive radar seeker (PRS) of anti-radiation missiles (ARM) works in tracking radars is explained and demonstrated in this paper. The performance of PRS that the tracking location can fall at a point completely outside the decoys under coherent decoy jamming by using phase-amplitude sensing model, angle measurement error with different SNR compared with theoretic calculation results is given. The numerical results are presented to verify the effectiveness of this model and the coherent decoys jamming ARM theory.

Streszczenie. W artykule zaprezentowano jednopulsowy pasywny poszukiwacz radaru PRS w broni anty-radiacyjnej. Właściwości systemu PRS są analizowane przy wykorzystaniu modelu amplitudowo-fazowego, na podstawie którego obliczany jest błąd pomiaru kąta. (Badania błędu pomiaru kąta w antyradiacyjnej broni PRS)

**Keywords:** PRS, ARM, Monopulse angle measurement system, Phase-amplitude-comparison. **Słowa kluczowe:** PRS, pasywny wyszukiwacz radaru.

### Introduction

PRS is the most important component of ARM, most of the ARM PRS are monopulse angle measurement system, monopulse, or simultaneous-lobing tracking, obtains an angle error estimate from a single pulse return. Unlike sequential-lobing system, monopulse radar doesn't require a long integration time or demodulation process, resulting in an immunity to noise jamming effects. This inherent ECCM against many on-board jamming techniques explains why so many missiles have adopted monopulse tracker for their guidance seekers. Also many sequential-lobing trackers have been modified using monopulse techniques to reduce their vulnerability to jamming. [1-2].

The angle measurement methods used in PRS including: amplitude sensing, phase sensing, amplitude-phase (A-P) sensing and phase-amplitude (P-A) sensing [3]. PRS with amplitude sensing system would point to the power center of sources, when the PRS satisfies some requirement and the received signals' accumulation number is enough, the PRS would point the biggish power source. The A-P sensing angle measurement system uses interferometer to obtain precise angle message, and uses amplitude sensing method to eliminate the phase ambiguity [4]. The P-A sensing angle measurement uses cross beam to form a network, then transforms the phase message to amplitude message to improve the measurement precision.

The monopulse system PRS has great advantage in tracking single source, the antenna beam of PRS can't be too narrow in order to track target availably, which would bring multi-sources into the beam, in this situation, how the PRS would work under multi-sources, many researches have done on the tracking direction of PRS according to the theoretic calculation under non-coherent multi-sources, but little research have done under coherent multi-sources, especially through an angle measurement model [5-6]. In this paper, how the PRS would work under dual-coherent sources through an angle measurement model would be presented and analyzed in detail.

## **Basic Form of Monopulse System**

We can see all kinds of different real monopulse systems along with the development of radar technology. Basic composing part including: angle sensitivity equipment, angle message convertor, angle discriminator, as shown in Fig.1.

Angle sensing module is sensitive with the location message of the target, then, transfers the location message to angle message. Basic module of angle sensing including:

amplitude sensing, phase sensing and A-P, P-A sensing, and phase sensitivity is implemented in this paper.





Angle convertor module is used for transforming the angle message obtained by angle sensing equipment to the combination relationship of amplitude and phase of two independent channels, which meet the different form need of angle discrimination behind.

Angle discrimination module is used for pairing the combination relationship signals transferred from angle convertor module with the direction of angle to get a single angle message.

#### **Angle Measurement Model of PRS**

Most of the PRS used now are A-P sensing or P-A sensing system. A-P sensing system uses two parallel or gradient antennas to form a cross network which is used for the amplitude sensing processing of two information exchange channels, which eliminates the phase ambiguity. P-A sensing system uses cross beam to form a network, transforms the phase difference message to amplitude sensing message to improve the angle measurement precision [3].



Fig.2. Phase interferometer angle measurement system

1, Problem of Phase Ambiguity:

Phase interferometer has high resolution in angle measurement.

From Fig.2 we have:

(1) 
$$\varphi = \frac{2\pi d}{\lambda} \sin \theta$$

and

(2) 
$$\Delta \varphi = \frac{\partial \varphi}{\partial \theta} \Delta \theta + \frac{\partial \varphi}{\partial \lambda} \Delta \lambda + \frac{\partial \varphi}{\partial d} \Delta d$$

then

(3) 
$$\delta_{\Delta\theta} = \frac{\lambda \Delta \varphi}{2\pi d \cos \theta} + \frac{\Delta \lambda}{\lambda} \tan \theta$$

Supposed that the frequency is changeless, we have:

(4) 
$$\delta_{\Delta\theta} = \frac{\lambda \Delta \varphi}{2\pi d \cos \theta}$$

From Eq.4, we have: the angle measurement precision  $\delta_{\Delta\theta}$  is related with d, the resolution of angle is higher when the length of *d* becomes larger. Form Eq.1, we can see when  $d > \lambda/2$ ,  $\varphi = 2N\pi + \phi$ , N=1,2,3,..., and  $\varphi$  has multi-values, we can't find the real direction.

When the  $\phi$  changes from  $-\pi$  to  $\pi$ , the largest nonambigutiy angle is

(5) 
$$\theta_{\rm u} = 2 \arcsin(\lambda/2d)$$

From Eq.5 we can see, if we want to enlarge the nonambiguity angle, we should shorten the d, which is conflicting with the requirement of angle resolution. When  $\theta$ is small, from Eq.1, we have  $\varphi \approx 2\pi d\theta/\lambda$ , angle ambiguity appears outside  $-\lambda/2d < \theta < \lambda/2d$ , if the space between antennas increases to 3d, then  $\varphi \approx 6\pi d\theta/\lambda$ , angle ambiguity appears outside  $-\lambda/6d < \theta < \lambda/6d$ , the slope of the curve becomes triple as its primary one, which is advantageous to improve the angle scale resolution.



Fig.3. Relationship between  $\varphi$  and  $\theta$ 

As shown in Fig.3 above, we ofen use multi-baseline to resolve the angle ambiguity, and the contradiction between non-ambiguity visual range and anlge measurement resolution can be resolved. The non-ambiguity visual range is determined by the short baseline interferometer, signal generated as  $\varphi = 2\pi d\theta/\lambda$ , and the angle measurement resolution determined by the long baseline, which generated as  $\phi = 2\pi \cdot 3d (\sin \theta - n\lambda/6d)/\lambda$ , (n=0,1,2).

From the disscussion above, we know multi-baseline interferometer can be used to resolve the angle ambiguity, but for the limitation of the size of PRS, multi-baseline antennas can't be all fixed in PRS. for example, the antenna is too large for to be fixed in PRS when the frequency is low. So we use P-A sensing principle to resolved the phase ambiguity in ARM PRS [3].

## 2,P-A Sensing System Model:

Phase-amplitude sensing system is composed of two interferometer antennas with a certain distance, and phase information can be transferred to amplitude information through a cross beam network as shown in Fig.4, after frequency conversion, intermediate frequency amplifier, demodulation and video amplifier, signal precessing, the angle direction of recieved signals can be obtained [3] [5]. Supposed the orientation pattern of two antennas is  $F_A(\Theta)$  and  $F_B(\Theta)$ , then one of the output branch of cross beam network is:

(6)  

$$F_{A}(\theta) = F(\theta) + F(\theta) e^{j\left[\left(\varphi - \frac{\pi}{2}\right)\right]}$$

$$= F(\theta) e^{j\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right)} \left[ e^{-j\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right)} + e^{j\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right)} \right]$$

$$= 2F(\theta) \cos\left[\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right)\right] e^{j\frac{1}{2}\left(\varphi - \frac{\pi}{2}\right)}$$

Then the amplitude of it is:

(7) 
$$|F_A(\theta)| = 2F(\theta)\cos\left[\frac{1}{2}\left(\frac{2\pi d\sin\theta}{\lambda} - \frac{\pi}{2}\right)\right]$$

When the phase  $\varphi = \frac{2\pi d \sin \theta}{\lambda} = \frac{\pi}{2}$ ,  $F_A(\theta)$  has the largest value, and the  $\theta$  is:

(8) 
$$\theta_0 = \arcsin(\lambda/4d)$$

When  $\theta = 0^{\circ}$ 

(9) 
$$F_{A}(\theta) = \sqrt{2}F(\theta) = 0.707F_{A}(\theta_{0})$$

Same as the caculation of  $F_A(\theta)$ , for the other output branch we have:

(10) 
$$|F_B(\theta)| = 2F(\theta)\cos\left[\frac{1}{2}\left(\frac{2\pi d\sin\theta}{\lambda} + \frac{\pi}{2}\right)\right]$$

When the  $F_{B}(\theta)$  has the largest value,  $\theta$  is:

(11) 
$$-\theta_0 = \arcsin\left(\lambda/4d\right)$$

Which is symmetrical with  $F_{A}(\theta)$ :

(12) 
$$F_{B}\left(-\theta_{0}\right) = F_{A}\left(\theta_{0}\right)$$

When  $\theta = 0^0$ , we have:

(13) 
$$F_B\left(-\theta_0\right) = \sqrt{2}F\left(\theta\right) = 0.707F_B\left(\theta_0\right)$$

So the cross point of  $F_A(\theta)$  and  $F_B(\theta)$  is at  $\theta = 0^0$ , and it locates at the value of 0.707. then we have the range of non-ambiguity angle is:

(14) 
$$\theta_{v} = 2\theta_{0} = 2 \arcsin(\lambda/4d)$$

In this situation, the phase-amplitude sensing system become a combination of amplitude-sensing and phase – sensing systems used only one pair of antennas.



Fig.4. Phase-amplitude sensing system model

In low frequency sect,  $2\theta_0$  becomes much more larger, angle ambiguity can be eliminated by the random move of PRS antennas. When in high frequency, two pairs of antennas can be used, antennas with smaller distance used to eliminate ambiguity, antennas with larger distance used to advance precision. Supposed the incident angle of two sources is  $\theta_1$  and  $\theta_2$ .

Then the output of the sum channel of PRS is:

(15) 
$$U_{\Sigma}(t) = [F(\theta_1 - \theta_0) + F(\theta_1 + \theta_0)] \cdot S_0(t) + [F(\theta_2 - \theta_0) + F(\theta_2 + \theta_0)] \cdot S_1(t)$$

Output of minus channel is:

(16) 
$$U_{\Delta}(t) = [F(\theta_1 - \theta_0) - F(\theta_1 + \theta_0)] \cdot S_0(t) + [F(\theta_2 - \theta_0) - F(\theta_2 + \theta_0)] \cdot S_1(t)$$

Where  $S_0(t)$  and  $S_1(t)$  are signals receive by two antennas, after mixing and IF amplifying, supposed te two channels has the same characteristics, then we have:

(17) 
$$U_{\Sigma o}(t) = F_{\Sigma}(\theta_1) \cdot S_0(t) + F_{\Sigma}(\theta_2) \cdot S_1(t)$$

(18) 
$$U_{\Delta o}(t) = F_{\Delta}(\theta_1) \cdot S_0(t) + F_{\Delta}(\theta_2) \cdot S_1(t)$$

Where  $F_{\Sigma}(\theta)=F(\theta-\theta_0)+F(\theta+\theta_0)$ ,  $F_{\Delta}(\theta)=F(\theta-\theta_0)-F(\theta+\theta_0)$ , after through limitr and discriminator, the output signal is:

(19) 
$$u_{out} = K |U| \cos(\alpha_1 - \alpha_2)$$

where

$$(20) \begin{cases} \alpha_{1} = tg^{-1} \frac{F_{\Sigma}(\theta_{1}) \cdot A_{0} \cdot \sin \theta_{0}(t) + F_{\Sigma}(\theta_{2}) \cdot A_{1} \cdot \sin \theta_{1}(t)}{F_{\Sigma}(\theta_{1}) \cdot A_{0} \cdot \cos \theta_{0}(t) + F_{\Sigma}(\theta_{2}) \cdot A_{1} \cdot \cos \theta_{1}(t)} \\ \alpha_{2} = tg^{-1} \frac{F_{\Delta}(\theta_{1}) \cdot A_{0} \cdot \sin \theta_{0}(t) + F_{\Delta}(\theta_{2}) \cdot A_{1} \cdot \sin \theta_{1}(t)}{F_{\Delta}(\theta_{1}) \cdot A_{0} \cdot \cos \theta_{0}(t) + F_{\Delta}(\theta_{2}) \cdot A_{1} \cdot \cos \theta_{1}(t)} \end{cases}$$

And  $\mathcal{G}_{0}(t)$ ,  $\mathcal{G}_{1}(t)$  are the instantaneous phase of  $S_{0}(t)$ 

and  $S_1(t)$ . PRS reaches balance when  $u_{out} = 0$ , that is:

(21) 
$$\alpha_1 - \alpha_2 = \pm \frac{\pi}{2}$$

When the target departures from the axe of PRS very little, we can approximatively consider the orientation pattern of the antenna as:

(22) 
$$F(\theta \pm \theta_0) \approx F(\theta)(1 \mp \mu \theta)$$

Where 
$$\mu$$
 is a constant, then we have:

(23) 
$$\beta^2 \theta_1 + \theta_2 + \beta \cdot \cos(\theta_0(t) - \theta_1(t)) \cdot (\theta_1 + \theta_2) = 0$$

The point direction of PRS under two coherent decoys can be written as below:

(24)  
$$\theta = \frac{-\Delta\theta (1+\beta\cos\Delta\varphi)}{1+2\beta\cos\Delta\varphi+\beta^2} + \theta_1$$
$$or \quad \frac{\Delta\theta (1+\beta\cos\Delta\varphi)}{1+2\beta\cos\Delta\varphi+\beta^2} + \theta_2$$

where  $\Delta \varphi = \mathcal{G}_0(t) - \mathcal{G}_1(t)$ ,  $\beta = A_0/A_1 \cdot \theta = \theta_2$ ,  $\theta_1 = \theta + \Delta \theta$ .

Angle measurement system of PRS starts as a amplitude-sensing angle measurement, signals received by two amplitude-sensing channels provide a rough angle message, which makes the antennas of PRS track targets through antenna servo system and finish the collimation process. When the collimation axe of PRS turns into main petal linear area of phase-sensing system, the PRS begins to measure angle and track target accurately.

# Simulations

When the ARM is far away from sources, the spread angle of two sources to PRS is very small, and the incident angle of each source to two antennas can be considered as the same. Supposed that  $\theta_1 = 15^0$ , and  $\theta_2 = 15.2^0$ , frequencies of two decoys are both 6GHz, the length of baseline is d = 0.15m, orientation pattern of two antennas defined as below:

(25) 
$$F(\theta) = \frac{\sin\frac{\pi d}{\lambda}\theta}{\frac{\pi d}{\lambda}\theta} = \frac{\sin x}{x}$$

Simulation 1: When  $\beta$ = 0.7, 0.8, 0.9, the P-A sensing angle measurement characteristic of PRS along with the phase difference at PRS under coherent decoys is shown in Fig.5.



Fig.5. Angle measurement curve of PRS

Simulation2: When SNR=20dB, the error between result obtained by P-A sensing angle measurement system and theoretic calculation is shown in Fig.6.



Fig.6. Angle measurement error at SNR=20 dB under P-A sensing system

Simulation3: As discussed in reference [8], to realize coherent decoys jamming ARM, the phase difference between two sources at the PRS must be controlled in the range of about  $180^{\circ} \pm 20^{\circ}$ , when the phase difference at the PRS changes from  $(160^{\circ}, 200^{\circ})$ , the average angle measurement error along with difference SNR is as shown in Fig.7, the Monte Carlo number is 100.



Fig.7. Average angle measurement error along with SNR

From Fig.5 we can see when the phase difference of two coherent sources at PRS is nearby 180 degree, the direction of PRS would point outside the sources, which is the same as theoretic calculation [8-12]. From Fig.6 we can see when the phase difference close by 180 degree, the angle measure error becomes larger when the phase difference gets more close to 180 degree. From Fig.7, we can see that the angle measuremenst error becomes smaller along with the SNR gets higher, when the SNR is higher than 26 dB, the error would be smaller than the spread angle of two sources to the PRS. The Average

angle measurement error shows the validity of the P-A sensing angle measurement model.

#### Conclusions

In this paper, P-A sensing angle measurement system in PRS are presented to resolve the angle ambiguity of phase interferometer, PRS tracking target model based on this systems is built, then the P-A sensing tracking model under two coherent decoys is analyzed in detail, numerical simulation results are given under this model which prove that the point direction of PRS can be outside the coherent sources, and the angle measurement error given shows the validity of this PRS angle measurement model. The results can provide a platform for ARM jamming system, reference to the application of radar and decoy, could improve the radar survive ability and the ARM attack efficiency.

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