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Robust Dynamic Output Feedback H∞ Control for Uncertain Switched Singular Systems

***Abstract**. This paper considers the problems of dynamic output feedback H∞ control for uncertain switched singular system with parametric uncertainties. A switching rule and a switched dynamic output feedback controller are designed to guarantee that the closed-loop system is asymptotically stable with a prescribed H∞ disturbance attenuation level γ. Such sufficient conditions are derived via a series of strict linear matrix inequalities (LMIs). Finally, a numerical example is given to illustrate the effectiveness of the proposed method.*

***Streszczenie.** W artykule analizuje się problem dynamiki system sterowania H∞ dla system pojedynczego z niepewnymi przełączeniami. Badano zasady przełączania i dynamikę przełączania gwarantującą stabilną prace systemu. Przedstawiono przykład numeryczny ilustrujący skuteczność proponowanej metody. (**Odporny układ sterowania typu H∞ dla systemu z niepewnymi przełączeniami**)*

Keywords: switched systems, singular systems, robust *H∞* control, dynamic output feedback, linear matrix inequalities (LMIs).
Słowa kluczowe: odporny system sterowania, sprzężenie zwrotne, stabilność.

1. Introduction

In the past few decades, singular systems have been paid much attention, because they often appear in practice such as electric power systems, electrical networks, energy systems, social economic systems and biological systems and other areas[1-4].

On the other hand, there has been increasing interest in stability analysis and design for switched systems [5-7].There are two basic problems in stability and design of switched systems: (i) find conditions for stability under arbitrary switching; (ii) identify and construct the stabilizing switching laws. There are many existing works studying these problems for switched linear time-invariant systems. Many researchers have studied these problems[8-11]. However, the design of switching rule is a challenging problem.

The importance of both types of systems, suggested to attempt a further step towards the study of switched singular linear systems. Some related results have been reported[12-15]. However, to the best of our knowledge, the problem of dynamic output feedback robust H_∞ control for uncertain switched singular linear systems with parameter uncertainties has not been fully investigated.

In this paper, the problem of dynamic output feedback robust H_∞ control for uncertain switched singular linear systems with parameter uncertainties is considered. A switching rule and a switched dynamic output feedback controller are obtained such that the resulting closed-loop system is stable with a prescribed level of H_∞ disturbance attenuation level γ for all admissible parameter uncertainties.

Notations: We use standard notations throughout this paper. M^{T} is the transpose of the matrix M . $M>0$ ($M<0$) means that M is positive definite (negative definite). $M\ge 0$ ($M\le 0$) means that M is positive semi-definite (negative semi-definite). $L_2[0,T]$ ($0\le T<\infty$) denotes the space of square integrable functions on $[0,T]$ and $\|\omega\|_{L_2[0,T]}=\left(\int_0^T \omega^{\mathrm{T}}(t)\omega(t)dt\right)^{1/2}$ *for* $\forall\omega\in L_2[0,T]$. The symbol (*) denotes generically symmetric blocks.

2. Preliminaries and problem formulation

Consider the switched singular linear system with parameter uncertainties described by

$$(1)\quad \Sigma_1:\begin{cases} E\dot{x}(t)=(A_{\sigma(t)}+\Delta A_{\sigma(t)})x(t)+B_{1\sigma(t)}\omega(t)+(B_{2\sigma(t)}+\Delta B_{\sigma(t)})u(t)\\ z(t)=C_{1\sigma(t)}x(t)+D_{\sigma(t)}u(t)\\ y(t)=C_{2\sigma(t)}x(t)\end{cases}$$

Where: $x(t)\in\mathbb{R}^{n_x}$ is the state, $\omega(t)\in\mathbb{R}^{n_\omega}$ is the exogenous input with $\omega(t)\in L_2[0,\infty)$, $z(t)\in\mathbb{R}^{n_z}$ is the controlled output, $u(t)\in\mathbb{R}^{n_u}$ is the control input, $y(t)\in\mathbb{R}^{n_y}$ is the measured output. $\sigma:[0,\infty)\to\overline{\mathbb{N}}=\{1,2,\cdots,N\}$ is a piecewise constant function of time called switching signal. Moreover, we say that the i -th subsystem Σ_i is active at time t when $\sigma(t)=i$. The matrix $E\in\mathbb{R}^{n_x\times n_x}$ may be singular and $rankE=r_p<n_x$. $A_i,\ B_{1i},\ B_{2i},\ C_{1i},\ C_{2i},\ D_i,\forall i\in\overline{\mathbb{N}}$ are known real constant matrices with appropriate dimensions, and $\Delta A_i(t)$ and $\Delta B_i(t)$ are real-valued matrix functions representing time-varying parameter uncertainties. The parameter uncertainties are assumed to be of the following form

$$(2)\qquad \begin{bmatrix}\Delta A_i(t) & \Delta B_i(t)\end{bmatrix}=G_i\Sigma_i(t)\begin{bmatrix}F_{1i} & F_{2i}\end{bmatrix},\forall i\in\overline{\mathbb{N}}$$

Where G_i, F_{1i}, F_{2i} are known constant matrices with appropriate dimensions and $\Sigma_i(t)\in\mathbb{R}^{j\times k}$ is an unknown matrix function satisfying

$$(3)\qquad \Sigma_i^{\mathrm{T}}(t)\Sigma_i(t)\le I_k$$

Let us consider the nominal system of the system (1)

$$(4)\qquad \Sigma_2:\begin{cases}E\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}\omega(t)\\ z(t)=C_{\sigma(t)}x(t)\end{cases}$$

The unforced form of system (4) is the following autonomous switched singular linear system

$$(5)\qquad E\dot{x}(t)=A_{\sigma(t)}x(t)$$

The control problem is to design a switching rule $\sigma:[0,\infty)\to\overline{\mathbb{N}}$ and the dynamic output feedback controllers $K_i(s)\,(i\in\overline{\mathbb{N}})$ for the i -th sub-system (1)

$$(6)\qquad \Sigma_K:\begin{cases}E_K\dot{\tilde{x}}(t)=A_{K_i}\tilde{x}(t)+B_{K_i}y(t)\\ u(t)=C_{K_i}\tilde{x}(t)+D_{K_i}y(t), i\in\overline{\mathbb{N}}\end{cases}$$

with $\tilde{x}(t)\in\mathbb{R}^{n_{\tilde{x}}}$ is the controller state, $rankE_K=r_k<n_{\tilde{x}}$ and $A_{K_i},B_{K_i},C_{K_i},D_{K_i}$ are the state description form of the controller such that the closed-loop system

$$\Sigma_c:\begin{cases}E_c\dot{\xi}=(A_{c\sigma(t)}+\Delta A_{c\sigma(t)})\xi+B_{c\sigma(t)}\omega,\\ z=C_{c\sigma(t)}\xi.\end{cases} \tag{7}$$

with $\xi=[x^{\mathrm{T}}\quad \tilde{x}^{\mathrm{T}}]^{\mathrm{T}}$, $A_{ci}=A_i^o+B_{2i}^oK_iC_{2i}^o$, $\Delta A_{ci}=\Delta A_i^o+\Delta B_i^oK_iC_{2i}^o$,

$B_{ci}=B_{1i}^o=\begin{bmatrix}B_{1i}\\0\end{bmatrix}$, $C_{ci}=C_{1i}^o+D_i^oK_iC_{2i}^o$, $C_{1i}^o=\begin{bmatrix}C_{1i} & 0\end{bmatrix}$, $D_i^o=\begin{bmatrix}D_i & 0\end{bmatrix}$,

$E_c=\begin{bmatrix}E & 0\\0 & E_K\end{bmatrix}$, $A_i^o=\begin{bmatrix}A_i & 0\\0 & 0\end{bmatrix}$, $\Delta A_i^o=\begin{bmatrix}\Delta A_i & 0\\0 & 0\end{bmatrix}$, $B_{2i}^o=\begin{bmatrix}B_{2i} & 0\\0 & I\end{bmatrix}$,

$\Delta B_i^o=\begin{bmatrix}\Delta B_i & 0\\0 & 0\end{bmatrix}$, $C_{2i}^o=\begin{bmatrix}C_{2i} & 0\\0 & I\end{bmatrix}$, $K_i=\begin{bmatrix}D_{K_i} & C_{K_i}\\B_{K_i} & A_{K_i}\end{bmatrix}$

is stable with H_∞ disturbance attenuation level γ for all admissible uncertainties which satisfy (3), i.e.

1) With $\omega(t)=0$, the closed-loop system is asymptotically stable for all admissible uncertainties.

2) With zero-initial condition $\xi(0)=0$, $\|z\|_{L_2[0,T]}<\gamma\|\omega\|_{L_2[0,T]}$ for all nonzero $\omega\in L_2[0,T]\,(0\le T<\infty)$ and all admissible uncertainties.

The following Lemmas are necessary for our further discussion.

Lemma 1[14] Let X and Y be matrices or vectors of the same dimensions, then

$$X^{\mathrm{T}}Y+Y^{\mathrm{T}}X\le X^{\mathrm{T}}PX+Y^{\mathrm{T}}P^{-1}Y,\forall P>0$$

Lemma 2[15] Given a symmetrical matrix $H\in\mathbb{R}^{m\times m}$ and two matrices P, Q of column dimension m, consider the problem of finding some matrix X of compatible dimensions such that

$$H+P^{\mathrm{T}}XQ+Q^{\mathrm{T}}X^{\mathrm{T}}P<0$$

Denote by $P^\perp$ and $Q^\perp$ any matrices whose columns form bases of the null space of P and Q, respectively. That is to say $PP^\perp=0, QQ^\perp=0$. Then the above matrix inequality is solvable for X if and only if

$$P^{\perp\mathrm{T}}HP^\perp<0, Q^{\perp\mathrm{T}}HQ^\perp<0$$

3. Main results

In this section, we give some sufficient conditions which guarantee the closed-loop system (7) is asymptotically stable with H_∞ disturbance attenuation level γ.

Theorem 1 Given any constant $\gamma>0$, the uncertain switched singular linear system (1) is asymptotically stable with H_∞ disturbance attenuation level γ via switched dynamic output feedback for all admissible uncertainties which satisfy (3), if there exist a matrix X_c and scalars $\alpha_i\ge 0$ with $\sum_{i=1}^{N}\alpha_i=1\,(\forall i\in\bar{\mathbb{N}})$ and positive scalar ε such that the following inequalities are true.

$$E_c^{\mathrm{T}}X_c=X_c^{\mathrm{T}}E_c\ge 0 \tag{8}$$

$$\begin{bmatrix}A_c^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_c & * & * & * & *\\ \gamma^{-1}B_c^{\mathrm{T}}X_c & -I & 0 & 0 & 0\\ C_c & 0 & -I & 0 & 0\\ \varepsilon^{-1}G_c^{\mathrm{T}}X_c & 0 & 0 & -I & 0\\ \varepsilon F_c & 0 & 0 & 0 & -I\end{bmatrix}<0 \tag{9}$$

where:

$A_c=A^o+B_2^oKC_2^o$, $B_c=B_1^o=\left[\sqrt{\alpha_1}B_{11}^o,\sqrt{\alpha_2}B_{12}^o,\cdots,\sqrt{\alpha_N}B_{1N}^o\right]$,

$C_c=C_1^o+D^oKC_2^o$, $F_c=F_1^o+F_2^oKC_2^o$,

$G_c=G^o=\left[\sqrt{\alpha_1}G_1^o,\sqrt{\alpha_2}G_2^o,\cdots,\sqrt{\alpha_N}G_N^o\right]$,

$A^o=\sum_{i=1}^{N}\alpha_iA_i^o$, $B_2^o=\left[\alpha_1B_{21}^o,\alpha_2B_{22}^o,\cdots,\alpha_NB_{2N}^o\right]$,

$C_1^o=\left[\sqrt{\alpha_1}(C_{11}^o)^{\mathrm{T}},\sqrt{\alpha_2}(C_{12}^o)^{\mathrm{T}},\cdots,\sqrt{\alpha_N}(C_{1N}^o)^{\mathrm{T}}\right]^{\mathrm{T}}$,

$C_2^o=\left[(C_{21}^o)^{\mathrm{T}},(C_{22}^o)^{\mathrm{T}},\cdots,(C_{2N}^o)^{\mathrm{T}}\right]^{\mathrm{T}}$, $F_{1i}^o=\begin{bmatrix}F_{1i} & 0\end{bmatrix}$,

$D^o=diag\left\{\sqrt{\alpha_1}D_1^o,\sqrt{\alpha_2}D_2^o,\cdots,\sqrt{\alpha_N}D_N^o\right\}$, $F_{2i}^o=\begin{bmatrix}F_{2i} & 0\end{bmatrix}$,

$G_{ci}=G_i^o=\begin{bmatrix}G_i\\0\end{bmatrix}$, $K=diag\{K_1,K_2,\cdots,K_N\}$,

$F_1^o=\left[\sqrt{\alpha_1}(F_{11}^o)^{\mathrm{T}},\sqrt{\alpha_2}(F_{12}^o)^{\mathrm{T}},\cdots,\sqrt{\alpha_N}(F_{1N}^o)^{\mathrm{T}}\right]^{\mathrm{T}}$,

$F_2^o=diag\left\{\sqrt{\alpha_1}F_{21}^o,\sqrt{\alpha_2}F_{22}^o,\cdots,\sqrt{\alpha_N}F_{2N}^o\right\}$, $F_{ci}=F_{1i}^o+F_{2i}^oK_iC_{2i}^o$.

In this case, the dynamic output feedback controller gain matrix can be obtained as

$$K=diag\{K_1,K_2,\cdots,K_N\},\ K_i=\begin{bmatrix}D_{K_i} & C_{K_i}\\B_{K_i} & A_{K_i}\end{bmatrix}, i\in\bar{\mathbb{N}} \tag{10}$$

The switching rule $\sigma(t)$ is taken as

$$\begin{aligned}\sigma(t)=\arg\min_{i\in\bar{\mathbb{N}}}\{&\xi^{\mathrm{T}}(A_{ci}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_{ci}+C_{ci}^{\mathrm{T}}C_{ci}+\gamma^{-2}X_c^{\mathrm{T}}B_{ci}B_{ci}^{\mathrm{T}}X_c+\\&\varepsilon^{-2}X_c^{\mathrm{T}}G_{ci}G_{ci}^{\mathrm{T}}X_c+\varepsilon^2F_{ci}^{\mathrm{T}}F_{ci})\xi\}\end{aligned} \tag{11}$$

Proof: We first show that the closed-loop system of the system (7) is asymptotically stable. By Schur complement lemma, the inequality (9) is equivalent to the following inequality.

$$A_c^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_c+C_c^{\mathrm{T}}C_c+\gamma^{-2}X_c^{\mathrm{T}}B_cB_c^{\mathrm{T}}X_c+\varepsilon^{-2}X_c^{\mathrm{T}}G_cG_c^{\mathrm{T}}X_c+\varepsilon^2F_c^{\mathrm{T}}F_c<0$$

Hence, for any nonzero state $\xi\in\mathbb{R}^{n_x+n_{\tilde{x}}}$, we have

$$\begin{aligned}\sum_{i=1}^{N}\alpha_i(&\xi^{\mathrm{T}}(A_{ci}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_{ci}+C_{ci}^{\mathrm{T}}C_{ci}+\gamma^{-2}X_c^{\mathrm{T}}B_{ci}B_{ci}^{\mathrm{T}}X_c+\\&\varepsilon^{-2}X_c^{\mathrm{T}}G_{ci}G_{ci}^{\mathrm{T}}X_c+\varepsilon^2F_{ci}^{\mathrm{T}}F_{ci})\xi)<0\end{aligned}$$

Furthermore, by means of the switching rule (11), it follows that

$$\begin{aligned}\xi^{\mathrm{T}}(&A_{c\sigma(t)}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_{c\sigma(t)}+C_{c\sigma(t)}^{\mathrm{T}}C_{c\sigma(t)}+\gamma^{-2}X_c^{\mathrm{T}}B_{c\sigma(t)}B_{c\sigma(t)}^{\mathrm{T}}X_c+\\&\varepsilon^{-2}X_c^{\mathrm{T}}G_{c\sigma(t)}G_{c\sigma(t)}^{\mathrm{T}}X_c+\varepsilon^2F_{c\sigma(t)}^{\mathrm{T}}F_{c\sigma(t)})\xi<0.\end{aligned} \tag{12}$$

Let $\{(t_k,i_k)|i_k\in\bar{\mathbb{N}};k=0,1,\cdots;0=t_0\le t_1\le\cdots\}$ be switching sequence in the interval $[0,\infty)$ that is generated by the switching rule (11). Setting common Lyapunov function $V(\xi)=\xi^{\mathrm{T}}E_c^{\mathrm{T}}X_c\xi$, it follows that

$$\begin{aligned}\dot{V}(\xi)=&\xi^{\mathrm{T}}(A_{ci_k}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_{ci_k})\xi+\xi^{\mathrm{T}}(\Delta A_{ci_k}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}\Delta A_{ci_k})\xi+\\&\omega^{\mathrm{T}}(B_{ci_k}^{\mathrm{T}}X_c\xi)+(B_{ci_k}^{\mathrm{T}}X_c\xi)^{\mathrm{T}}\omega\quad t\in[t_k,t_{k+1}),(k=0,1,\cdots)\end{aligned} \tag{13}$$

By means of (2) and (3) and Lemma 1, we have

$$\begin{aligned}\dot{V}(\xi)\le&\xi^{\mathrm{T}}(A_{c\sigma(t)}^{\mathrm{T}}X_c+X_c^{\mathrm{T}}A_{c\sigma(t)}+\gamma^{-2}X_c^{\mathrm{T}}B_{c\sigma(t)}B_{c\sigma(t)}^{\mathrm{T}}X_c+\\&\varepsilon^{-2}X_c^{\mathrm{T}}G_{c\sigma(t)}G_{c\sigma(t)}^{\mathrm{T}}X_c+\varepsilon^2F_{c\sigma(t)}^{\mathrm{T}}F_{c\sigma(t)})\xi+\gamma^2\omega^{\mathrm{T}}\omega\\&t\in[t_k,t_{k+1}),(k=0,1,\cdots)\end{aligned} \tag{14}$$

Noting that $\omega(t)=0$, by means of (8) and (12), we get $V(\xi)\ge 0$ and $\dot{V}(\xi)<0$ for $\forall t\ge 0$ under the switching rule (11). Hence, the asymptotic stability of system (7) with $\omega(t)=0$ follows immediately.

Secondly, we will investigate the H_∞ disturbance attenuation level γ of system (7). Assume $\xi(0)=0$ and for $\forall T>0$ introduce the performance

$$J_T=\int_0^T(z^{\mathrm{T}}(t)z(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t))dt.$$

Let $\{(t_k,i_k)\,|\,i_k\in\bar{\mathbb{N}};k=0,1,\cdots,s;0=t_0\le t_1\le\cdots\le t_s=T\}$ be switching sequence in the interval $[0,T]$ that is generated by the switching rule (11). Setting common Lyapunov function $V(\xi)=\xi^{\mathrm{T}}E_c^{\mathrm{T}}X_c\xi$ and noting that $\xi(t_0)=\xi(0)=0$, then for $\forall\omega\in L_2[0,T]$

$$\begin{aligned} J_T &= \sum_{k=0}^{s-1}\left(\int_{t_k}^{t_{k+1}}(z^{\mathrm T}z-\gamma^2\omega^{\mathrm T}\omega+\dot V)dt-(V(\xi(t_{k+1}))-V(\xi(t_k)))\right) \\ &= \sum_{k=0}^{s-1}\int_{t_k}^{t_{k+1}}(z^{\mathrm T}z-\gamma^2\omega^{\mathrm T}\omega+\dot V)dt-V(\xi(T)) \\ &\le \sum_{k=0}^{s-1}\int_{t_k}^{t_{k+1}}\xi^{\mathrm T}(A_{ci_k}^{\mathrm T}X_c+X_c^{\mathrm T}A_{ci_k}+C_{ci_k}^{\mathrm T}C_{ci_k}+\gamma^{-2}X_c^{\mathrm T}B_{ci_k}B_{ci_k}^{\mathrm T}X_c+ \\ &\qquad \varepsilon^{-2}X_c^{\mathrm T}G_{ci_k}G_{ci_k}^{\mathrm T}X_c+\varepsilon^2F_{ci_k}^{\mathrm T}F_{ci_k})\xi dt \end{aligned}$$

By (12), for $\forall t\ge 0$ we get $J_T<0$, that is to say

$$\|z\|_{L_2[0,T]}<\gamma\|\omega\|_{L_2[0,T]},\forall\omega\in L_2[0,T]$$

holds true. This completes the proof.

Next, we will now establish, that the nonlinear inequalities (8), (9) are equivalent to some LMIs conditions. Without loss of generality, we assume that E and E_K are

$$E=\begin{bmatrix} I_{r_p} & 0\\ 0 & 0\end{bmatrix}, E_K=\begin{bmatrix} I_{r_k} & 0\\ 0 & 0\end{bmatrix} \tag{15}$$

The following Lemmas are necessary for our further discussion.

Lemma 3 Consider a plant (1), the switched controller as in (6) and matrices E and E_K as in (15). Then the inequalities (8), (9) equivalently can be written as

$$E_c^{\mathrm T}X_c=X_c^{\mathrm T}E_c\ge 0, Q^{\perp\mathrm T}H_{X_c}Q^{\perp}<0, P^{\perp\mathrm T}T_{X_c}P^{\perp}<0 \tag{16}$$

where:

$$H_{X_c}=\begin{bmatrix} (A^o)^{\mathrm T}X_c+X_c^{\mathrm T}A^o & * & * & * & *\\ \gamma^{-1}(B_1^o)^{\mathrm T}X_c & -I & * & * & *\\ C_1^o & 0 & -I & * & *\\ \varepsilon^{-1}(G^o)^{\mathrm T}X_c & 0 & 0 & -I & *\\ \varepsilon F_1^o & 0 & 0 & 0 & -I\end{bmatrix}$$

$$T_{X_c}=\begin{bmatrix} A^oX_c^{-1}+X_c^{-\mathrm T}(A^o)^{\mathrm T} & * & * & * & *\\ \gamma^{-1}(B_1^o)^{\mathrm T} & -I & * & * & *\\ C_1^oX_c^{-1} & 0 & -I & * & *\\ \varepsilon^{-1}(G^o)^{\mathrm T} & 0 & 0 & -I & *\\ \varepsilon F_1^oX_c^{-1} & 0 & 0 & 0 & -I\end{bmatrix} \tag{17}$$

Proof: We make use of the fact that the controller data occurs in (9) in an affine way, i.e. (9) can be written as

$$H_{X_c}+P_{X_c}^{\mathrm T}KQ+Q^{\mathrm T}K^{\mathrm T}P_{X_c}<0 \tag{18}$$

where $P_{X_c}=\left[(B_2^o)^{\mathrm T}X_c\quad 0\quad (D^o)^{\mathrm T}\quad 0\quad \varepsilon(F_2^o)^{\mathrm T}\right]$, $Q=\left[C_2^o\quad 0\quad 0\quad 0\quad 0\right]$.

By Lemma 2

$H_{X_c}+P_{X_c}^{\mathrm T}KQ+Q^{\mathrm T}K^{\mathrm T}P_{X_c}<0\Leftrightarrow P_{X_c}^{\perp\mathrm T}H_{X_c}P_{X_c}^{\perp}<0$ and $Q^{\perp\mathrm T}H_{X_c}Q^{\perp}<0$.

Now let $P=\left[(B_2^o)^{\mathrm T}\quad 0\quad (D^o)^{\mathrm T}\quad 0\quad \varepsilon(F_2^o)^{\mathrm T}\right]$ and $S=diag\{X_c,I,I,I,I\}$, we can obtain

$$P_{X_c}^{\perp}=S^{-1}P^{\perp} \text{ and } (S^{-1})^{\mathrm T}H_{X_c}S^{-1}=T_{X_c}.$$

Then, from the above, we have

$$P_{X_c}^{\perp\mathrm T}H_{X_c}P_{X_c}^{\perp}<0\Leftrightarrow P^{\perp\mathrm T}(S^{-1})^{\mathrm T}H_{X_c}S^{-1}P^{\perp}<0 \text{ i.e. } P^{\perp\mathrm T}T_{X_c}P^{\perp}<0$$

This completes the proof of Lemma 3.

Although we have removed the controller matrices in this characterization, it is also not computationally attractive since the inequalities (16) contain the matrix X_c as well as the inverse X_c^{-1}. This problem can be overcome by an explicit parameterization of X_c and X_c^{-1}. A possible solution X_c of (16) is necessarily non-singular and $E_c^{\mathrm T}X_c=X_c^{\mathrm T}E_c\ge 0$ implies that X_c and X_c^{-1} can be written as

$$X_c=\left[\begin{array}{cc|cc} X_1 & 0 & N_1 & 0\\ X_3 & X_4 & N_3 & N_4\\ \hline N_1^{\mathrm T} & 0 & L_1 & 0\\ N_7 & N_8 & L_3 & L_4\end{array}\right], X_c^{-1}=\left[\begin{array}{cc|cc} Y_1 & 0 & M_1 & 0\\ Y_3 & Y_4 & M_3 & M_4\\ \hline M_1^{\mathrm T} & 0 & S_1 & 0\\ M_7 & M_8 & S_3 & S_4\end{array}\right] \tag{19}$$

$$X_1=X_1^{\mathrm T}, L_1=L_1^{\mathrm T}, Y_1=Y_1^{\mathrm T}, S_1=S_1^{\mathrm T}$$

and $X_1, Y_1\in\mathbb{R}^{r_p\times r_p}, X_4, Y_4\in\mathbb{R}^{(n_x-r_p)\times(n_x-r_p)}, L_1, S_1\in\mathbb{R}^{r_k\times r_k}$, $L_4, S_4\in\mathbb{R}^{(n_{\bar k}-r_k)\times(n_{\bar k}-r_k)}$, other the sub-matrices of appropriate dimension. Due to this partition of X_c and X_c^{-1}, we have the following lemma.

Lemma 4 Assume the existence of matrices X_c and X_c^{-1} as in (19) such that (16) hold true. Define

$$A_i=\begin{bmatrix} A_{i11} & A_{i12}\\ A_{i21} & A_{i22}\end{bmatrix}, B_{1i}=\begin{bmatrix} B_{i11}\\ B_{i12}\end{bmatrix}, G_i=\begin{bmatrix} G_{i1}\\ G_{i2}\end{bmatrix}, C_{1i}=\begin{bmatrix} C_{i11} & C_{i12}\end{bmatrix}, F_{1i}=\begin{bmatrix} F_{i11} & F_{i12}\end{bmatrix}$$

$$A_{i11}\in\mathbb{R}^{r_p\times r_p}, B_{i11}\in\mathbb{R}^{r_p\times n_\omega}, C_{i11}\in\mathbb{R}^{n_z\times r_p}, G_{i1}\in\mathbb{R}^{r_p\times r_p}, F_{i11}\in\mathbb{R}^{r_p\times n_\omega}$$

Then $Q^{\perp\mathrm T}H_{X_c}Q^{\perp}<0, P^{\perp\mathrm T}T_{X_c}P^{\perp}<0$ equivalently can be written as

$$\begin{bmatrix} \bar A_{21} & C_2\\ \bar A_{22} & \\ \bar B_{12} & 0\\ 0 & 0\\ \bar G_2 & 0\\ 0 & 0\end{bmatrix}^{\perp\mathrm T}\underbrace{\begin{bmatrix} A^{\mathrm T}X_0+X_0^{\mathrm T}A & * & * & * & *\\ \gamma^{-1}B_1^{\mathrm T}X_0 & -I & 0 & 0 & 0\\ C_1 & 0 & -I & 0 & 0\\ \varepsilon^{-1}G^{\mathrm T}X_0 & 0 & 0 & -I & 0\\ \varepsilon F_1 & 0 & 0 & 0 & -I\end{bmatrix}}_{H_0}\begin{bmatrix} \bar A_{21} & C_2\\ \bar A_{22} & \\ \bar B_{12} & 0\\ 0 & 0\\ \bar G_2 & 0\\ 0 & 0\end{bmatrix}^{\perp}<0 \tag{20}$$

$$X_0=\begin{bmatrix} X_1 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix} \bar A_{12}^{\mathrm T} & B_2^{\mathrm T}\\ \bar A_{22}^{\mathrm T} & \\ \bar C_{12}^{\mathrm T} & D^{\mathrm T}\\ \bar F_{12}^{\mathrm T} & \varepsilon F_2^{\mathrm T}\\ 0 & 0\\ 0 & 0\end{bmatrix}^{\perp\mathrm T}\underbrace{\begin{bmatrix} AY_0+Y_0^{\mathrm T}A^{\mathrm T} & * & * & * & *\\ C_1Y_0 & -I & 0 & 0 & 0\\ \varepsilon F_1Y_0 & 0 & -I & 0 & 0\\ \varepsilon^{-1}G^{\mathrm T} & 0 & 0 & -I & 0\\ \gamma^{-1}B_1^{\mathrm T} & 0 & 0 & 0 & -I\end{bmatrix}}_{T_0}\begin{bmatrix} \bar A_{12}^{\mathrm T} & B_2^{\mathrm T}\\ \bar A_{22}^{\mathrm T} & \\ \bar C_{12}^{\mathrm T} & D^{\mathrm T}\\ \bar F_{12}^{\mathrm T} & \varepsilon F_2^{\mathrm T}\\ 0 & 0\\ 0 & 0\end{bmatrix}^{\perp}<0 \tag{21}$$

$$Y_0=\begin{bmatrix} Y_1 & 0\\ 0 & 0\end{bmatrix}$$

where:

$A=\sum_{i=1}^{N}\alpha_iA_i, B_1=\left[\sqrt{\alpha_1}B_{11},\sqrt{\alpha_2}B_{12},\cdots,\sqrt{\alpha_N}B_{1N}\right]$

, $B_2=\left[\alpha_1B_{21},\alpha_2B_{22},\cdots,\alpha_NB_{2N}\right]$, $\bar A_{12}=\sum_{i=1}^{N}\alpha_iA_{i12}$, $\bar A_{21}=\sum_{i=1}^{N}\alpha_iA_{i21}$,

$\bar A_{22}=\sum_{i=1}^{N}\alpha_iA_{i22}$, $\bar B_{12}=\left[\sqrt{\alpha_1}B_{112},\sqrt{\alpha_2}B_{212},\cdots,\sqrt{\alpha_N}B_{N12}\right]$,

$C_1=\left[\sqrt{\alpha_1}C_{11}^{\mathrm T},\sqrt{\alpha_2}C_{12}^{\mathrm T},\cdots,\sqrt{\alpha_N}C_{1N}^{\mathrm T}\right]^{\mathrm T}$, $C_2=\left[C_{21}^{\mathrm T},C_{22}^{\mathrm T},\cdots,C_{2N}^{\mathrm T}\right]^{\mathrm T}$,

$\bar C_{12}=\left[\sqrt{\alpha_1}C_{112}^{\mathrm T},\sqrt{\alpha_2}C_{212}^{\mathrm T},\cdots,\sqrt{\alpha_N}C_{N12}^{\mathrm T}\right]^{\mathrm T}$, $D=diag\left\{\sqrt{\alpha_1}D_1,\sqrt{\alpha_2}D_2,\cdots,\sqrt{\alpha_N}D_N\right\}$,

$G=\left[\sqrt{\alpha_1}G_1,\sqrt{\alpha_2}G_2,\cdots,\sqrt{\alpha_N}G_N\right]$, $\bar G_2=\left[\sqrt{\alpha_1}G_{12},\sqrt{\alpha_2}G_{22},\cdots,\sqrt{\alpha_N}G_{N2}\right]$,

$F_1=\left[\sqrt{\alpha_1}(F_{11})^{\mathrm T},\sqrt{\alpha_2}(F_{12})^{\mathrm T},\cdots,\sqrt{\alpha_N}(F_{1N})^{\mathrm T}\right]^{\mathrm T}$,

$F_2=diag\left\{\sqrt{\alpha_1}F_{21},\sqrt{\alpha_2}F_{22},\cdots,\sqrt{\alpha_N}F_{2N}\right\}$,

$\bar F_{12}=\left[\sqrt{\alpha_1}(F_{112})^{\mathrm T},\sqrt{\alpha_2}(F_{212})^{\mathrm T},\cdots,\sqrt{\alpha_N}(F_{N12})^{\mathrm T}\right]^{\mathrm T}$.

Proof: We introduce the shorthand notation

$X_l=\begin{bmatrix} X_3 & X_4\end{bmatrix}, Y_l=\begin{bmatrix} Y_3 & Y_4\end{bmatrix}$ and $X_c=\begin{bmatrix} X & N_u\\ N_l & L\end{bmatrix}, X_c^{-1}=\begin{bmatrix} Y & M_u\\ M_l & S\end{bmatrix}$

for the indicated block partition in (19). The matrices H_{X_c}, T_{X_c} in (17) then become

$$H_{X_c}=\begin{bmatrix} A^{\mathrm T}X+X^{\mathrm T}A & * & * & * & * & *\\ N_u^{\mathrm T}A & 0 & * & * & * & *\\ \gamma^{-1}B_1^{\mathrm T}X & \gamma^{-1}B_1^{\mathrm T}N_u & -I & * & * & *\\ C_1 & 0 & 0 & -I & * & *\\ \varepsilon^{-1}G^{\mathrm T}X & \varepsilon^{-1}G^{\mathrm T}N_u & 0 & 0 & -I & *\\ \varepsilon F_1 & 0 & 0 & 0 & 0 & -I\end{bmatrix} \tag{22a}$$

(22b) $$T_{X_c}=\begin{bmatrix} AY+Y^{\mathrm{T}}A^{\mathrm{T}} & * & * & * & * & * \\ M_u^{\mathrm{T}}A^{\mathrm{T}} & 0 & * & * & * & * \\ \gamma^{-1}B_1^{\mathrm{T}} & 0 & -I & * & * & * \\ C_1Y & C_1M_u & 0 & -I & * & * \\ \varepsilon^{-1}G^{\mathrm{T}} & 0 & 0 & 0 & -I & * \\ \varepsilon F_1Y & \varepsilon F_1M_u & 0 & 0 & 0 & -I \end{bmatrix}$$

$Q^{\perp}$ and $P^{\perp}$ can be expressed as

(23) $$Q^{\perp\mathrm{T}}=\begin{bmatrix} C_2^{\perp\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & I & I & I \end{bmatrix},\quad P^{\perp\mathrm{T}}=\begin{bmatrix} B_2^{\mathrm{T}\perp\mathrm{T}} & 0 & 0 & D^{\mathrm{T}\perp\mathrm{T}} & 0 & F_1^{\mathrm{T}\perp\mathrm{T}} \\ 0 & 0 & I & 0 & I & 0 \end{bmatrix}$$

Due to the zero column in (23) and (22a, b) the inequalities in (16) are equivalent to

(24) $$\begin{bmatrix} C_2^{\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} A^{\mathrm{T}}X+X^{\mathrm{T}}A & * & * & * & * \\ \gamma^{-1}B_1^{\mathrm{T}}X & -I & 0 & 0 & 0 \\ C_1 & 0 & -I & 0 & 0 \\ \varepsilon^{-1}G^{\mathrm{T}}X & 0 & 0 & -I & 0 \\ \varepsilon F_1 & 0 & 0 & 0 & -I \end{bmatrix}}_{H'} \begin{bmatrix} C_2^{\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix}<0$$

(25) $$\begin{bmatrix} B_2^{\mathrm{T}\perp} & 0 \\ D^{\mathrm{T}\perp} & 0 \\ \varepsilon F_2^{\mathrm{T}\perp} & 0 \\ 0 & I \\ 0 & I \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} AY+Y^{\mathrm{T}}A^{\mathrm{T}} & * & * & * & * \\ C_1Y & -I & 0 & 0 & 0 \\ \varepsilon F_1Y & 0 & -I & 0 & 0 \\ \varepsilon^{-1}G^{\mathrm{T}} & 0 & 0 & -I & 0 \\ \gamma^{-1}B_1^{\mathrm{T}} & 0 & 0 & 0 & -I \end{bmatrix}}_{T'} \begin{bmatrix} B_2^{\mathrm{T}\perp} & 0 \\ D^{\mathrm{T}\perp} & 0 \\ \varepsilon F_2^{\mathrm{T}\perp} & 0 \\ 0 & I \\ 0 & I \end{bmatrix}<0$$

With $P_H=\begin{bmatrix} C_2 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $P_T=\begin{bmatrix} B_2^{\mathrm{T}} & D^{\mathrm{T}} & \varepsilon F_2^{\mathrm{T}} & 0 & 0 \end{bmatrix}$ the inequality (24) and (25) can be written as

(26) $$P_H^{\perp\mathrm{T}}H'P_H^{\perp}<0,\ P_T^{\perp\mathrm{T}}T'P_T^{\perp}<0$$

If we additionally introduce $Q_H=\begin{bmatrix} I_{n_x} & 0 & 0 & 0 & 0 \end{bmatrix}$ and $Q_T=\begin{bmatrix} I_{n_x} & 0 & 0 & 0 & 0 \end{bmatrix}$ the inequalities

(27) $$Q_H^{\perp\mathrm{T}}H'Q_H^{\perp}<0,\ Q_T^{\perp\mathrm{T}}T'Q_T^{\perp}<0$$

are trivially fulfilled. By view of Lemma 2 the inequalities (26), (27) then become $\exists\beta,\delta$ makes

(28) $$H'+P_H^{\mathrm{T}}\beta Q_H+Q_H^{\mathrm{T}}\beta^{\mathrm{T}}P_H<0,\ T'+P_T^{\mathrm{T}}\delta Q_T+Q_T^{\mathrm{T}}\delta^{\mathrm{T}}P_T<0$$

with matrices β,δ of suitable dimension. Now we can split H_0 from H' (and analogous for T_0):

(29) $$H'=H_0+\begin{bmatrix} \bar{A}_{21} & \bar{A}_{22} & \bar{B}_{12} & 0 & \bar{G}_2 & 0 \end{bmatrix}^{\mathrm{T}}X_l\begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}X_l^{\mathrm{T}}\begin{bmatrix} \bar{A}_{21} & \bar{A}_{22} & \bar{B}_{12} & 0 & \bar{G}_2 & 0 \end{bmatrix}$$

In conjunction with the corresponding inequality in (28) we end up with

$$H_0+\begin{bmatrix} \bar{A}_{21} & \bar{A}_{22} & \bar{B}_{12} & 0 & \bar{G}_2 & 0 \\ C_2 & & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} X_l \\ \beta \end{bmatrix}\begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}^{T}\begin{bmatrix} X_l \\ \beta \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \bar{A}_{21} & \bar{A}_{22} & \bar{B}_{12} & 0 & \bar{G}_2 & 0 \\ C_2 & & 0 & 0 & 0 & 0 \end{bmatrix}<0$$

A final application of Lemma 2 renders the Lemma 4.

The inequalities (20), (21) are linear inequalities in X_1, Y_1. However, these inequalities are based on the assumption that matrices X_c, X_c^{-1} as in (19) actually exists. This problem partly is addressed in the following lemma.

Lemma 5[15] Suppose that $X_{11}=X_{11}^{\mathrm{T}}, Y_{11}=Y_{11}^{\mathrm{T}}\in\mathbb{R}^{n\times n}$ with $X_{11}>0, Y_{11}>0$ are given. Let r be a non-negative integer. Then there exists matrices $X_{12}, Y_{12}\in\mathbb{R}^{n\times r}$, $X_{22}=X_{22}^{\mathrm{T}}, Y_{22}=Y_{22}^{\mathrm{T}}\in\mathbb{R}^{r\times r}$, and

(30) $$\begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}>0,\ \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}^{-1}=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{bmatrix}$$

If and only if

(31) $$\begin{bmatrix} X_{11} & I \\ I & Y_{11} \end{bmatrix}\geq 0 \text{ and } rank\begin{bmatrix} X_{11} & I \\ I & Y_{11} \end{bmatrix}\leq n+r.$$

Lemma 6 A parameterization of X_c and X_c^{-1} as in (12) with $E_c^{\mathrm{T}}X_c=X_c^{\mathrm{T}}E_c\geq 0$ is possible if and only if

(32) $$\begin{bmatrix} X_1 & I \\ I & Y_1 \end{bmatrix}\geq 0,\quad \begin{matrix} X_1>0 \\ Y_1>0 \end{matrix}$$

hold true.

Proof: From $E_c^{\mathrm{T}}X_c=X_c^{\mathrm{T}}E_c$ we get the parameterization (19). Due to $rank(E_c^{\mathrm{T}}X_c)=r_p+r_k$, $E_c^{\mathrm{T}}X_c\geq 0$ is equivalent to $\begin{bmatrix} X_1 & N_1 \\ N_1^{\mathrm{T}} & L_1 \end{bmatrix}>0$ The parameterization (19) furthermore implies $\begin{bmatrix} X_1 & N_1 \\ N_1^{\mathrm{T}} & L_1 \end{bmatrix}\begin{bmatrix} Y_1 & M_1 \\ M_1^{\mathrm{T}} & S_1 \end{bmatrix}=I$, i.e. $Y_1>0$. Application of Lemma 5 then renders the inequalities (32). The rank condition in (31) is always fulfilled since we have $n=n_x=r_k=r$.

Theorem 2 Consider a plant (1), the switched dynamic output feedback controller as in (6) and matrices E and E_c as in (15). The robust H_∞ control problem to render the closed loop system (7) stabilization with H_∞ disturbance attenuation level γ has a solution if and only if the linear matrix inequalities (20), (21), (32) have a solution X_1 and Y_1.

Proof: The theorem is a straightforward consequence of Lemma 3, 4, and 6 except one technical detail: in Lemma 4 the decoupled LMIs (20), (21) are derived under the nonlinear coupling condition due to (19). The coupling between X_1, Y_1 is captured by the LMIs from Lemma 6 but for the remaining sub-matrices in (19) the point is open. An analysis of the proof of Lemma 3 shows, that the original inequality conditions due to the generalized bounded real lemma also affects the sub-matrices X_l, Y_l (due to (29) and the corresponding inequality for Y_l). However, the reformulation

(33) $$X_c\Pi_1=\Pi_2,\ \Pi_1=\left[\begin{array}{cc|cc} Y_1 & 0 & I_{r_p} & 0 \\ Y_3 & Y_4 & 0 & I_{n_x-r_p} \\ \hline M_1^{\mathrm{T}} & 0 & 0 & 0 \\ M_7 & M_8 & 0 & 0 \end{array}\right],\quad \Pi_2=\left[\begin{array}{cc|cc} I_{r_p} & 0 & X_1 & 0 \\ 0 & I_{n_x-r_p} & X_3 & X_4 \\ \hline 0 & 0 & N_1^{\mathrm{T}} & 0 \\ 0 & 0 & N_7 & N_8 \end{array}\right]$$

of (19) shows, that any restriction of X_l, Y_l does not affects the existence of a matrix X_c such that (19) or (33) holds true: If $X_j, Y_j, j\in\{1,3,4\}$ are given, we always can choose the matrices $M_k, N_k, k\in\{1,7,8\}$ such that Π_1,Π_2 and therefore X_c are non-singular, i.e. such that (19) holds true.

4. Controller algorithm

In this section, switched dynamic output feedback robust H_∞ controller design consists of the following steps:

a) Solution of the LMIs (20), (21) and (32) in Theorem 2 for X_1, Y_1.

b) Parameterization of the LMIs (24), (25) with X_1, Y_1 from a) and solution for X_l, Y_l.

c) The matrices $M_k, N_k, k \in \{1, 7, 8\}$ in (33) must be chosen such that Π_1, Π_2 are non-singular. The matrix X_c then can be computed as $X_c = \Pi_2 \Pi_1^{-1}$.

d) The scalars $\alpha_i \geq 0$ with $\sum_{i=1}^{N} \alpha_i = 1 (\forall i \in \bar{\mathbb{N}})$ can be chosen randomly. Substituting X_c from c) into (18) and solution for the switched dynamic output feedback robust H_∞ controller gain matrix K by efficient numerical methods, such as LMI box in Matlab.

5. Numerical example

Consider the uncertain switched singular linear system (1) with $N = 2$, $\sigma(t):[0,\infty) \to \{1,2\}$ and parameters as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & -4 \\ -1 & -100 \end{bmatrix}, A_2 = \begin{bmatrix} -100 & 2 \\ 2 & 1 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.7 \\ 1.5 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} -4 \\ 0.2 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, B_{22} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}^{\mathrm{T}}, C_{11} = C_{21} = \begin{bmatrix} -1 & 2 \end{bmatrix},$$

$$C_{12} = C_{22} = \begin{bmatrix} 2 & 1 \end{bmatrix}, D_1 = 2, D_2 = 1, G_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}^{\mathrm{T}}, G_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}^{\mathrm{T}},$$

$$F_{11} = \begin{bmatrix} 0.8 & 0 \end{bmatrix}, F_{12} = \begin{bmatrix} 1 & -1 \end{bmatrix}, F_{21} = 0.2, F_{22} = 0.3,$$

$$\Sigma_1(t) = \sin(t), \Sigma_2(t) = \cos(t)$$

Setting $\alpha_1 = 0.6, \alpha_2 = 0.4, \varepsilon = 1$, $\gamma = 1$. We choose

$$\xi(0) = \begin{bmatrix} 3 & -1 & 2 & 1 \end{bmatrix}^{\mathrm{T}}$$

By Theorem 2, using Matlab LMI Control Toolbox to solve the LMIs (20), (21) and (32), we obtain the solution as follows:

$$X_1 = \begin{bmatrix} 2.4952 & -0.0805 \\ -0.0805 & 2.0062 \end{bmatrix}, Y_1 = \begin{bmatrix} 2.8204 & -0.0030 \\ -0.0030 & 2.0087 \end{bmatrix}.$$

Therefore, by formula (18), two gain matrices can be obtained as

$$K_1 = \left[\begin{array}{c|cc} -0.4891 & 0.0251 & 0.0209 \\ \hline -7.8482 & -96.8061 & -19.0481 \\ -3.1077 & 23.2422 & -79.6014 \end{array}\right], K_2 = \left[\begin{array}{c|cc} -0.9600 & 0.1247 & -0.1540 \\ \hline -27.6270 & -4.3455 & -18.0171 \\ 25.8217 & 1.3917 & -71.8225 \end{array}\right].$$

6. Conclusion

The problem of dynamic output feedback robust H_∞ control for uncertain switched singular linear systems with parameter uncertainties has been studied. The sufficient conditions for stabilization with are presented in terms of a series of strict LMIs. The proposed switching rule and the switched dynamic output feedback controllers guarantee that the closed-loop system is asymptotically stable with H_∞ disturbance attenuation level γ.

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