

# Modeling of Thermal effects on Magnetic Hysteresis using the Jiles-Atherton Model

**Abstract.** The present paper deals with a temperature dependent modelling approach for the generation of hysteresis loops of ferromagnetic materials. The physical model is developed to study the effect of temperature on the magnetic hysteresis loop using JA model. The thermal effects were incorporated through temperature dependent hysteresis parameters of JA model. The temperature-dependent JA model was validated against measurements made on the ferrite material and the results of proposed model were in good agreement.

**Streszczenie.** Zaprezentowano metodę modelowania pętli histerezy z uwzględnieniem wpływu temperatury. Do tego celu wykorzystano model Jiles-Atherton włączając do modelu parametry zależne od temperatury. Model sprawdzono na materiałach ferrytowych. **(Modelowanie wpływu temperatury na pętlę histerezy przy wykorzystaniu modelu Jiles-Atherton)**

**Keywords:** Jiles-Atherton model, Magnetic hysteresis, Temperature, Modeling.

**Słowa kluczowe:** Model Jiles-Atherton, pętla histerezy.

## 1. Introduction

Magnetic hysteresis is encountered in the operation of most Electrical Engineering devices (motors, transformers, magnetic recording, and components for power electronics). The development of a model can accurately describe this cycle and its variation depending on the operating regime that is still an issue that inspires researchers.

Amongst these models, we find the Jiles-Atherton model which is based on energy considerations, that is on the movement of Bloch walls within the material. Its inconvenience lies in the identification of parameters and especially in case of temperature change. All machines are electric seat Joule losses, hysteresis and eddy currents, making these devices heat up. For instance, the temperature can be very high in case of heating induction furthermore, the integration of the temperature in the hysteresis model is imperative for a reliable estimate of losses related to this phenomenon.

Taking into account the variation of the five parameters model depending on the temperature during the integration of this model in a computer code by finite element field is a formidable task, especially in the study of magnetic electromagnetic device.

In this paper, we propose the introduction of temperature effect for the generation of hysteresis loops in the Jiles-Atherton model.

In this paper, we adopt a different approach of [1]. We introduced the two thermal behavior of the spontaneous magnetization  $M_s$  and coercive  $H_c$  in the parameter  $k$  of the Jiles-Atherton model.

## 2. The Jiles Atherton model

According to the Jiles–Atherton (JA) model [2], the total magnetization of a ferromagnetic material can be represented as the sum of contributions of irreversible,  $M_{irr}$ , and reversible,  $M_{rev}$ , magnetization components [2]:

$$(1) \quad M = M_{rev} + M_{irr}$$

The reversible component is the combination of the reversible translation and rotation of the walls in ferromagnetic materials. But the irreversible component represents the irreversible displacement of the magnetic domains. The differential equation describing the dependence of magnetization on magnetic field can be constructed as

$$(2) \quad M = M_{irr} + c(M_{an} - M_{irr})$$

$$(3) \quad M = M_{an} - k\delta \frac{dM_{irr}}{dH_e}$$

where  $M_{an}$  is the anhysteretic curve, which follows the Langevin function

$$(4) \quad M_{an} = M_s \left( \coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right)$$

where  $H_e = H + \alpha M$  is the effective field taking into account the domain interactions,  $\alpha$  is the molecular field parameter,  $k$  parameter is linked to the coercitive field,  $c$  Reversibility coefficient and  $\delta$  parameter is  $\delta = +1$  when  $\frac{dH}{dt} > 0$  and  $\delta = -1$  when  $\frac{dH}{dt} < 0$  [3].

The differential susceptibility according to the JA model can be expressed as follows [4].

$$(5) \quad \frac{dM}{dH} = \frac{(1-c) \frac{dM_{irr}}{dH_e} + c \frac{dM_{an}}{dH_e}}{1 - \alpha c \frac{dM_{an}}{dH_e} - \alpha(1-c) \frac{dM_{irr}}{dH_e}}$$

## 3. Temperature Dependence of JA

Thermal effects can be incorporated into the model through the temperature dependence of hysteresis parameters in (5): spontaneous magnetization  $M_s$ , parameter  $\alpha$ , reversibility factor  $c$  and parameter  $k$ .

### a. Spontaneous magnetization $M_s$

The temperature dependence of spontaneous magnetization  $M_s$  can be expressed using Weiss theory of ferromagnetism [5].

$$(6) \quad M_s(T) = M_s^{Ta} \left( 1 - \exp\left(-\frac{T - T_c}{\tau_{M_s}}\right) \right)$$

Where  $M_s^{Ta}$  is the value of spontaneous magnetization at room temperature,  $T_c$  is the Curie temperature, and  $\tau_{M_s}$  is the constant defined on the experimental curve of spontaneous magnetization with temperature.

### b. Parameter $a$

The parameter  $a$ , is treated as a constant in this model because its variation with temperature is negligible [6].

### c. Parameter $\alpha$

The parameter  $\alpha$ , can be expressed in an isotropic material as [7]

$$(7) \quad \alpha = \frac{3a}{M_s} - \frac{1}{\chi_{an}}$$

Anhyseretic susceptibility  $\chi_{an}$  is generally very high, so the second term of the expression (7) is negligible compared to the first term of (7), and therefore the expression of the  $\alpha$  parameter became

$$(8) \quad \alpha = \frac{3a}{M_s}$$

$$(9) \quad \alpha(T) = \frac{3a}{M_s^{Ta}} \frac{1}{(1 - \exp(-\frac{T-T_c}{\tau_{M_s}}))}$$

$$(10) \quad \alpha(T) = \alpha(T^{Ta}) \frac{1}{(1 - \exp(-\frac{T-T_c}{\tau_{M_s}}))}$$

where  $\alpha^{Ta}$  is the value of parameter  $\alpha$  at room temperature [8].

c. Parameter c

The factor c, is expressed as [7]

$$(11) \quad c = \frac{3a\chi_{in}}{M_s}$$

Assuming constant initial susceptibility  $\chi_{in}$  and substituting the expression for  $M_s$  from (6), we get

$$(12) \quad c(T) = \frac{3a}{M_s^{Ta}} \chi_{in} \frac{1}{(1 - \exp(-\frac{T-T_c}{\tau_{M_s}}))}$$

$$(13) \quad c(T) = c^{Ta} \frac{1}{(1 - \exp(-\frac{T-T_c}{\tau_{M_s}}))}$$

where  $c^{Ta}$  is the value of parameter  $c$  at room temperature.

e. Parameter k

The parameter k, can be expressed in an isotropic material as [7].

$$(14) \quad k = \frac{M_{an}(H_c)}{(1-c)} \left[ \alpha + \frac{1-c}{x_c - c \frac{dM_{an}(H_c)}{dH}} \right]$$

$$(15) \quad M_{an}(H_c) = M_s \left( \coth\left(\frac{H_c}{a}\right) - \frac{a}{H_c} \right)$$

$$(16) \quad \frac{dM_{an}(H_c)}{dH} = \left[ \frac{M_s}{a} - \frac{M_{an}(H_c)}{a} \left( \frac{M_{an}(H_c)}{M_s} + \frac{2a}{H_c} \right) \right] (1 + \alpha\chi_c)$$

where  $H_c$  is the coercive field and  $\chi_c$  is the susceptibility. The coercive field is a value strongly depends on the temperature [9]. Due to the exponential decay of coercive field in a ferromagnetic material [10], the coercive field should vary exponentially with temperature according to the equation

$$(17) \quad H_c(T) = H_c^{Ta} \exp\left(-\frac{T}{\tau_{H_c} T_c}\right)$$

where  $H_c^{Ta}$  is the value of coercive field at room temperature and  $\tau_{H_c}$  is the constant defined on the

experimental curve of coercive field with temperature.

Assuming constant coercive susceptibility  $\chi_c$  and substituting the expression for  $H_c$  from (17), we get

$$(18) \quad M_{an}(T) = M_s(T) \left( \coth\left(\frac{H_c(T)}{a}\right) - \frac{a}{H_c(T)} \right)$$

$$(19) \quad \frac{dM_{an}(T)}{dH} = \left[ \frac{M_s(T)}{a} - \frac{M_{an}(T)}{a} \left( \frac{M_{an}(T)}{M_s(T)} + \frac{2a}{H_c(T)} \right) \right] (1 + \alpha(T)\chi_c)$$

The substitution of equations (6), (10), (13), (17), (18) and (19) in (14), gives the following expression

$$(20) \quad k(T) = \frac{M_{an}(T)}{(1-c(T))} \left[ \alpha(T) + \frac{1-c(T)}{x_c - c(T) \frac{dM_{an}(T)}{dH}} \right]$$

### 3.1. Parameter identification

The hysteresis parameters governing the magnetization processes  $M_s$ ,  $k$ ,  $a$ ,  $c$  and  $\alpha$  can be identified from the magnetic properties such as initial susceptibility  $\chi_{in}$ , anhyseretic susceptibility  $\chi_{an}$ , coercivity  $H_c$ , and remanence  $M_r$ . The identification procedures were calculated using an established procedure [11-12] which is sufficient for modelling purposes. The measured spontaneous magnetization  $M_s$  and coercive field as a function of temperature were used to estimate the  $\tau_{M_s}$  and  $\tau_{H_c}$ , by fitting the analytical model shown in (6) and (17). This procedure will to describe hysteretic behaviour at any temperature up to Curie point.

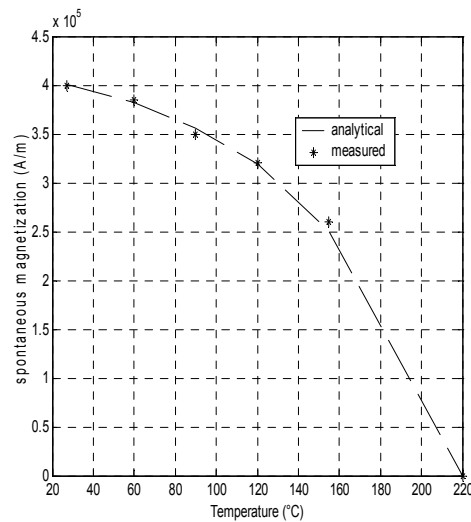


Fig.1. Variation of  $M_s$  with temperature

After identifying the necessary parameters to generate the hysteresis loop according to the temperature  $M_s^{Ta}$ ,  $\alpha^{Ta}$ ,  $c^{Ta}$ ,  $H_c^{Ta}$  and  $a$ , the evolution curve of  $M_s$  according to the temperature (fig 1) was used to identify the constant  $\tau_{M_s}$ . The increase in temperature decreases the spontaneous magnetization, first slowly then more rapidly as one approaches the Curie point (Fig. 1). The parameter  $\alpha$  and the parameter  $c$  increase monotonically with temperature, as shown in Fig. 3 and Fig.4 respectively.

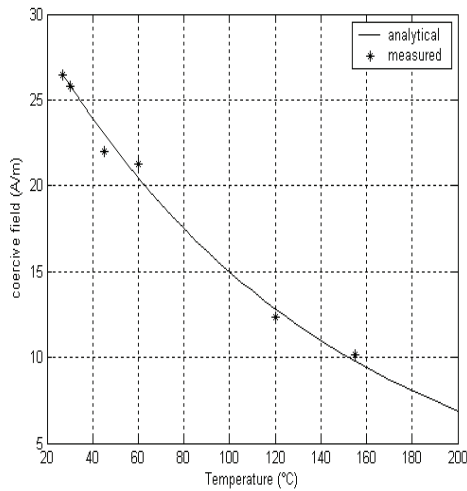


Fig. 2. Variation of  $H_c$  with temperature

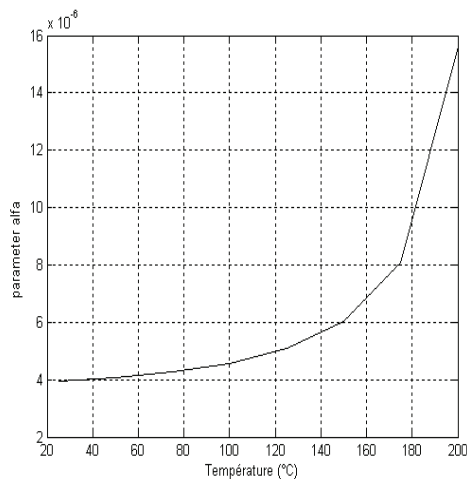


Fig. 3. Calculated temperature dependence of parameter  $\alpha$

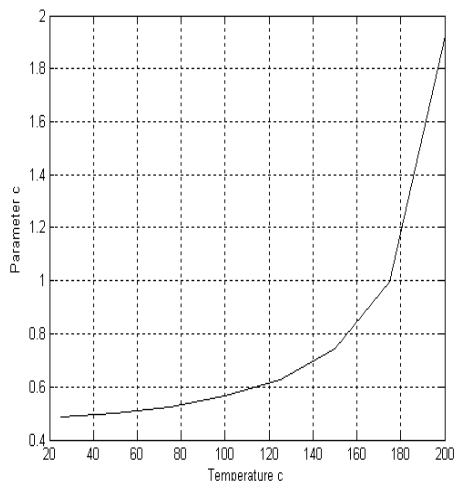


Fig 4. Calculated temperature dependence of parameter  $c$

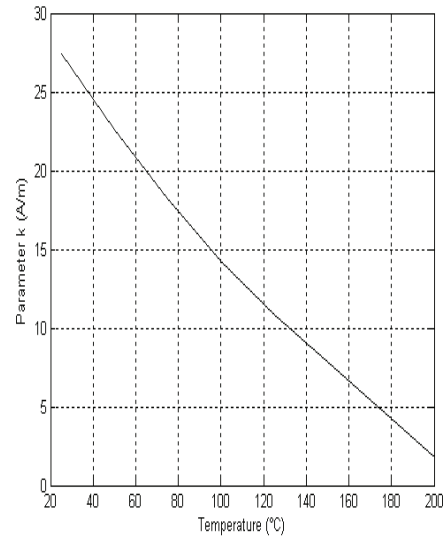


Fig. 5. Calculated temperature dependence of parameter  $k$

The evolution curve of the coercive field  $H_c$  according to the temperature was used to identify  $\tau_{H_c}$  constant. The coercive field decreases exponentially with temperature as shown in Fig 2. The two laws of thermal behavior of  $M_s$  and  $H_c$  were introduced in the expression of the parameter  $k$ . The variation of this parameter with temperature is shown in Figure 5.

#### 4. Comparison with experiments

Fig. 6 illustrates the experimental setup for measuring magnetic hysteresis loops at different temperatures using ring sample. The sample is a torus with primary and secondary windings. Excitation is applied using an arbitrary function generator which allows us to impose current or voltage on the primary winding. A computer remotely controls these devices. Using the Ampere and Faraday laws, the field  $H$  and the magnetic flux density  $B$  are calculated from the measurement of the current in the primary winding and the secondary winding voltage.

The sample under test was placed in the middle of an environment chamber, which could maintain a constant temperature over a wide range from 25 °C to 160 °C.

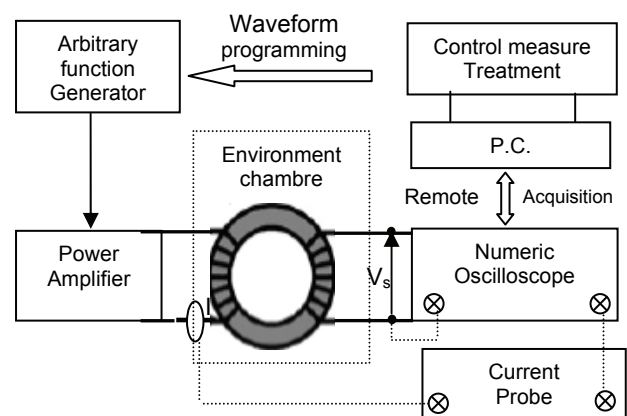


Fig. 6. Experimental setup for measuring magnetic hysteresis loops

The temperature dependent JA model was validated versus experimental data of ferrite 3F3 material with Curie point at 220°C. The hysteresis loops were measured using the experimental setup of Fig. 6 at various temperatures ranging from 27 °C to 155 °C and compared to those calculated numerically.

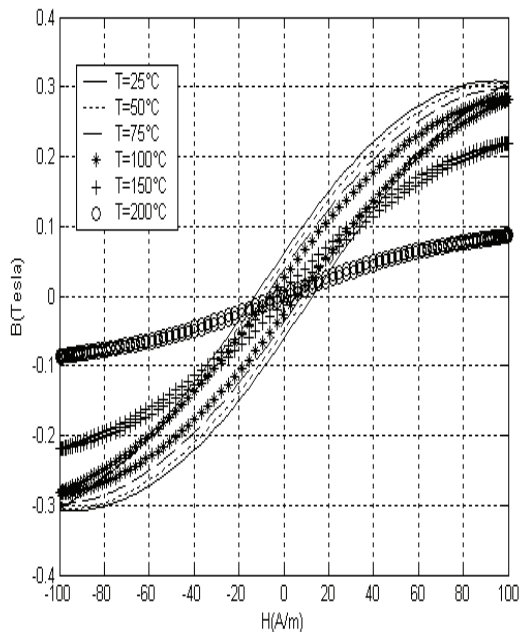


Fig7. Calculated temperature dependence of magnetic hysteresis loops in a 3F3 material

The measured and calculated hysteresis loops of 3F3 material at temperatures 27 °C, 60 °C, and 155 °C are compared in Fig. 8. The hysteresis loop gradually flattens as the temperature approaches the Curie point. After the Curie point, the material becomes paramagnetic and does not exhibit hysteresis. The calculated hysteresis loops start to deviate from the measured loops at higher temperatures. This behaviour could be due to the assumption that the  $\chi_c$  susceptibility is independent of temperature in the used model.

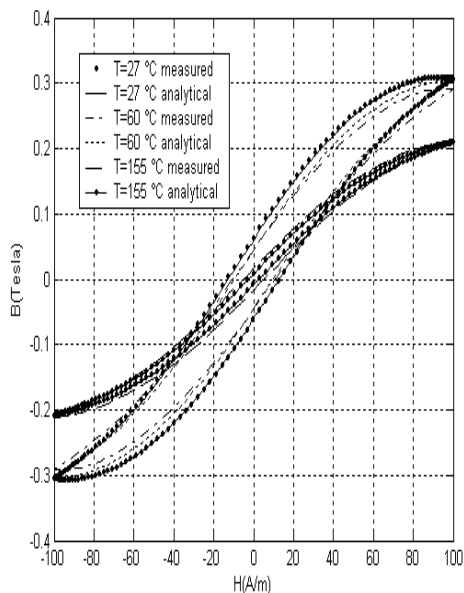


Fig.8. Variation of magnetic hysteresis loops with temperature in a 3F3 material with Curie point at  $T_c=220$  °C.

## Conclusion

In this work, a JA based model of hysteresis depending on temperature has been developed. Thermal effects have been introduced in the model of JA through two important values which enter into the construction of the magnetic hysteresis loop: spontaneous magnetization  $M_s$  and coercive  $H_c$ , the evolution curve of MS according to temperature was used to calculate the parameters  $\alpha$  and  $c$  as a function of temperature, the evolution of  $M_s$  and  $H_c$  as a function of temperature was used to calculate the thermal behavior of the parameter  $k$ . The identification procedure for the proposed model is to calculate the model parameters at room temperature, and to determine the two constants,  $\tau_{M_s}$  and  $\tau_{H_c}$ , from the evolution of  $M_s$  and  $H_c$  as a function of temperature. The model can be further improved if an analytical model of temperature dependence of the coercive susceptibility  $\chi_c$  is implemented.

The temperature-dependent JA model was validated versus measurements made on ferrite (3F3). The comparison showed that the temperature-dependent Jiles Atherton model was in good agreement with the measurements.

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Abdelaziz Ladjimi, département de génie électrotechnique et Automatique, université de Guelma, Algérie.