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Discrete Sliding Mode Control for DC-DC Converters with Uncertainties

Abstract*. A discrete-time sliding mode controller is presented for DC-DC converters. A criterion for the existence of the sliding surface is derived based on the LMI technique combined with the Lyapunov method. In addition, the design of sliding mode control is presented according to the reachability condition. The control law can effectively constrain the input disturbance and reduce the effects from the parameter variations while the bounds of the uncertainties are unknown. Finally, simulation results show the feasibility and effectiveness of the proposed method*

Streszczenie. *Zaprezentowano kontroler ślizgowy o czasie dyskretnym do przekształtników DC-DC. Kryterium obecności powierzchni ślizgowej jest wyprowadzone bazując na technice LMI i metodzie Lapunowa. (**Dyskretne sterowanie ślizgowe dla przekształtników DC-DC z niepewnościami**).*
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Keywords: discrete-time system, sliding mode control, DC-DC converter, uncertainties
Słowa kluczowe: systemy czasu dyskretnego, sterowanie ślizgowe

Introduction

In the past decades, the study of DC-DC converters has attracted much attention due to their extensive applications such as power electronics, automatic control and so on [1]. A lot of research achievements related to DC-DC converters have been reported in [2, 3] e.g., for continuous-time cases, and [4,5] for discrete-time ones. Different control algorithms such as many linear control methods are used to make DC-DC converters obtain a robust output voltage [6,7]. However, DC-DC converters are nonlinear and time variant systems, these linear control techniques are too suitable to obtain a good dynamic response [8]. Since DC-DC converters constitute a case of variable structure systems, the sliding mode control (SMC) technique can be a possible option to control this kind of objects [9]. The notable advantage of SMC is its intrinsic nature of robustness against uncertainties and external disturbances, which make it become an effective control strategy for DC-DC converters [10]. Up to now, most of the research results are about continuous-time sliding mode control.

In fact, the research of discrete sliding mode control (DSMC) has recently stirred considerable interests because the practical realization of the sliding mode controller is often done by computers and microcontrollers [11,12]. To the authors' best knowledge, little works on discrete sliding mode control theory for DC-DC converters have been available in the literature so far. In [13], a discrete sliding mode controller was presented for DC-DC converters, however, the nominal systems were studied while uncertainties were not considered. In [14], discrete global sliding mode control for buck converter with input disturbances were discussed. In practical systems, parameters uncertainties and input disturbances often exist. Motivated by this, in this paper, we propose the sliding mode control of discrete-time linear systems with unmatched uncertainties. First, the sliding surface is designed by using LMI technique when the unmatched uncertainties exist. The sufficient condition for the existence of stable sliding surface is derived in terms of LMIS. Moreover the design of sliding mode control is presented also. Finally, illustrative examples are presented to show the feasibility and effectiveness of the proposed method.

System model

The circuit principle diagram of DC-DC converter is shown in Fig.1, consisting of one switch, a fast diode and RLC components.

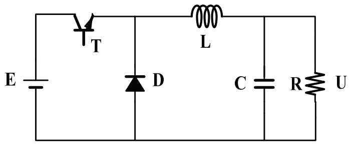

Fig. 1. Circuit principle diagram of DC-DC converter

Let $x_1(t) = Uref - U$, $x_2(t) = \dot{x}_1(t)$, the state equation of the DC-DC converter can be described as

(1) $\dot{x}(t) = \bar{\bar{A}}x(t) + \bar{\bar{B}}u(t)$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad \bar{\bar{A}} = \begin{bmatrix} 0 & 1 \\ -1/LC & -1/CR \end{bmatrix},$$

$$\bar{\bar{B}} = \begin{bmatrix} 0 \\ -E/LC \end{bmatrix}.$$

Due to the widespread use of computers for control purpose, the study of discrete-time system is more significant.

Adopting zero order hold device, we can get discrete-time system as follows:

(2) $x(k+1) = Gx(k) + Hu(k)$

where $G = e^{AT}$, $H = (\int_0^T e^{At} dt) \times B$ and T is sampling time.

Merely out of custom, we replace matrix G with matrix $\bar{A}$ and replace matrix H with matrix $\bar{B}$, then (2) can be rewritten as:

(3) $x(k+1) = \bar{A}x(k) + \bar{B}u(k)$

In the working process, uncertainties cannot be avoided, so it is rational to take into account the uncertainties. So the discrete-time system model can be amended as

(4) $x(k+1) = (\bar{A} + \Delta\bar{A})x(k) + (\bar{B} + \Delta\bar{B})u(k)$

Assumption 1: $\Delta\bar{A}$ is unmatched uncertainty and $\Delta\bar{B}$ has the structure satisfies the following form $\Delta\bar{B} = \bar{B} \times \Delta\tilde{B}$.

For analyzing the system (4), we will transform it to its regular form. The system is controllable and $\bar{B} = [B_1, B_2]^{\mathrm{T}}$ satisfies $\det(B_2) \neq 0$, so there exists a linear nonsingular transformation:

(5) $$z = Mx = \begin{bmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix} x$$

which transforms the system dynamics (4) to its regular form

(6) $$z(k+1) = (A+\Delta A)z(k) + (B+\Delta B)u(k)$$

where

$$z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \qquad A = M\overline{A}M^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$B = M\overline{B} = \begin{bmatrix} 0 \\ I_m \end{bmatrix}, \qquad \Delta B = M\Delta\overline{B},$$

$$\Delta A = M\Delta\overline{A}M^{-1} = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix}, \qquad C = \overline{C}M^{-1}.$$

That is

(7) $$z_1(k+1) = (A_{11}+\Delta A_{11})z_1(k) + (A_{12}+\Delta A_{12})z_2(k)$$

(8) $$\begin{aligned} z_2(k+1) = {} & (A_{21}+\Delta A_{21})z_1(k) + (A_{22}+\Delta A_{22})z_2(k) \\ & + B(I+\Delta\tilde{B})u(k) \end{aligned}$$

Design of sliding mode for discrete-time system

Traditional design methods of sliding surface include pole placement method and LQR method etc. These methods design sliding surface based on nominal system, which do not possess ideal robustness when system includes unmatched uncertainties. Some literatures have been developed to deal with the problem of designing stable sliding surface for continuous-time systems with unmatched uncertainties. But almost all of previous works discuss continuous-time systems. In this section we consider the design of discrete-time sliding mode. For a class of linear systems with unmatched uncertainties a discrete-time sliding mode is designed and the sliding mode parameters are solved by means of LMI technique in order to improve system robustness.

The sliding surface is usually defined as following

(9) $$s(k) = Cz(k) = \begin{bmatrix} -K & I_m \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} = 0$$

where $C \in R^{m\times n}, K \in R^{m\times(n-m)}$.

In sliding mode, $s(k) = Cz(k) = 0$. Now we can easily get the following equation:

(10) $$z_2(k) = Kz_1(k)$$

Substituting (10) into (7) yields:

(11) $$z_1(k+1) = (A_{11} + \Delta A_{11} + A_{12}K + \Delta A_{12}K)z_1(k)$$

Supposing uncertainties ΔA_{11} and ΔA_{12} are admissibly norm-bounded and structure as following:

Assumption 2:

(12) $$\Delta A_{11} = DF(k)E_1, \Delta A_{12} = DF(k)E_2$$

where D, E_1 and E_2 are known real constant matrices of appropriate dimensions, and $F(k)$ is time-varying matrix and satisfies $F(k)^T F(k) \le I$, where I is the identity matrix.

Substituting (12) into (11) yields:

(13) $$z_1(k+1) = (A_{11} + A_{12}K + DF(E_1 + E_2K))z_1(k)$$

Finding the parameter K which can guarantee system (13) asymptotic stability we can get the sliding mode surface.

For analyzing the asymptotic stability of the sliding mode dynamics (11), we introduce a lemma as follows

Lemma 1[15]: Given constant matrices D, E and $F(k)$ of appropriate dimensions, the following inequality holds $DF(k)E + E^{\mathrm{T}}F^{\mathrm{T}}(k)D^{\mathrm{T}} \le \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E$ where $F(k)$ satisfies $F^T(k)F(k) \le I$ and $\varepsilon > 0$.

The main result on the asymptotic stability of the sliding mode dynamics (11) is summarized in the following theorem.

Theorem1: If there exists a symmetric and positive definite matrix X, some matrix W and some scalar $\varepsilon > 0$ such that the following LMI (14) is satisfied, then the sliding mode dynamics (11) is asymptotically stable

(14) $$\begin{bmatrix} -X & * & * & * \\ A_{11}X + A_{12}W & -X & * & * \\ 0 & \varepsilon D^T & -\varepsilon I & * \\ E_1X + E_2W & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

where $X = P^{-1}$, $W = KX$ and $*$ denotes the transposed elements in the symmetric positions.

Proof: Consider Lyapunov function candidate

(15) $$v(k) = z_1^{\mathrm{T}}(k)Pz_1(k)$$

where P is a positive definite symmetrical matrix. The difference of $v(k)$ is

(16) $$\begin{aligned} \Delta v(k) &= v(k+1) - v(k) \\ &= z_1^{\mathrm{T}}(k+1)Pz_1(k+1) - z_1^{\mathrm{T}}(k)Pz_1(k) \\ &= z_1^{\mathrm{T}}(k)\Phi^{\mathrm{T}}P\Phi z_1(k) - z_1^{\mathrm{T}}(k)Pz_1(k) \\ &= z_1^{\mathrm{T}}(k)(\Phi^{\mathrm{T}}P\Phi - P)z_1(k) \end{aligned}$$

where $\Phi = A_{11} + A_{12}K + DF(E_1 + E_2K)$.

So the sliding mode dynamics (11) is asymptotically stable if the following inequality holds

(17) $$\Phi^{\mathrm{T}}P\Phi - P < 0$$

Applying Schur complement to (17), (17) is equivalent to

(18) $$\begin{bmatrix} -P & \Phi^{\mathrm{T}} \\ \Phi & -P^{-1} \end{bmatrix} < 0$$

That is

(19) $$\Gamma + \overline{D}F\overline{E} + \overline{E}^T F^T \overline{D}^T < 0$$

where $\Gamma = \begin{bmatrix} -P & (A_{11} + A_{12}K)^T \\ A_{11} + A_{12}K & -P^{-1} \end{bmatrix}$, $\overline{D} = \begin{bmatrix} 0 \\ D \end{bmatrix}$, $\overline{E} = \begin{bmatrix} E_1 + E_2 & 0 \end{bmatrix}$.

According to Lemma 1, the matrix inequality (19) if there exists a constant $\varepsilon > 0$ such that the following inequality holds.

(20) $$\Gamma + \varepsilon\overline{D}\overline{D}^T + \varepsilon^{-1}\overline{E}^T\overline{E} < 0$$

That is

$$\Gamma + \begin{bmatrix} 0 & (E_1 + E_2K)^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \varepsilon I & 0 \\ 0 & \varepsilon^{-1}I \end{bmatrix} \begin{bmatrix} 0 & (E_1 + E_2K)^T \\ D & 0 \end{bmatrix}^T < 0$$

(21)

By applying Schur complement to (21), (21) is equivalent to

(22) $$\begin{bmatrix} -P & (A_{11} + A_{12}K)^T & 0 & E_1 + E_2K \\ * & -P^{-1} & D & 0 \\ * & * & -\varepsilon^{-1}I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0$$

Denoting $X = P^{-1}$, $W = KX$ and taking the congruence transformation with $diag(P^{-1}, I, \varepsilon I, I)$ yield (14).

Remark 1: Because (14) is a LMI, it is possible to compute the value of matrix X and W by matlab software, so C can be gained, that is, the asymptotically stable sliding surface $s(k)$ can be gained.

Design of sliding mode controller

In the previous section, when the uncertain system (6) in the sliding mode, the sliding mode surface was designed to guarantee the asymptotic stability of the reduced order system in terms of LMI. Next, we should find output feedback control law to drive state trajectories of the system to arrive at the switch band in limit time and maintain in the switch band. This means that the control law is designed to guarantee system satisfy the reaching condition.

System (6) can be rewritten as follows

$$\begin{aligned} z(k+1) &= (A+\Delta A)z(k)+(B+\Delta B)u(k) \\ &= Az(k)+Bu(k)+\varphi(k) \end{aligned} \tag{23}$$

where

$$\varphi(k)=\Delta Az(k)+\Delta Bu(k)=z(k+1)-Az(k)-Bu(k) \tag{24}$$

Choose the discrete-time approximate law:

$$s(k+1)-s(k)=-\varepsilon T\operatorname{sgn}s(k)-qTs(k) \tag{25}$$

Substituting (9) and (23) into (25), we can get the control law:

$$u(k)=-(CB)^{-1}\begin{bmatrix} CAz(k)+C\varphi(k) \\ -(1-qT)s(k)+\varepsilon T\operatorname{sgn}s(k) \end{bmatrix} \tag{26}$$

Because the uncertain items are contained in (26), so this control law can not be realized in practice. But $\varphi(k-1)$ can be computed by the following equation.

$$\varphi(k-1)=z(k)-Az(k-1)-Bu(k-1) \tag{27}$$

Assumption 3: The motion character of the uncertain items is slower than the sampling frequency. That means $\varphi(k)$ can be replaced by $\varphi(k-1)$, and then the control law is considered:

$$u(k)=u(k-1)-(CB)^{-1}\begin{bmatrix} CAz(k)-CAz(k-1) \\ +qTs(k)+\varepsilon T\operatorname{sgn}s(k) \end{bmatrix} \tag{28}$$

Simulation study

In this section, we validate the effectiveness and performance of the proposed discrete-time variable sliding mode controller by some numerical simulations.

We choose the parameters of DC-DC converter as follows:

$E=12V$, $Uref=5V$, $L=44\mu H$, $C=220\mu F$, $T=0.001$, $R=30\Omega$.

So we can calculate

$$A=\begin{bmatrix} 1 & -38.76 \\ 0.083 & -2.37 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\Delta A=\begin{bmatrix} \sin(k/8) & 3.88*\sin(k/8) \\ 0 & 1.1289 \end{bmatrix},\ \Delta B=\begin{bmatrix} -0.2 \\ 0 \end{bmatrix}.$$

We choose the other parameters of converter as follows:

$D=1$, $F=\sin(k/8)$, $E_1=1$, $E_2=3.88$.

By the means of LMI toolbox we can have $C=[-0.0256 \quad 1]$.

Taking the initial value $x(0)=[0 \quad 0]^T$, $q=10$, $\varepsilon=0.5$, $T=0.001$, the simulation results are given in the following figures.

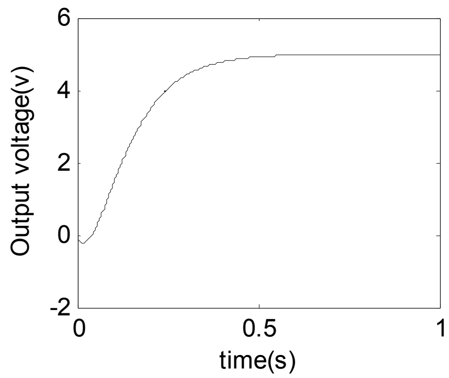

Fig. 2. Output voltage waveform with DSMC

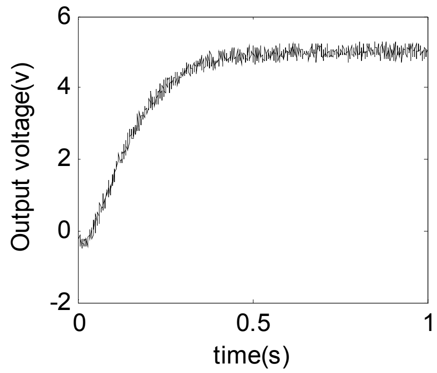

Fig. 3. Output voltage waveform with PI control

Simulation results of converter based control with DSMC and PI control are given in Fig. 2 and Fig. 3 respectively. It can be seen that the DSMC proposed has stronger stability and robustness than PI control.

Conclusion

A discrete-time variable sliding mode controller has been proposed to overcome the influence which unmatched uncertainties bring about to system, and assure DC-DC converter have good robustness. The output can track the reference well, and the uncertainties almost do not affect the output.

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