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Degenerate Chebyshev Approximation Of The Recursive Digital Filter Group Delay Response

Abstract. *Multiple-pole transfer functions approximating a constant group delay in a degenerate minimax sense is presented in this paper. The introduction of constraints at the origin and at extremes of the group delay near the cutoff frequency, has improved passband and stopband performances significantly, while by using the multiple pole transfer function, the less sensitivity to the filter coefficients change is achieved simultaneously.*

Streszczenie. *Zaprezentowano wielobiegunową funkcję transferu uwzględniającą grupowe opóźnienie w filtrach cyfrowych. Wprowadzenie wymuszenia znacznie poprawia parametry zarówno w paémie przenoszenia jak i zaporowym. Uzyskano też mniejszą wrażliwość na zmianę parametrów filtru. (**Zmodyfikowana aproksymacja Czebyszewa uwzględniającą opóźnienie w filtrach cyfrowych**)*

Keywords: Chebyshev approximation, Group delay response, Mutation of zero and pole location, Multiple pole transfer function, Coefficient rounding, Filter group delay sensitivity.
Słowa kluczowe: filtr Czebyszewa, opóźnienie, filtry cyfrowe.

Introduction

Transfer functions of low-pass recursive digital filters with equiripple (Chebyshev) approximation of the constant group delay, has been described in papers [1] and [2]. These functions are over-redundant, in the sense that the group delay bandwidth is needlessly considerably wider then θ_{3dB} filter passband. This redundancy can be minimized by combining the technique of increasing multiplicity of the real pole and the nearest pole to the unit circle, using the special type of the equal-ripple (or constrained Chebyshev) delay approximation, as proposed in [3] and [4]. As a benefit, the stopband magnitude performance is improved, but the redundancy still exists and the group delay bandwidth exceeds 20% of θ_{3dB} the filter passband. This redundancy can be exploited by introducing constraint at extremes of the group delay near cutoff frequency. As a additional benefit both the stopband and the passband characteristics are improved. This paper reports results obtained using this technique, which can be classified as the z-domain technique.

Approximation Technique

The all-pole transfer function of the recursive digital filter can be written [5], [6] as follows:

$$F_n(x,z) = \frac{h_0 z^n}{(z-r_0)^{\mu} A}$$

where:

(1) $$A = \prod_{i=1}^{m-1}(z - r_i e^{\pm j\varphi_i}) \prod_{i=m}^{m+1}(z - r_i e^{\pm j\varphi_i})^v$$

where: $n = \mu + 2\nu + m - 1$ is the denominator degree (m is an odd number since poles appear in a conjugated pairs; in this case $m = 3$), h_0 is a real constant selected so that to fulfill conditions: $|F_n(x,1)|$=1, $x = [r_0, r_1, \varphi_1, ..., r_{m-1}, \varphi_{m-1}, r_m, \varphi_m]^T$ is a parameter vector. Standard curves are obtained for $\mu = 0$, [2], and they are denoted by a in the Figures enclosed below. In our method, for n odd, $\mu = 1$, otherwise $\mu = 2$. Multiple pole transfer functions are obtained by selecting $\nu > 1$. The group delay constraint at the origin represents the decreasing of the group delay error at the origin $\varepsilon(0)$ towards the lower limit value $-\varepsilon_m$, while the constraint in the stopband represents the group delay error decreasing towards the $\varepsilon_{mi} = 0.001\varepsilon_m$.

The group delay of the function (1) at frequencies $z = e^{j\theta}$ is:
(2)
$$\tau_n(x,\theta) = -n + \mu\frac{1-r_0\cos\theta}{1-2r_0\cos\theta + r_0^2}$$
$$+ \sum_{i=1}^{m-1}\frac{1-\cos(\theta-\varphi_i)}{1-2r_i\cos(\theta-\varphi_i)+r_i^2} + \sum_{i=m}^{m+1}\frac{1-\cos(\theta-\varphi_i)}{1-2r_i\cos(\theta-\varphi_i)+r_i^2}$$

where: $\theta = \omega T$, T is the sampling period, $r_{2i} = r_{2i-1}$ and $\varphi_{2i} = -\varphi_{2i-1}$ for $i > 1$. Unknown elements of the coefficient vector x are determined by solving the following system of nonlinear equations:
(3)
$$\begin{aligned}
&\tau_n(x,\theta) - \tau_0 = \varepsilon(0)\tau_0 \\
&\tau_n(x,\theta_i) - \tau_0 = (-1)^{i+1}\varepsilon_m\tau_0, \quad i = 1,2...,n \\
&\tau_n(x,\theta_i) - \tau_0 = (-1)^{i+1}\varepsilon_{mi}\tau_0, \quad i = n+1,...,m+2 \\
&\tau_n(x,\theta_i) - \tau_0 = \varepsilon_m\tau_0
\end{aligned}$$

where $\theta_i (i = 1,2,...,m+2)$ are extreme or critical points, obtained for maximum relative delay errors ε_m and ε_{m_i}. $\varepsilon(0)$ can vary in the range $-\varepsilon_m < \varepsilon(0) < \varepsilon_m$, and it influences over the passband and the stopband, while ε_{m_i} can vary between $-\varepsilon_m$ and ε_m in the passband and $k\varepsilon_m$ for extremes in the stopband. There is an additional condition that must be satisfied: $\tau_n(x,\theta_c) - \tau_0 = \varepsilon_m\tau_0$, because the group delay characteristic, behind and near θ_{3dB}, looses equiripple minimax properties. The desired bandwidth can be obtained by the variation of the parameter τ_0, and the desired shape of the group delay characteristic near θ_{3dB} can be modified by error variation ε of the first extreme before θ_{3dB} and the first extreme after $\theta_{3dB}(\varepsilon_{im})$. Initial solutions can be found applying procedure described by Deczky [2] or Stojanovic and Micic [3].

Discussion of results

Fig.1 shows the amplitude characteristic of eight order transfer functions. For $\varepsilon_m = 15\%$, $\mu = 0$, $\nu = 1$, $m = 7$, $k = 1$, $\varepsilon_{mi} = \varepsilon_m$, four simple conjugated complex pairs of poles are obtained (standard curve a); for $\varepsilon_m = 15\%$, $\mu = 2$, $\nu = 3$, $m = 1$, $k = 1$, $\varepsilon_{mi} = 0.001\varepsilon_m$ a double real pole and a triple conjugated complex pairs of pole are obtained (curve b). Both curves are normalized so that $\theta_{3dB}(\varepsilon_{im}) = 0.2\pi$ and the constraint at the origin is $\varepsilon(0) = -0.9\varepsilon_m$. Fig. 1 shows that the stopband attenuation of the amplitude characteristic of the transfer function [3] (curve b), compared with the amplitude characteristic of the classical transfer function [2] (curve a) at frequency $\theta = 0.4\pi$, is greater for approximately 13dB. For $\varepsilon_m = 15\%$, $\mu = 2$, $\nu = 2$, $m = 3$, $k = 5.6$, $\varepsilon_{mi} = 0.001\varepsilon_m$, a double real pole, a simple and a double conjugated complex pairs of poles are obtained (curve c). It can be seen that the amplitude characteristic of the proposed transfer function c (compared with the standard curve a), is approximately 20 dB greater, on the same frequency $\theta = 0.4\pi$.

Fig.2 shows magnified details of the passband part, representing the critical frequency range in which presented curves significantly differ (after this range, curves are almost identical!). It is obvious, also, that curve marked by c is flatter then the previous, marked by a and b respectively, pointing that the curve c have better characteristics in the passband.

Fig.3 shows corresponding group delay errors. In the case a, redundancy is obvious [2]. Using the technique [3], (case b), this redundancy is significantly reduced, but still exists.

Case c shows that this redundancy still exist, but there is a certain modeling of the group delay characteristic behind θ_{3dB} , in a way that relative group delay error do not exceeds 15% at the normalized cutoff frequency θ_{3dB}.

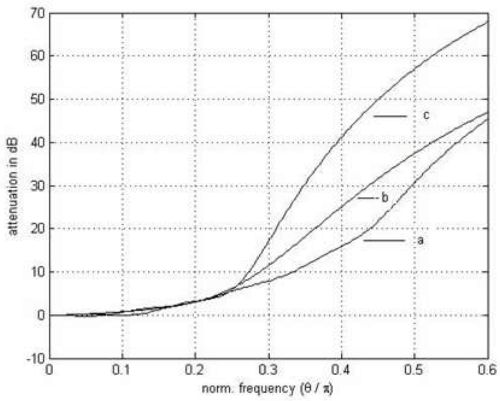

Fig. 1. Amplitude characteristics for the eight order filter (group delay error $\varepsilon_m = 15\%$): (a) $\mu = 0$, $\nu = 1$, $m = 7$, $k = 1$, $\varepsilon_{mi} = \varepsilon_m$; (b) $\mu = 2$, $\nu = 3$, $m = 1$, $k = 1$, $\varepsilon_{mi} = 0.001\varepsilon_m$; (c) $\mu = 2$, $\nu = 2$, $m = 3$, $k = 5.6$, $\varepsilon_{mi} = 0.001\varepsilon_m$;

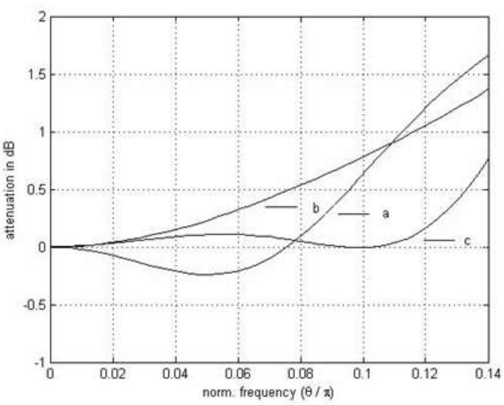

Fig. 2. Magnified passband part details of the amplitude characteristic for the eight order filter (group delay error $\varepsilon_m = 15\%$): (a) $\mu = 0$, $\nu = 1$, $m = 7$, $k = 1$, $\varepsilon_{mi} = \varepsilon_m$; (b) $\mu = 2$, $\nu = 3$, $m = 1$, $k = 1$, $\varepsilon_{mi} = 0.001\varepsilon_m$; (c) $\mu = 2$, $\nu = 2$, $m = 3$, $k = 5.6$, $\varepsilon_{mi} = 0.001\varepsilon_m$

Sensitivity of The Group Delay Response

As well known, for the practical implementation of digital filters, there is a need for circuits performing delaying, adding and multiplying digital numbers. Design accuracy of the filter transfer function or the group delay characteristic, strongly depends on the length of the digital word representing coefficients of the digital filter. Rabiner L.R. and B. Gold [7] have

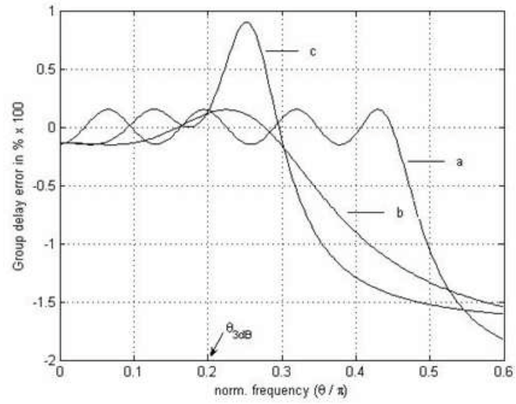

Fig. 3. Group delay characteristics for the eight order filter (group delay error $\varepsilon_m = 15\%$): (a) $\mu = 0$, $\nu = 1$, $m = 7$, $k = 1$, $\varepsilon_{mi} = \varepsilon_m$; (b) $\mu = 2$, $\nu = 3$, $m = 1$, $k = 1$, $\varepsilon_{mi} = 0.001\varepsilon_m$; (c) $\mu = 2$, $\nu = 2$, $m = 3$, $k = 5.6$, $\varepsilon_{mi} = 0.001\varepsilon_m$

shown that rounding after multiplication, in fact add a noise, so they proposed the procedure for calculating the value of this noise. J. F. Kaiser [8] is considered also how digital filter coefficients rounding can changes the location of poles and zeros in the z-plane. He has shown also, that when filter coefficients change, the parallel and the cascade filter implementation has smaller influence on the location of the poles and zeros, compared with the direct filter implementation. These conclusions can be useful also for analyzing sensitivity characteristics of the group delay, but they are not sufficient for the quantitative estimation of the influence of the digital filter coefficients small mutation to the group delay characteristic. According to references [9]-[12] the influence of the digital filter coefficients to the group delay characteristic, is not adequately considered.

This is the reason to start this chapter with analyzing sensitivity of the group delay characteristic, in the sense of the mutation of zeros and poles location, and accordingly, the mutation of the digital filter coefficients. If words with fixed length are used for the digital filter realization, the adopted structure has the direct influence to the noise generated by the digital filter. Direct implementation has the largest sensitivity to the smaller mutation of the coefficients, while in the sense of the sensitivity, results obtained using parallel and cascade implementation are better [13], [14]. There is another reason for using cascade implementation of transfer function: the investigation is easy when the cascade form is used. Other forms demand more extensive calculating. Hence, the cascade form of the transfer function will be considered in this chapter. For the examination of the influence of the pole magnitude and angle on calculating filter group delay sensitivity, the half-log sensitivity is used:

$$S_{r_{pj}}^{\tau(\theta)} = r_{pj}\frac{\partial \tau(\theta)}{\partial r_{pj}} \tag{4}$$

and

$$S_{\theta_{pj}}^{\tau(\theta)} = \theta_{pj}\frac{\partial \tau(\theta)}{\partial \theta_{pj}} \tag{5}$$

where r_{pj} and θ_{pj} are the magnitude and the angle of the complex poles, respectively. When the sensitivity of the concrete filter is considered, then values for parameters r_{pj} and θ_{pj} are known values, so the sensitivity depends only on digital frequency θ. Using Taylor series and ignoring higher order derivatives, after a few simple calculations, one can obtain the group delay changes of the filter (in seconds) with respect to relative changes of the pole magnitude and angle:

$$\Delta\tau_j(\theta) = S^{\tau(\theta)}_{r_{pj}} \frac{\Delta r_{pj}}{r_{pj}} + S^{\tau(\theta)}_{\theta_{pj}} \frac{\Delta\theta_{pj}}{\theta_{pj}} \quad (6)$$

The example of the eight-order digital filter described in the previous section will be considered, in order to illustrate obtained results. Thus, let us consider coefficients change of this eight order filter. The assumption is that the word length is eight bits and the fixed-point arithmetic is used. The transfer functions for eight order lowpass filters are:

$a)\quad \mu = 0, \quad \nu = 1, \quad m = 7$ (standard error curve):

$$F_8(z) = \frac{0.074713z^2}{z^2-1.559372z+0.634085} \times \frac{0.0329836z^2}{z^2-1.307432z+0.637295} \quad (7)$$

$$\frac{0.788281z^2}{z^2-0.859764z+0.648045} \times \frac{1.365259z^2}{z^2-0.326810z+0.692069}$$

$b)\quad \mu = 2, \quad \nu = 3, \quad m = 1:$

$$F_8(z) = \frac{0.188950z^2}{(z-0.565316)^2} \times \frac{0.119500z^6}{(z^2-0.889565z+0.382121)^3} \quad (8)$$

$c)\quad \mu = 2, \quad \nu = 2, \quad m = 3:$

$$F_8(z) = \frac{0.153196z^2}{(z-0.608597)^2} \times \frac{0.340132z^2}{z^2-1.145130z+0.485253} \quad (9)$$

$$\times \frac{0.857643z^4}{(z^2-0.762290z+0.688380)^2}$$

Coefficient rounding at eight bits is performed. Bandwidth is $\theta_{3dB} = 0.2\pi$ and the maximal group delay error is $\varepsilon_{max} = 15\%$, in all cases. Using these and calculated values and basing on (6), the expression for $\Delta\tau(\theta)$,as a function of frequency, is obtained. The shape of this function is shown in Fig. 4. Results, obtained on this way, can be compared with the standard error group delay curve (described in previous chapters) on a following manner: $\Delta\tau$ is obtained as the difference of the group delay when rounding is not performed, and the group delay when rounding is performed (representing the real group delay characteristic of the considered digital filter.

In the first case of comparison, from Fig.4 and Fig.5, it is obvious that curves (a) have same shapes, but differences exist as the consequence of using the arithmetic with rounding numbers, and also due to the mentioned ignoring of higher derivatives.

In the second case (b), it is obvious that recursive digital filters, proposed in the reference [3], are less sensitive to the coefficients change, compared to filters having standard group delay error curve [2], [15]. This is even more noticeable in third case (c). According to this, performed investigations presented in this paper, obviously indicate the fact that the multiple pole transfer function is less sensitive to the coefficients change compared to the simple pole transfer function. Described technique is very useful in the case when there is a need to determine the necessary number of bits for filter coefficients in the practical realization of a digital filter. Example illustrates difference between the sensitivity performance of the filter designed by method [2],[15] and the filter with multiple poles [3], [4].

Conclusions

The base characteristic of presented filters is a quasi-equiripple approximation of the constant group delay with

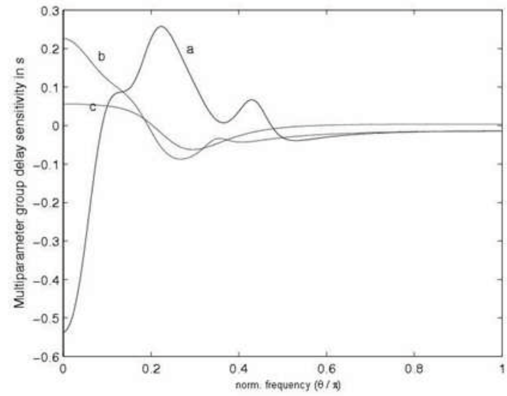

Fig. 4. Multiparameter group delay sensitivity for the eight-order digital filter with the group delay error $\varepsilon_{max} = 15\%$, obtained using the arithmetic with rounding numbers: (a) $\mu = 0$, $\nu = 1$, $m = 7$(standard error curve); (b) $\mu = 2$, $\nu = 3$ $m = 1$; (c) $\mu = 2$, $\nu = 2$ $m = 3$;

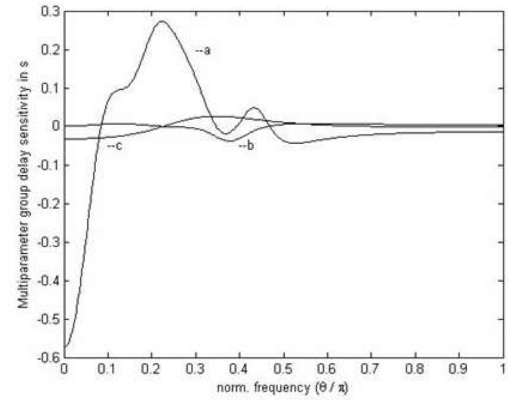

Fig. 5. Multiparameter group delay sensitivity for the eight-order digital filter with the group delay error $\varepsilon_{max} = 15\%$, obtained by the computer simulation of real filters: (a) $\mu = 0$, $\nu = 1$, $m = 7$(standard error curve); (b) $\mu = 2$, $\nu = 3$ $m = 1$; (c) $\mu = 2$, $\nu = 2$ $m = 3$;

constraints at the origin and in the stopband. These filters are derived from filters which are described in [3], so they keep all benefits cited [3]. The improvement of the filter performance in proposed approximation is enabled by introducing mentioned group delay constraints at the origin and in the stopband. On that way, the amplitude characteristic is improved both in the passband and in the stopband, while by using the multiple pole transfer functions, the less sensitivity to the filter coefficients change is achieved simultaneously.

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REFERENCES

[1] J. Thiran, "Equal-ripple delay recursive digital filters", in *IEEE Trans on Circuit Theory*, Vol. 18, Issue 6 pp. 664-669, November 1971.
[2] A. Deczky, "Recursive digital filters having equiripple group delay", in *IEEE Trans on Circuits and Systems*, Vol. 21, Issue 1, pp. 131-136 January 1974.
[3] V. Stojanovic, A. Micic, "Multiple-pole transfer function for recursive digital filter design" , in *AEU Int. J. Electron. Commun.*, Vol. 46, pp. 431-434 November 1992.
[4] V. Stojanovic, A. Micic, "Multiple-pole transfer function with equiripple group delay and magnitude for recursive filter design", *AEU Int. J. Electron. Commun.*, Vol. 47 pp. 114-118, February 1993.

[5] G. Mollova, R. Unbehauen, "Design of Recursive Filters with Constant Group Delay and Chebyshev Attenuation ", in *Proc. 23rd Int. Conference on Acoustic, Speech, And Signal Processing - ICASSP'98*, May 12-15, 1998, Seattle, WA, USA v.3, pp.1257-1260.

[6] G. Mollova "Digital Filters Design with Arbitrary Magnitude and Phase Specifications", in *Contents of Proceedings Electronics*, Book 2, pp.65-70. 1998.

[7] L. Rabiner, B. Gold *Theory and application of digital signal processing*, Prentice-Hall, INC. Englewood Cliffs, New Jersey, 1975.

[8] J. Kaiser, , "Design methods for digital filters", *Proc. of the first allerton conference on Circuit And System Theory, 1963.*

[9] V. Cappelini, A. Constantinides, P. Emiliani *Digital filters and their applications*, Academic press, London, 1978.

[10] S. Holford, P. Agathoklis, "The use of model reduction techniques for designing IIR filters with linear phase in the passband", *IEEE Trans. Signal Processing.*, Vol. 44, pp. 2396-2404, Oct. 1996 .

[11] B. Weisburn, T. Parks, B. Shenoy, "Error criteria for filter design", *Proc. of the IEEE International Conference on Acoustics, Speech, And Signal Processing, Vol. 3, page III: 5658211;568, IEEE ICASSP-94, Adelaide, Australia, April 198211, 22 1994.*

[12] XI Zhang, K. Suzuki, T. Yoshikawa, "Complex Chebyshev approximation for IIR digital filters based on eigen-value problem", *IIEEE Trans. on Circuit and II: Analog And Digital Signal Processing*, Vol. 47, No. 12, pp.1429-1436, December 2000.

[13] J. Bandler, M. Rizk, H. Tromp, "Efficient calculation of exact group delay sensitivities", *, IEEE Trans. on Microwave Theory and Techniques*, Vol. MTT-24,No.4, pp. 188-194, April 1976.

[14] G. Allen, , "Perturbation effects on filters having multiple poles", *Acoustics, Speech, Snd signal Processing, IEEE International Conference on, 1994, ICASSP-94 vol. 6, pp. 589-592,19-22 April 1994*

[15] A. Deczky *Equiripple and minimax (Chebyshev) approximations for recursive digital filters* , Addison-Wesley, 1989.

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