

# Multilateration and Flip Ambiguity Mitigation in Ad-hoc Networks

**Abstract.** Many approaches to localization in ad-hoc networks are based on the distance estimation between nodes. Due to signal propagation characteristics and measurement errors, the distance estimations are not accurate. Subsequently, the entire localization process produces node positions with a certain error. Our work is focused on the mitigation of distance estimation errors caused by a phenomenon called flip ambiguity. Unlike other works in the field focusing on the trilateration, we focus on the multilateration approach as well. We propose an enhanced multilateration algorithm mitigating the flip ambiguity and show its benefits (higher accuracy) by comparing it with both the traditional multilateration and trilateration algorithms in a simulated scenario.

**Streszczenie.** Wiele przybliżeń lokalizacji w sieci ad-hoc opartych jest na oszacowaniu odległości między węzłami. Ze względu na charakterystykę propagacji sygnału oraz błędy pomiaru, ocena odległości nie jest dokładna. Nasza praca skupia się na ograniczeniu błęd oszacowania odległości powodowanego przez zjawisko zwane niejednoznacznością typu flip ambiguity. W odróżnieniu od innych prac skupiających się na trilateracji, my skupimy się również na multilateracji. Proponujemy uwydatniony algorytm multilateracji, który ogranicza błędy. (Korygowanie błęd typu flip ambiguity w sieciach typu Ad-hoc)

**Keywords:** Distance, Flip ambiguity, Localization, Multilateration, Wireless Sensor Network

**Słowa kluczowe:** bezprzewodowe sieci czujników. Sieci ad-hoc.

## Introduction

Area of wireless sensor networks covers networks with envisioned hundreds of nodes. These nodes can be dispersed randomly or manually. Regardless the mean of placement, for the application purposes the nodes have to be aware of their position in the network or at least the position in relation to their neighbors. Otherwise, the application (no matter if the nodes are collecting data or providing some information) is not able to fully satisfied given requirements [1],[2].

Position information can be set into the nodes manually or be determined by a certain automated process. Considering a large number of nodes in a network, the manual setting would be very demanding and expensive. Furthermore, in the case of mobile nodes, the automated-localization is the only possibility [3].

There have been proposed several localization approaches including GPS (Global Positioning System [4]). However, adding a GPS receiver into each node is expensive, and thus, usually just a few nodes in a network have GPS location capabilities if the GPS system is used. These nodes play then an important role in the localization process because they are used as an infrastructure for localization of the other nodes. These nodes are called reference nodes or anchors and the coordinates of the other nodes are derived from them.

A lot of localization algorithms use distance estimation and lateration to calculate coordinates of the unlocalized nodes. Because of the erroneous distance estimation, the determined position has a certain error as well. The position error has to be taken into account and if it is comparable to the ranging error then, the position determination is acceptable. In some cases, however, the position error significantly exceeds the ranging error due to a completely false position estimation. This is caused by inappropriate geometric relations among the anchors used for the calculation (see Fig.2). This phenomenon is called flip ambiguity and causes the largest estimation error when using lateration.

Our work focuses on the large-scale errors during position estimation (caused by flip ambiguity) and their elimination using an adaptation of the robust criterion described by Kannan [5][6] in the multilateration approach. Multilateration is a variation of lateration using more than three nodes for position estimation (see Section Multilateration for other information). The multilateration approach seems to be advantageous because it uses more information related to the

position of the unknown node. It means that even if the distances from the anchor nodes are not accurate a larger number of references should eliminate a false estimation. We first analyze the dependency of position accuracy on the number of anchor nodes used and then we implement our proposed multilateration principle into an incremental localization algorithm which performs localization iteratively in several rounds depending on the network size, radio range and the start position of the incremental localization algorithm. Our approach is range-based without any anchor infrastructure. It is based on the fact that more reference nodes can give more precise position estimation while the problem of flip ambiguities is minimized by controlling a geometrical arrangement of the reference nodes set. The accuracy of the localization process is evaluated by mean absolute error (MAE), global energy ratio (GER) and global distance error (GDE).

The rest of the paper is organized as follows. Next section briefly discusses related work in the field of ad-hoc localization. Section 3 focuses on the lateration principle with the flip ambiguity phenomenon. Section 4 describes multilateration with performed simulations following by the section devoted to the proposed enhanced multilateration algorithm and the flip ambiguity mitigation. Furthermore, the section discusses the results of the simulations and points out pros and cons of the investigated approach. All the simulations were performed in Matlab simulation tool. And finally, the last section concludes the paper.

## Related Work

Localization in ad-hoc networks is a complex process and depending on the application various requirements can be defined. Therefore, there are also a lot of algorithms with different approaches. Their classification can be found, for example, in works presented in [7]-[9].

There are two major approaches of localization algorithms - centralized and distributed. Centralized localization means that data obtained from the ranging phase are migrated to a certain powerful station where they are processed. The results (mostly the position information or coordinates) are then transmitted back to the respective nodes. The big advantage of the centralized localization is that the computational demands are moved to a powerful station capable of complex computing which relieves the other nodes in the network. On the other hand, there is an extra traffic caused by the localization related data transmission to and from the central station. The representatives of centralized

algorithms are multidimensional scaling improvement (MDS-MAP) [10], simulated annealing localization [11] and RSSI based centralized localization [12].

In distributed algorithms the computation is performed at each node. Therefore, the computational tasks have to be designed for constrained computing and memory resources. This is the drawback of such localization because the algorithms cannot be too complex. Lower complexity of the algorithm means also smaller possibility of a precise estimation (e.g. through optimization). On the other hand, the distributed character allows higher scalability and independence on a certain network structure or a central point. Ad-hoc positioning system (APS) [13], n-hop multilateration [14], anchor free algorithm (AFL) [15] or S-MRL [16] are examples of distributed localization algorithms. Furthermore, there are proposals combining both approaches using algorithms with centralized and distributed features as well. These hybrid approaches include, for instance, combination of MDS technique with proximity based map (PDM) [17] and combination of MDS with APS system [18].

Since lateration is one of the most used algorithms for position determination its description and analysis under various conditions appears in a lot of papers from different fields (e.g. [19], [20]). The influence of anchor position geometries is discussed, for example, in works [5] and [21]-[25] and empirical studies are presented in [26]. For our work we come out mostly of [5] and, therefore, we introduce shortly its approach with the robust criterion here. The main idea of the enhancement of localization accuracy is an identification of flip ambiguities and excluding them from the following iteration steps. That prevents dissemination of large scale-errors over the network. After a position is found, it is evaluated against a certain condition. And only if the condition is fulfilled the node is further used in localization process as a new reference node. The approach is applicable in both centralized and distributed algorithms.

Most of the studies use trilateration approach to identify the geometry problem leading to flip ambiguities. The analysis of flip ambiguities is based on the quadrilateral, structure of four nodes, which is formed by three reference nodes and an unlocalized node. The quadrilateral is tested against the robust criterion and only if it is identified as not susceptible to flip ambiguity then it is used in the localization process.

### Localization based on lateration

Basically, the localization process consists of two phases. The first is called ranging. The information about distances, angles, number of hops or other important relations between nodes is gathered in this phase. The second phase, employing a certain localization algorithm, processes this information to estimate the position of the unlocalized nodes. Some localization protocols include other phases such a dissemination of the collected information or refinement process enhancing accuracy of the estimation by error reduction.

The procedures of the ranging phase depend on the measurement technique used. Generally, there are distance measurements, bearing measurements, time measurements etc. Every measurement features some error, which is impossible to eliminate completely and, therefore, subsequent localization algorithm has to take the measurement inaccuracy into account [27]. These ranging errors represent the main error source in the position estimation [28], [29].

Information obtained in the first phase is then processed by a certain localization algorithm. Many of them have been proposed up to now and new ones are still being introduced

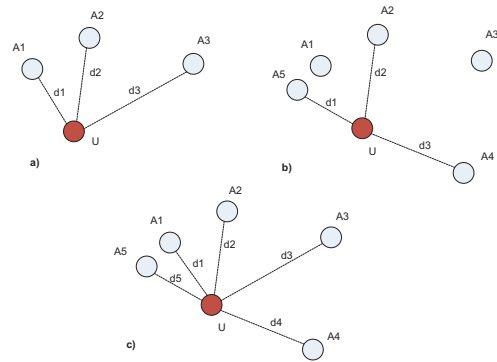


Fig. 1. Trilateration, trilateration with selection and multilateration

with the intention to find an optimal algorithm for a general use or for a particular type of applications [30]-[36].

Our work is focused on the range-based and anchor-free localization algorithms. Range-based algorithms use the distance or angle estimation, range-free algorithms use the hop count or other relative information instead. Anchor based localization works with an infrastructure of nodes which know their position a priori. These nodes spread their coordinates over the neighborhood in order to allow other nodes being localized in the predetermined coordinate system. Anchor-based algorithms generally feature higher accuracy depending also on the node and anchor density in a network. Anchor-free systems form a relative coordinate system which supports other procedures such a geographical routing or data gathering. The relative coordinate system can be subsequently converted into a global system (e.g. GPS) introducing a certain transformation.

Considering range-based localization algorithms with distance measurement, lateration is the most common way how to calculate a node position. Using its basic form in two dimensional space we talk about trilateration. Trilateration is used to calculate the coordinates of an unlocalized node from the known distances to three reference nodes with known coordinates. Because of the erroneous distance estimations the result of trilateration is erroneous as well. Whenever possible, to reduce the influence of distance errors, more than three reference nodes are used for coordinate calculation. One approach is based on selection of the most appropriate triple of reference nodes [21], another one uses more reference nodes at the same time. In that case we talk about multilateration [22]. The three different lateration approaches can be seen in Fig.1.

In the presented scenario A1 to A5 represent reference nodes (anchors) and U stands for Unknown referring to the unlocalized node with unknown coordinates. We assume that all the anchors are in the radio range of the Unknown. In the first case (case a)) the Unknown has only three anchors in its radio neighborhood and, thus, the trilateration is used. In the case b) there are more than three reference nodes available and the Unknown can choose the best triple of them (based on the predetermined conditions; discussed later in the next section). The best triple is depicted by the dashed lines d1-d3 in the figure. The last part of the figure demonstrates multilateration when all the anchor nodes are used for the position estimation.

The basic formula to identify the distance between the reference node and an unlocalized node in Euclidian space is:

$$(1) \quad d_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2.$$

For each reference node included in lateration a corresponding equation is used. In our notation unlocalized node coordinates are  $(x,y,z)$ , coordinates of  $i$ -th reference node are  $(x_i,y_i,z_i)$  and  $d_i$  stands for the distance between the unlocalized and the reference node. This set of quadratic equations can be solved by subtracting the last equation from the others one by one which gives a set of linear equations. Subsequently, LU decomposition or a certain iterative algorithm (e.g. LSE) can be used for solving the set. When assuming  $M$  suitable reference nodes, the equation set can be presented as

$$(2) \quad \mathbf{Ax} = \mathbf{b}$$

where

$$(3) \quad \mathbf{A} = 2 \begin{bmatrix} x_1 - x_M & y_1 - y_M & z_1 - z_M \\ x_2 - x_M & y_2 - y_M & z_2 - z_M \\ \vdots & \vdots & \vdots \\ x_{M-1} - x_M & y_{M-1} - y_M & z_{M-1} - z_M \end{bmatrix}$$

$$(4) \quad \mathbf{b} = \begin{bmatrix} x_1^2 - x_M^2 + y_1^2 - y_M^2 + z_1^2 - z_M^2 - d_1^2 - d_M^2 \\ x_2^2 - x_M^2 + y_2^2 - y_M^2 + z_2^2 - z_M^2 - d_2^2 - d_M^2 \\ \vdots \\ x_{M-1}^2 - x_M^2 + y_{M-1}^2 - y_M^2 + z_{M-1}^2 - z_M^2 - d_{M-1}^2 + d_M^2 \end{bmatrix}$$

and vector of unknown coordinates:

$$(5) \quad \mathbf{x} = [x \ y \ z]^T.$$

Provided that all the reference coordinates are accurate as well as the distances to each reference node, the position of the unlocalized node can be correctly calculated. However, measurement methods, environment uncertainty, noise and other negative effects introduce an error in distance estimation and subsequently in position estimation. Apart from the acceptable shift of estimated coordinates from the true position (the shift is still in surrounding of the true position), lateration can produce an error in much larger scale. This happens when the relative locations of the reference nodes used for lateration are unsuitable. In that case, the estimation is faraway from the true position and the error is unacceptable. This phenomenon is called flip ambiguity and it is mainly dependent on the relative position of all nodes participating in lateration. Fig.2 presents an example of two different arrangements of anchors and an unlocalized node. In both cases the lateration algorithm was executed twenty times and the result position was displayed in the figure with the mark (+). The anchors are represented by blue dots and the true position of the unlocalized node (U) by a black dot. It is obvious that in case a) all the estimations occur in the neighborhood of the true position. However, in the second example with different anchors arrangement, the estimations are crowded around two distinctive areas. That means that there is a high probability of a large error in the position estimation.

The reason of flip ambiguities resides in the geometrical organization of the anchor nodes and the unlocalized node. When anchors positions are almost collinear, they form a mirror through which the node position can be reflected. That causes the most significant error in the position estimation.

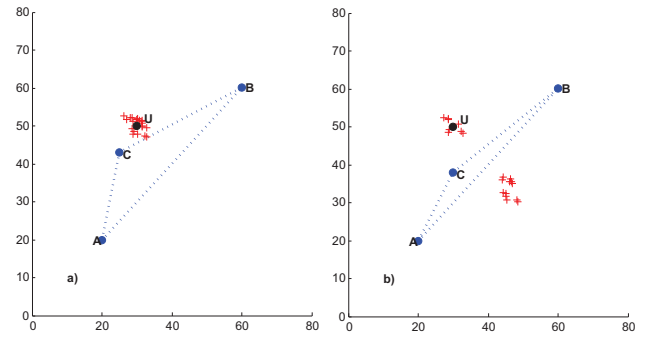


Fig. 2. Phenomenon of flip ambiguity. The position calculation was executed 20 times. Each estimated position is depicted by a red mark (+).

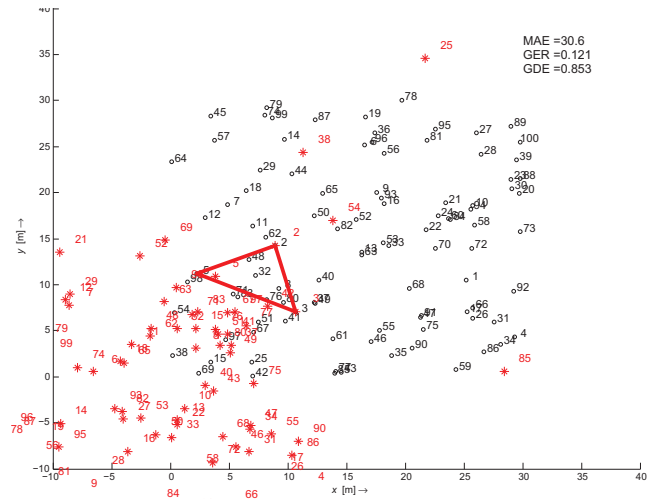


Fig. 3. Effect of flip ambiguity on the localization of other nodes in the first iteration step. The circles stand for the true position of nodes while the red asterisks (\*) represent the estimated positions.

Moreover, when a localization protocol is used the large error is likely to propagate and degrade enormously the overall localization accuracy when flipped estimations are presented in subsequent iterative steps. This is demonstrated by a given example in Fig.3. The localization used is based on an iterative anchor-free algorithm in a random topology consisting of 100 nodes. After the initial three nodes are selected as the reference system the localization proceeds with trilateration. The accuracy of the algorithm is expressed by MAE (Mean absolute error), GER (Global error ration) and GDE (Global distance error).

For analysis we assume a disc transmission radio model with radius  $R$  around each node. Furthermore, the noise propagation model with Gaussian distribution  $\mathcal{N}(\mu, \sigma^2)$  is used. To keep the ranging error limited a threshold  $\bar{\epsilon} > 0$  is defined. It can be chosen as  $3\sigma$ . Then, we can assume that

$$(6) \quad |d_{AB} - \bar{d}_{AB}| \leq \bar{\epsilon}$$

where  $\bar{d}_{AB}$  is the estimated distance between node A and node B while  $d_{AB}$  is the real distance between them.

Once the positions of three references in the quadrilateral and the distances of the fourth node to those ones are known, the coordinates of the unlocalized node can be calculated. Following the notation in parallel with [5] the quadrilateral is called  $(\bar{\epsilon}, \delta_S)$ -robust if the fourth node is unambiguously localizable with a predefined accuracy level  $\delta_S > 0$  provided that ranging errors do not exceed the predefined

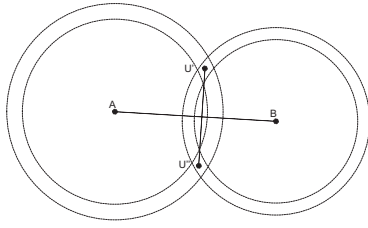


Fig. 4. Two possible locations of unlocalized node using only two references A and B threshold  $\bar{\epsilon}$ .

Considering a generic localization method, the algorithm proceeds in two phases. First, using two anchors A and B and the erroneous distance estimations to them ( $|AU|$ ,  $|BU|$ ), two areas of probable node position are determined. Using the third anchor and the distance to it, one of the two is selected (see Fig.4). The two possible areas of the Unknown occurrence include the correctly estimated position and the flipped position. The larger the ranging error is, the larger areas the lateration algorithm forms and that means a bigger chance of a false estimation consequently. The primary task of each localization algorithm is to determine a position within the predefined accuracy level  $\delta_S > 0$ . If the distance of the two estimations (calculated and flipped) is less than accuracy threshold ( $|U'U''| < \delta_S$ ) then the flip ambiguity is not an issue and it is called negligible flip ambiguity. The significant flip ambiguity exits when

$$(7) \quad (||CU'| - |CU''||)^2 \leq 4\bar{\epsilon}^2 \quad \text{when } |U'U''| > \delta_S$$

Hence, the robust criterion ( $\bar{\epsilon}$ ,  $\delta_S$ ) is defined as

$$(8) \quad (||CU'| - |CU''||)^2 > 4\bar{\epsilon}^2 \quad \text{when } |U'U''| > \delta_S$$

In the robust criterion, distances of an estimated position and a mirrored estimated position to the third node C and their difference is compared to the defined threshold. It means that the predetermined threshold  $\bar{\epsilon}$  has the most significant influence on the probability of flip ambiguity detection.

To evaluate the entire quadrilateral with all possible mirroring pairs of anchors, the criterion has to be applied on each combination of anchors within the quadrilateral:

$$(9) \quad (||ZU'| - |ZU''||)^2 > 4\bar{\epsilon}^2 \quad \text{when } |U'U''| > \delta_S$$

where Z stands for arbitrary anchor providing that the other two anchors form a possible mirroring axis.

### Localization using multilateration

So far, the problem of lateration was discussed considering only a quadrilateral (three anchors and one unknown node), respectively trilateration. Thus, the robust criterion was applied on the triple of anchor nodes in the unlocalized node neighborhood. However, providing the node neighborhood includes more than three reference nodes, they can be included in position calculation as well. Such approach is called multilateration. In [5] multilateration is described as follows: First, from the set of M reference nodes in the node neighborhood the three of them are selected as a RM set based on the robust criterion. During the selection all possible quadrilaterals (formed by unlocalized node and combination of three anchors in the neighborhood) are tested and the RM set must fulfill the following:

$$(10) \quad \max_{A,B,C \in M} (||ZU'| - |ZU''||)^2 \quad \text{where } Z \in \{A,B,C\}$$

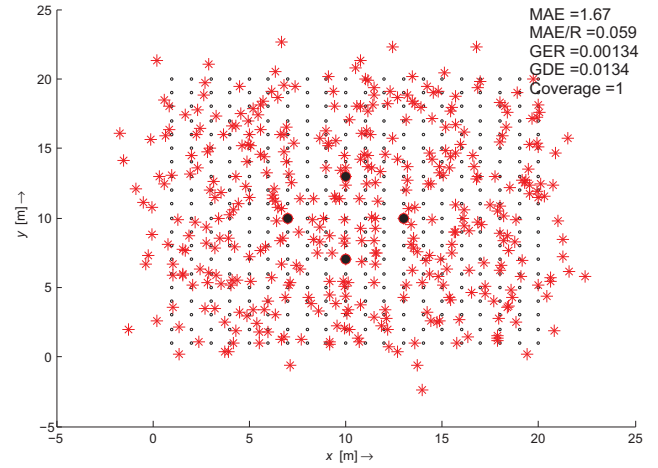


Fig. 5. Multilateration with 4 anchor nodes and 10 % ranging error in one-hop network

It means the best quadrilateral (evaluated by the robust criterion) is selected as a RM. Then, every other anchor node  $Y \in M$  is tested if it forms a robust quadrilateral with the unlocalized node. If so, it is included in the anchor set used for multilateration.

We confirmed the advantageous approach of multilateration by independent test in a scenario of one-hop network with increasing number of anchor nodes in the unlocalized node neighborhood. The aim of the simulation was to find out how considerably the accuracy increases with a larger number of anchor nodes used for multilateration. We used a field of 20x20 units with nodes located in a grid of the same size as the field. Only a predetermined number of nodes know their location and these nodes lie on a circle around the middle of the field in order to achieve the best arrangement and avoid the influence of anchor nodes positioning. In order to realize one-hop network (i.e. each node can reach all the anchors directly) the radio range of nodes is equal to the length of a diagonal of the field. Thus, all the nodes are localized simultaneously in one step. For the simulation the ranging error (the distance estimation error) was set to 10 % with the Gaussian distribution. Both the network deployment and the result of localization can be seen in Fig.5. The black spots represent the true position and the red asterisks stand for the estimated position. Moreover, this sample of the performed simulations uses 4 anchors which are marked as black circles around the middle of the field. We performed simulations with different numbers of anchors and the multilateration performance is expressed by MAE, GER and GDE (explained previously in the text). Apart from that the MAE related to the radio range is displayed as well. In Fig.6 there is a grid representing the field of nodes where each square of the grid represents one node. The color of the square is set according to the result of multilateration, particularly MAE. The color range of the color bar on the right side expresses MAE ranging from 0 to 8 units. As can be seen, MAE increases as the nodes are further from the anchors.

The results of simulations confirm the conclusion of other works in the field (e.g. [22]); all three error metrics decrease with increasing number of the anchors. The dependency of the metrics and number of anchors used can be seen in Fig.7. For better transparency only MAE and GER were chosen.

The accuracy of multilateration depends highly on mutual position of anchors too. Circular placement of anchors results in a better accuracy than a case when anchors are

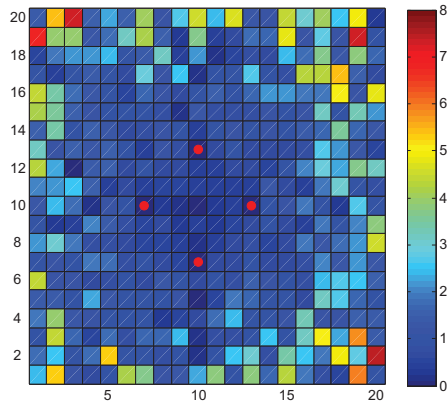


Fig. 6. Mean absolute error of nodes during the 4-nodes multilateration

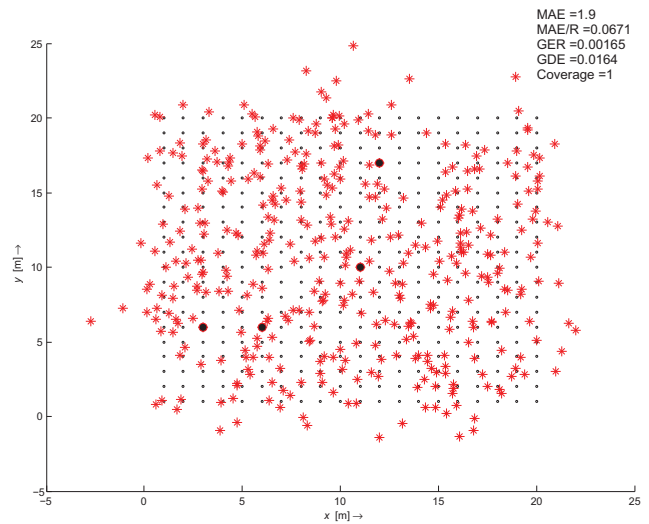


Fig. 8. Non-circular placement of anchor nodes in 4 anchors multilateration

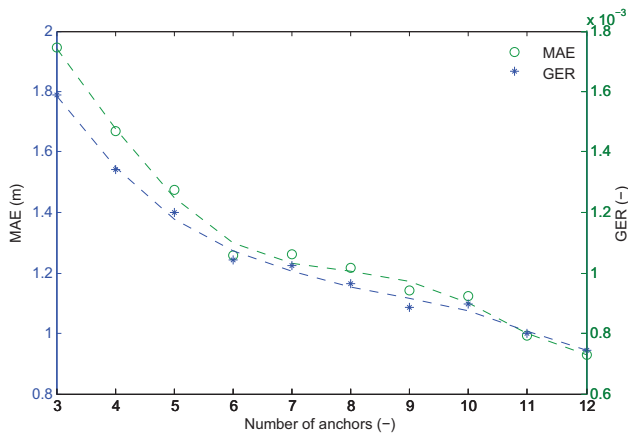


Fig. 7. Accuracy performance of multilateration with various number of anchors

almost collinear. Collinear anchor placement represents the extreme case but other deformations of polygon formed by anchors have a significant influence too. One example can be seen in Fig.8.

The simulations indicate worse results in both MAE and geometrical relation of the estimates. In words of relative error estimation the accuracy is about 8 % worse then in the previous case. It is also worth mentioning the errors of individual nodes in a geographical relation to the anchor polygon. This can be clearly seen in Fig.9. The nodes lying in a direction of longer axis of the polygon features larger MAE which is indicated by colors in the figure. It is analogous to trilateration and its geometrical characteristics. The best results of localization are achieved when the anchor nodes form an equilateral triangle. Moreover, the error distribution is even all around the anchor triangle.

### Proposed enhanced multilateration algorithm

Our approach to multilateration uses a different strategy. We do not separate the anchors in a neighborhood into an RM set and the rest but we take the whole anchor neighborhood as one consistent group M. To detect possible flip ambiguity we use an adapted robust criterion derived from (9):

$$(11) \quad (||ZU' - |ZU''||)^2 > \frac{|ZU'| + R}{R} \cdot 4\epsilon^2 \quad \text{when } |U'U''| > \delta_S$$

The adaptation of the criterion is based on the fact that the criterion condition is more strict for longer distances of the

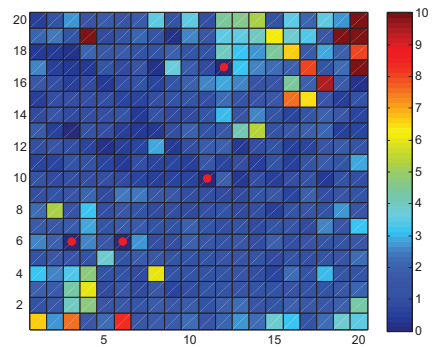


Fig. 9. Non-circular placement - uneven error distribution

unknown node from the anchor set.  $R$  stands for the range of nodes in the formula.

The algorithm tests all pairs of anchors (A, B) and the third node C in a relation to the unknown node U. If, for example, the first choice of the node C in M does not fulfill the condition and the next choice yes, the pair A, B is considered as a non-mirroring axis and, thus, there is no potential risk of flip ambiguity (the details are given in the following pseudocode).

Subsequently, the proposed algorithm of multilateration was incorporated in the complex incremental localization algorithm (ILA) to test its influence on the localization accuracy. ILA is simulated in a network of randomly distributed nodes. It is an anchor-free, range based algorithm that creates its own coordination system which can be subsequently transformed into a global one. Localization process is started by a randomly selected node. This node searches the surrounding in its radio range and finds its neighbors. Then, it evaluates possible pairs from the set of neighbors that could form a basic triangle. Evaluation is based on the geometrical characteristic of the tested triangle. The adapted robust criterion is used. When the start node finds two convenient neighbors it creates a local coordinate system.

The coordinate system is created as described in [37]. It is referred as Assumption Based Coordinates (ABC). The first assumption is that the initial node is located at coordinates (0,0,0). The second node in the triangle is provided with the coordinates  $(r_{01}, 0, 0)$  where  $r_{01}$  is the distance to the initial node. The coordinates of the following node are:

**Algorithm 1** Test of robust criterion for multilateration

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1: for each pair {A,B} where {A,B} ⊂ M do
2:   for each C where C ∈ M − {A,B} do
3:     test (ε̄, δS) - robust criterion of quadrilateral ABCU
4:     if robust criterion test passed then
5:       AB_passed = TRUE;
6:       break;
7:     else
8:       AB_passed = FALSE;
9:     end if
10:  end for
11:  if AB_passed = FALSE then
12:    TEST = FALSE; {The pair {A,B} is a possible
13:    mirroring axis. M is not the robust neighborhood.}
14:    break;
15:  else
16:    TEST = TRUE;
17:  end if
18: end for

```

$$(12) \quad x = \frac{r_{01}^2 + r_{02}^2 + r_{21}^2}{2r_{01}}$$

$$(13) \quad y = \sqrt{r_{02}^2 - x^2}$$

The enhanced multilateration algorithm was implemented as proposed and described previously. When more than three nodes are in a radio range of the unknown node it uses all of them for multilateration, otherwise, only trilateration with the robust criterion is applied.

For the simulations we used a network of hundred nodes spread randomly in the area. Every node covers a circular area with a radius  $R$ . To simulate the erroneous distance measurement the provided distances between nodes are affected by Gaussian distribution  $\mathcal{N}(R,4)$ . The selected error model is based on the work of Whitehouse and Culler [31] who showed that the estimation distance tends to overestimation. However, a proper calibration procedure leads to symmetric error distribution. Although we could use multi-hop connections, to keep the results more transparent, we used one-hop neighborhood scenario. The connectivity (number of neighbors in communication range) is specified by a radio range and a number of nodes in a predetermined area. The localization algorithm is a decentralized anchor free and range-based algorithm based on the Map-growing localization algorithm [32].

The standard scenario used for simulations is characterized by 100 nodes in an area of 30 m x 30 m with a range of 15 m. The standard deviation of the radio range error is 10 %.

In Fig.10 there is presented one example of our simulation results. It can be seen that the MAE error is less than 10 % which is a good result considering ranging errors. However, small coverage is a significant drawback of this algorithm. It means that only a small percentage of all the nodes is localized (slightly less than 20 %). The low coverage is caused by the strict robust criterion minimizing the flip ambiguity. Despite that, the flip ambiguity still occurs sometimes. The only one occurrence of the flipped estimation (node 31) in the first round of the algorithm causes meaningless results (see Fig.11).

We have repeated the described scenario in order to get statistically significant results and we got the follow-

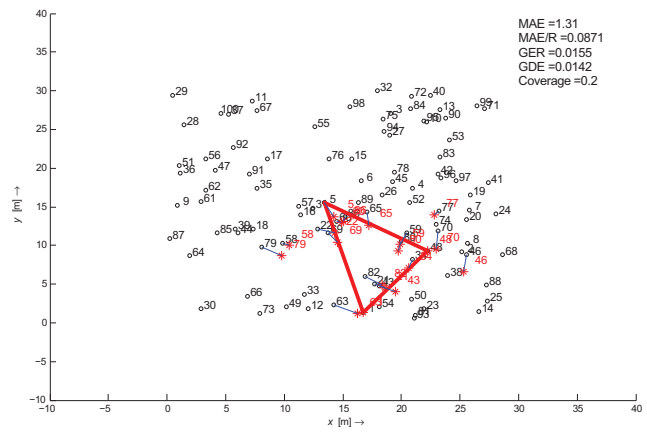


Fig. 10. Localization in random network of 100 nodes using multilateration and adapted robust criterion

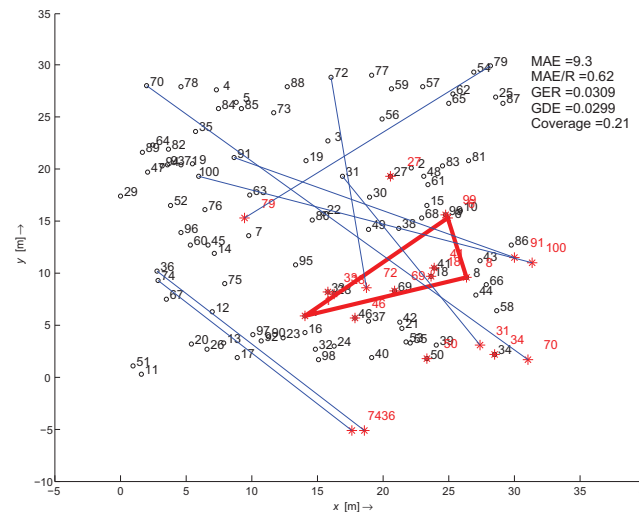


Fig. 11. Results of incremental localization with multilateration when flip ambiguity occurs

ing:  $\mathcal{N}_{MAE}(4.9787, 3.8020)$  and  $\mathcal{N}_{GER}(0.0492, 0.0395)$  with a global coverage ratio featuring normal distribution  $\mathcal{N}_{Coverage}(0.2095, 0.0705)$ . In order to compare the values with results of the localization algorithm employing trilateration with the robust criterion we have performed also those simulations ( $\mathcal{N}_{MAE}(6.9165, 3.5837)$ ,  $\mathcal{N}_{GER}(0.0255, 0.0106)$  and 100 % coverage).

Despite the lower coverage, the accuracy of the localization is higher when employing our approach. Therefore, we propose to use the enhanced multilateration as a preliminary step of the localization in order to obtain a set of highly precise position estimations. These accurately estimated positions can serve as an enriched set of reference nodes in the subsequent step. This has a significantly positive effect on overall localization results as can be seen from the performed simulations (see Fig.12).

We have also performed several simulations with traditional multilateration and trilateration approach with the robust criterion in order to compare it to our approach. After 10 simulations with each algorithm we got the following results: (i) multilateration  $\mathcal{N}_{MAE}(9.0906, 2.5354)$  and  $\mathcal{N}_{GER}(0.0141, 0.0040)$ , (ii) enhanced multilateration  $\mathcal{N}_{MAE}(7.6667, 2.1317)$  and  $\mathcal{N}_{GER}(0.01221, 0.0045)$ , (iii) trilateration  $\mathcal{N}_{MAE}(10.0562, 5.3051)$ ,  $\mathcal{N}_{GER}(0.0205, 0.0123)$ . All with the coverage of 100 %.

You can see the enhanced multilateration performs the best in both metrics MEA and GER. Traditional multilateration

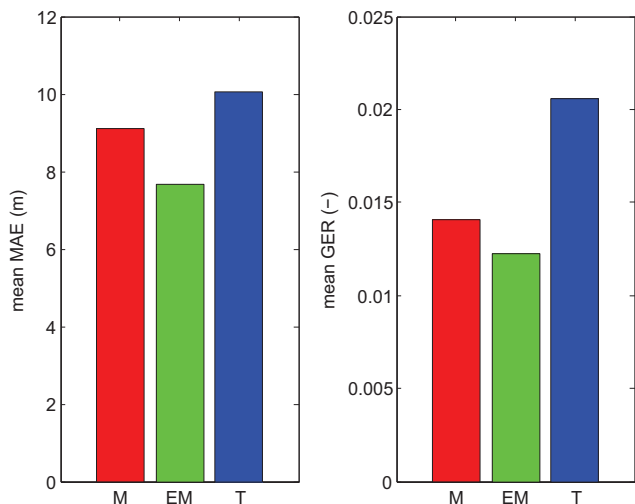


Fig. 12. Mean of MEA and GER of three examined approaches after 10 simulations with each algorithm; M - Multilateration; EM - Enhanced multilateration, T - Trilateration

with the robust criterion is slightly worse than our approach but better than the trilateration algorithm with the robust criterion.

### Conclusion

This paper is focused on the flip ambiguity of lateration and especially multilateration. The aim is to use multilateration for the position estimation and at the same time to mitigate the possibility of the flip ambiguity. Therefore, we proposed an enhanced multilateration algorithm, which uses all the anchor nodes in the neighborhood for a position estimation provided that there is a low probability of the flip ambiguity. To assure that we applied the modified robust criterion proposed in [5]. First, we conducted simulations of multilateration in one-hop grid network in order to investigate its characteristics. The results confirm the assumption that nodes can be localized more precisely if more anchor nodes are employed. Subsequently, we incorporated the proposed enhanced multilateration algorithm into an incremental anchor-free range based localization algorithm, performed several tests and compared the results with the trilateration approach.

The first set of simulations of multilateration confirms the benefit of using more than three anchors whenever possible (MAE is less than 10 %). However, when multilateration is implemented in a global incremental localization algorithm, it features a lower coverage (caused by the strict robust criterion mitigating the flip ambiguity). Despite the lower coverage, the accuracy of the position estimation is higher, and thus, it can be used as a preliminary step when a denser infrastructure of reference nodes is formed. This fact was confirmed during simulations conducted together with the traditional multilateration and trilateration algorithm. Our enhanced multilateration algorithm performs better considering both used metrics MEA and GER.

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