

# An algorithm for calculation of chosen differential parameters for a brushless motor with permanent magnet excitation with sine control (PMSM)

**Streszczenie.** W pracy przedstawiono algorytm wyznaczania pochodnych cząstkowych strumienia związanego z prądem i kątem obrotu silnika elektrycznego z magnesami trwałymi w wirniku i trójfazowym wzbudzeniem stojana. Ponieważ  $\Psi$  jest nieliniową funkcją wektorową, której wartości są obliczane na podstawie natężenia pola magnetycznego z wykorzystaniem specjalistycznego oprogramowania, analityczne obliczenie pochodnych niesie ze sobą poważne trudności i konieczne jest przeprowadzenie dodatkowych obliczeń. Obliczenia oparte na symulacji komputerowej przeprowadzono dla silnika IPMSg132 S4 PMSM. Rezultaty zaprezentowano w formie wykresów. (**Algorytm wyznaczania pochodnych cząstkowych strumienia związanego z prądem i kątem obrotu silnika elektrycznego z magnesami trwałymi w wirniku**)

**Abstract.** In the paper an algorithm for calculation of partial derivatives of flux associated to current and rotation angle of an electric motor with permanent magnets in the rotor and with a three-phase stator excitation has been presented. Because  $\Psi$  is a nonlinear vector function, whose values are calculated on the basis of magnetic field using a specialized piece of software, the analytic calculation of partial derivatives is related to certain difficulties and necessity to carry out a number of additional calculations. Simulation-based calculations have been carried out for the motor IPMSg132 S4 PMSM. The results have been presented in the form of charts.

**Słowa kluczowe:** proszę podać cztery terminy opisujące treść artykułu.

**Keywords:** proszę podać słowa kluczowe angielskie.

## Introduction

The subject of the analysis is a contactless electric motor with permanent magnets in the rotor and a multi-phase stator winding. The machines of this kind are referred to as Permanent Magnet Synchronous Motors in the international literature. Computer simulations of the motor operation are carried out for the motor IPMSg132 S4 PMSM produced by BOBRME "Komel".

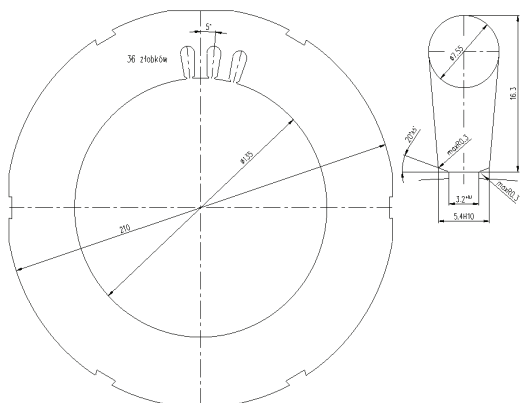


Fig.1. Dimensions of stator core for the motor IPMSg 132 S4 PMSM

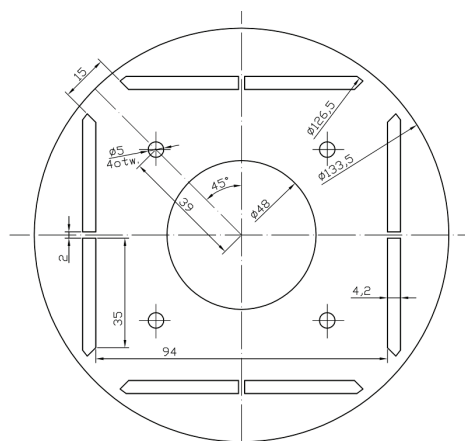


Fig. 2. Dimensions of rotor core for the motor IPMSg 132 S4 PMSM

Table 1. Nominal data concerning the motor IPMSg132 S4 PMSM

Type of the motor (designation)	IPMSg 132 S4 PMSM
Power	4 kW
Nominal voltage	3x400 V
Nominal current	7,5 A
Nominal velocity	1500 obr/min
Nominal moment	25,5 Nm

The cross-section of stator core and its dimensions are given in Figure 1, whereas in Figure 2 the cross-section of rotor core and its dimensions are shown.

## Computational analysis

The area being considered is depicted in Figure 3.

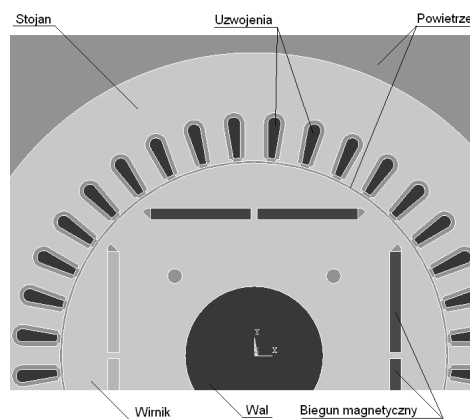


Fig.3. The area considered during numerical calculations taking into account magnetic properties of individual motor components

It should be remarked that areas of the same color will have the same magnetic properties. In order to discretize the considered area, the mesh has been created. It is shown in Fig. 4.

In the paper a method for calculation derivatives for  $\Psi = [\Psi_1, \Psi_2, \Psi_3]$  has been presented. The function may be represented using a set of three scalar functions:

$$(1) \quad \begin{aligned} \psi_1 &= \psi_1(i_1, i_2, i_3, \gamma), \\ \psi_2 &= \psi_2(i_1, i_2, i_3, \gamma), \\ \psi_3 &= \psi_3(i_1, i_2, i_3, \gamma), \end{aligned}$$

where  $\psi_1, \psi_2, \psi_3$  – magnetic coupling of stator circuits,  $i_1, i_2, i_3$  – currents in the stator circuits,  $\gamma$  – angle of rotor rotation.

In order to calculate the derivatives  $\frac{\partial \psi_1}{\partial i_1}, \frac{\partial \psi_1}{\partial i_2}, \frac{\partial \psi_1}{\partial i_3}, \frac{\partial \psi_1}{\partial \gamma}$

in a point with coordinates  $i_1, i_2, i_3, \gamma$ :

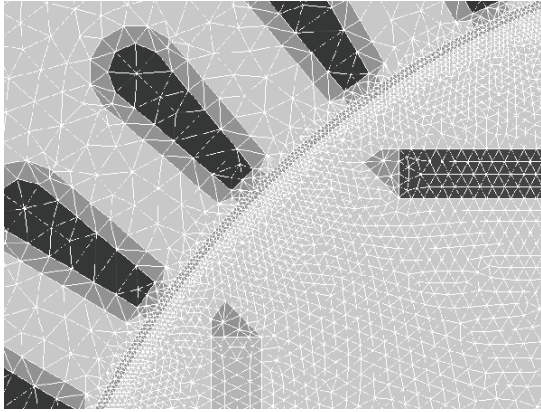


Fig.4. A fragment of finite element mesh

- the Taylor matrix has been determined for the scaled set of nodes and the inverse matrix has been found

$$(2) \quad T = \begin{bmatrix} 1 & 1/2 & -1/2 & -1/2 & -1/2 \\ 1 & -1/2 & 1/2 & -1/2 & -1/2 \\ 1 & -1/2 & -1/2 & 1/2 & -1/2 \\ 1 & -1/2 & -1/2 & -1/2 & 1/2 \\ 1 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/3 \\ 2/3 & -1/3 & -1/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 & -1/3 & 1/3 \\ -1/3 & -1/3 & 2/3 & -1/3 & 1/3 \\ -1/3 & -1/3 & -1/3 & 2/3 & 1/3 \end{bmatrix}$$

- the origin of coordinate system has been moved to the point, for which the derivative is calculated.

- the values of nodes coordinates in the physical coordinate system, referring to the geometry of the scaled set. Thus the  $n$ -th node ( $n=1, \dots, 5$ ) has dimensions

$$(3) \quad \begin{bmatrix} i_{1,n} \\ i_{2,n} \\ i_{3,n} \\ \gamma_n \end{bmatrix} = \begin{bmatrix} m_{i_1} & & & \\ & m_{i_2} & & \\ & & m_{i_3} & \\ & & & m_\gamma \end{bmatrix} \cdot \begin{bmatrix} x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \end{bmatrix} + \begin{bmatrix} i_{1,\xi} \\ i_{2,\xi} \\ i_{3,\xi} \\ \gamma_\xi \end{bmatrix}$$

where:  $diag(m_{i_1}, m_{i_2}, m_{i_3}, m_\gamma)$  – matrix of scaling coefficients,  $[x_{1,n}, x_{2,n}, x_{3,n}, x_{4,n}]^T$  – column of coordinates of  $n$ -th node of the scaled set,  $[i_{1,\xi}, i_{2,\xi}, i_{3,\xi}, \gamma_\xi]^T$  – column with coordinates, which determine the location of the origin of coordinate system referring to the scaled set in the linear space;

- discrete values of the function  $\vec{\psi}_1$  for all the nodes of the set have been found;

- the column of (scaled) derivatives has been determined

$$(4) \quad \vec{c} = T^{-1} \cdot \vec{\psi}_1 = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

- partial derivatives have been expressed in physical units

$$(5) \quad \frac{\partial \psi_1}{\partial i_1} = \frac{c_1}{m_{i_1}} \quad \frac{\partial \psi_1}{\partial i_2} = \frac{c_2}{m_{i_2}} \quad \frac{\partial \psi_1}{\partial i_3} = \frac{c_3}{m_{i_3}} \quad \frac{\partial \psi_1}{\partial \gamma} = \frac{c_4}{m_\gamma}$$

For the scalar functions  $\psi_2, \psi_3$  all calculations have been repeated.

After carrying out all the calculations, the results may be presented as follows:

$$(6) \quad L = \frac{\partial \vec{\psi}}{\partial \vec{i}} = \begin{bmatrix} \frac{\partial \psi_1}{\partial i_1} & \frac{\partial \psi_1}{\partial i_2} & \frac{\partial \psi_1}{\partial i_3} \\ \frac{\partial \psi_2}{\partial i_1} & \frac{\partial \psi_2}{\partial i_2} & \frac{\partial \psi_2}{\partial i_3} \\ \frac{\partial \psi_3}{\partial i_1} & \frac{\partial \psi_3}{\partial i_2} & \frac{\partial \psi_3}{\partial i_3} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

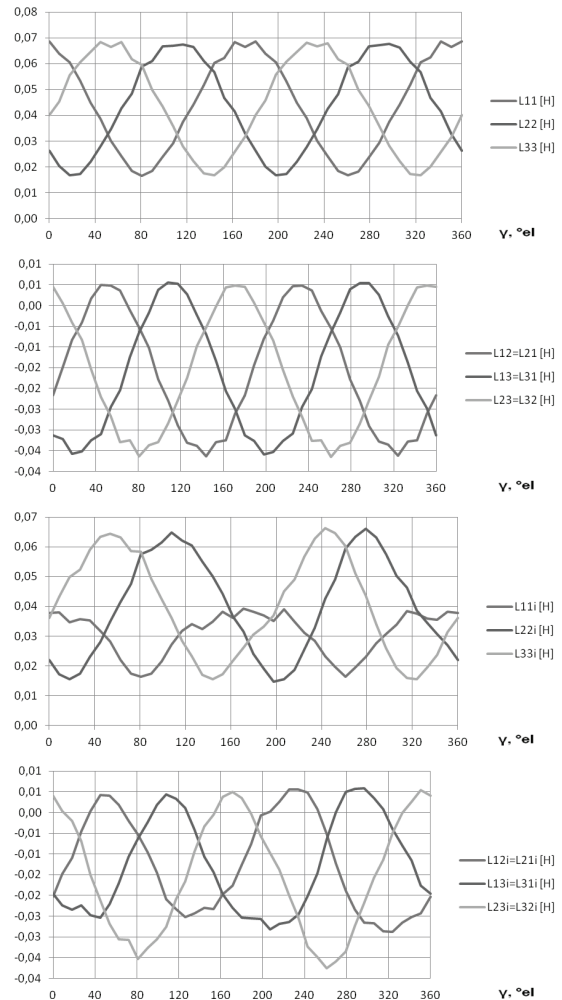


Fig.5. The dependencies  $L$  on rotation angle  $\gamma$ , for the idle state  $i_1=i_2=i_3=0$  and for stator currents  $i_1=10$  [A]  $i_2=i_3=0$  (indexed with „i”)

## Calculations of differential parameters for the machine IPMSg 132 S4 PMSM

In Figure 5 the differential parameters  $L = \frac{\partial \vec{\psi}}{\partial i}$ , and

$K = \frac{\partial \vec{\psi}}{\partial \gamma}$ , for the machine are depicted as functions of rotation angle  $\gamma$ , in the idle state.

The partial derivatives for the stable stator currents  $i_1=10$  [A]  $i_2=i_3=0$ , are marked with index "i", e.g.  $L_{11i}$ .

In Figure 6 the derivative  $L = \frac{\partial \vec{\psi}}{\partial i}$ , in dependence on current  $i_1$ , for the constant rotation angle of the rotor and for the lack of currents in the other phases, is shown.

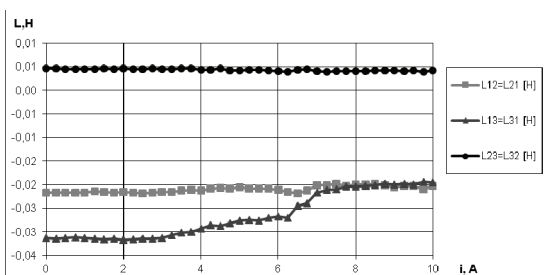
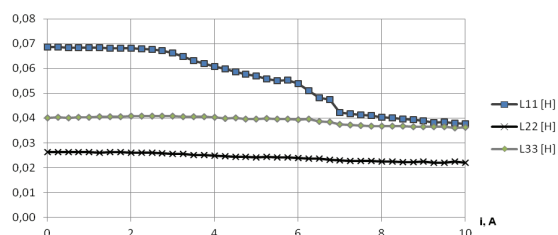


Fig.6. Dependencies of  $L$  on  $i_1$  current

## Conclusions

From the results of carried out simulations the following conclusions may be drawn:

- the inductances of the machine, either self-inductances or the coupled ones, are the functions of the double angle, similarly like in the case of a classical machine with extruded poles. The inductance matrix is symmetrical (Fig. 5);

- for current values larger than 4 [A] the magnetic circuit of the machine enters the saturation state, what leads to lower values of the differential inductance (derivative  $\frac{\partial \psi_1}{\partial i_1}$ ). The value of the derivative  $\frac{\partial \psi_2}{\partial i_1}$  as the function of

current  $i_1$  is practically constant due to weak coupling inductance between phases, which depends linearly on current (Fig. 6).

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